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Provincial Department of Education - NWP

10 E I

Second Term Test - Grade 13 - 2020

Index No : ....

Combined Mathematics I

Three hours only

Instructions:

- *This question paper consists of two parts.*
  Part A (Question 1 - 10) and **Part B** (Question 11 - 17)
- **Part A**
  *Answer **all** questions. Write your answers to each question in the space provided. you may use additional sheets if more space is needed.*
- **Part B**
  Answer five questions only. Write your answers on the sheets provided.
- *At the end of the time allocated, tie the answers of the two parts together so that* **Part A** *is on top of* **part B** *before handing them over to the supervisor.*
- *You are permitted to remove only* **Part B** *of the question paper from the Examination Hall.*

For Examiner's Use only

(10) Combined Mathematics I		
Part	Question No	Marks Awarded
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
	Total	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
	Total	
Paper 1 total		
Percentage		

Paper I	
Paper II	
Total	
Final Marks	

Final Marks

In Numbers	
In Words	

Marking Examiner	
Marks Checked by 1	
2	
Supervised by	

Combined Maths 13 - I (Part A)

Part A

01. Prove that $1+4+7+\ldots+(3n-2)=\dfrac{n(3n-1)}{2}$ for all $n \in N$, using the principle of mathematical induction.

02. Solve the inequality $|3x-1| > x+3$, using graphs. **Hence or otherwise**, find the range of the values of x which satisfies the inequality $|3x+5| > x+5$.

03. Evaluate. $\lim_{x \to \frac{\pi}{4}} \frac{4\sqrt{2}-(\cos x+\sin x)^5}{1-\sin 2x}$

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04 . Write the r^{rh} term, Ur of the series $\frac{1}{2!}, \frac{2}{3!}, \frac{3}{4!}, \frac{4}{5!}$ ....

If $f(r)=\frac{1}{r!}$, write $f(r)-f(r+1)$ in terms of Ur and hence evaluate $\sum_{r=1}^{n} Ur$.

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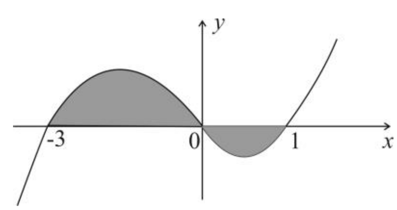
05. Find the equation of the normal drawn to the curve given by the parametric equations $x = (t+1)^2 t, \quad y = \frac{1}{2}t^3 + 3; t \geq -1$ at the point where $t = 2$.

06. The figure shows a rough sketch of the graph $y = x(x-1)(x+3)$. Fid the area of the finite region bounded by the curve and the x - axis.



07. Find the coordinates of the point on the parabola $y^2 = 8x$ which is at a minimum distance from the circle $x^2 + (y+6)^2 = 1$.

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08. The equation of the perpendicular bisector of the sides AB and AC of a triangle ABC are $x - y + 5 = 0$ and $x + 2y = 0$ respectively. If the point A is $(1, -2)$, then find the equation of the line BC.

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09. Find the equation of the circles which touches the axis of x and passes through the points $(1, -2)$ and $(3, -4)$.

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10. Find the solutions of the equation $\sin 6x + \cos 4x + 2 = 0$ within the range $0 \le x \le 2\pi$.

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Combined Maths 13 - I (Part B)

❖ **Answer only 05 questions**.

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11. *a.* *i.* If the difference between the roots of the quadratic equation $x^2 - 3 + k(2x+3) = 0$ is 2, find the value of k.

ii. If c is a real value in the quadratic equation $\frac{1}{x+1} + \frac{1}{x-1} = \frac{1}{c}$, show that the roots of this equation is real and distinct. Here $x \neq \pm 1$ and $c \neq 0$.

b. The polynomial $x^5 - 3x^4 + 2x^3 - 2x^2 + 3x + 1$ is denoted by $f(x)$.

i. Show that neither $(x+1)$ nor $(x-1)$ is a factor of $f(x)$.

ii. By substituting $x = 1$ and $x = -1$ in the identity $f(x) \equiv (x^2 - 1)q(x) + ax + b$, where $q(x)$ is a polynomial and a and b are constants, or otherwise, find the remainder when $f(x)$ is divided by $(x^2 - 1)$.

iii. Show that the remainder when $f(x)$ is divided by $(x^2 + 1)$ is $2x$.

iv. Find all the real roots of the equation $f(x) = 2x$.

12. *a.* If the p^{th} and q^{th} terms of a geometric progression are q and p respectively, then show that its $(p+q)^{th}$ term is $\left(\frac{q^p}{p^q}\right)^{\frac{1}{p-q}}$.

b. Prove that $1 + n^2 + n^4 \equiv (1+n^2)^2 - n^2$.

Write the r^{th} term Ur of the series $\frac{1}{1+1^2+1^4} + \frac{2}{1+2^2+2^4} + \frac{3}{1+3^2+3^4} + \ldots$

Using above identity or otherwise, find a function $f(r)$ so that $\frac{1}{2}\{f(r)-f(r+1)\}=Ur$ and hence show that $\sum_{r=1}^{n} Ur=\frac{n(n+1)}{2(n^2+n+1)}$.

13. *a.* Prove that,

$$\cos\alpha+\cos(\alpha+\beta)+\cos(\alpha+2\beta)+\ldots\quad+\cos[\alpha+(n-1)\beta]=\frac{\cos\left[\alpha+\left(\frac{n-1}{2}\right)\beta\right]\sin\left(\frac{n\beta}{2}\right)}{\sin\left(\frac{\beta}{2}\right)}$$

for all $n\in N$, using the principle of mathematical induction.

b. In how many ways can the all the letters of the word PERMUTATIONS be arranged to form different words. Among those formations find the number of words

i. starts with P and ends with S.

ii. where the vowels are all together

iii. where four letters are in between P and S.

c. A committee of 7 members has to be formed from 9 boys and 4 girls. In how many ways can this be done when the committee consists of

i. exactly 3 girls

ii. at least 3 girls

iii. at most 3 girls.

14. *a.* Let $f(x)=\frac{16(x+1)}{(x-1)^2(3x+1)}$ for $x\neq 1$ and $x\neq -\frac{1}{3}$.

Show that the derivative of $f(x)$, $f'(x)$ is given by $f'(x)=\frac{-32x(3x+5)}{(x-1)^3(3x+1)^2}$.

Write the equations of the asymptotes of $y=f(x)$.

Find the coordinates of the intersection points of the horizontal asymptote and the curve of $y = f(x)$.

Draw a rough sketch of the graph of $y = f(x)$ representing the turning points and asymptotes.

b. *i.* A tent is going to be formed as a right square pyramid. The distance to the each mid-point of each side of the square base from the top vertex is $3\sqrt{6}\, m$. If the area of the square base is A then, show that its volume V " is given by $V = \frac{A}{6}\sqrt{216 - A}$.

ii. Find the value of A such that V is the maximum and hence, find the height of tent and the length of a side of the base of the tent.

iii. If the same kind of cloth is used to make the base and faces of the tent, find the required amount of cloth to make the tent having a maximum space inside the tent.

15. *a.* Find the constants A and B such that $\frac{1}{(1-z)(1-2z)} \equiv \frac{A}{1-z} + \frac{B}{1+2z}$.

Using the substitution $t = \sin x$, show that $\int \frac{\sin x}{\sin 4x}\, dx = \frac{1}{4}\int \frac{dt}{(1-t^2)(1-2t^2)}$.

Hence, show that $\int \frac{\sin x}{\sin 4x}\, dx = P \ln\left|\frac{1+\sin x}{1-\sin x}\right| + Q \ln\left|\frac{1+\sqrt{2}\sin x}{1-\sqrt{2}\sin x}\right| + C$; C is an arbitrary constant and P, Q the constants to be determined.

b. If $f(x)$ is a function which is possible to integrate within the closed range $[a, b]$, prove that $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$.

It is given that $I = \int_a^b \sqrt{\frac{x-a}{b-x}}\, dx$ and $J = \int_a^b \sqrt{\frac{b-x}{x-a}}\, dx$. Prove that $I = J$.

Hence, prove that $I = \frac{\pi}{2}(b-a)$.

c. By using integration by parts, find $\int e^{3x} \sin 4x dx$.

16. *a*. Find the perpendicular distance to the line $ax+by+c=0$ from a point $P(x_1, y_1)$.

In a triangle ABC, $A\equiv(7,11)$ and the equation of the side BC is $3x-4y-2=0$. The ordinate of the mid-point of the side BC is 1 and the area of the triangle ABC is 30 square units. Find the coordinates of B and C.

b. Show that the general equation of the circle which touches the x- axis is $x^2+y^2+2gx+2fy+g^2=0$; g and f are real constants.

A variant circle touching x - axis passes through the point $A(-1,3)$. Show that the path of the other end of the diameter passes through A of the circle is given by $y=\frac{1}{12}(x+1)^2$.

17. *a*. Draw the rough sketches of the graphs of $y=2|\cos 2x|$ and $y=1+\sin x$ on the same diagram within the range $0\le x\le 2\pi$. Hence state the number of solutions of the equation $2|\cos 2x|=1+\sin x$ within the above range.

b. State and prove the **sine rule** for any triangle ABC in the usual notation.

If $\frac{\sin(A-B)}{\sin(A+B)}=\frac{a^2-b^2}{a^2+b^2}$; ($\hat{A}\neq\hat{B}$) in the usual notation, for any triangle ABC, show that the triangle is right-angled.

c. Find the values of x which satisfies the equation $\tan^{-1}x+\tan^{-1}2x=\frac{2\pi}{3}$.

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Provincial Department of Education - NWP

10	E	II

Second Term Test - Grade 13 - 2020

Index No : ....

Combined Mathematics II

Three hours only

Instructions:

* *This question paper consists of two parts.*
  Part A (Question 1 - 10) and **Part B** (Question 11 - 17)
* **Part A**
  Answer all questions. Write your answers to each question in the space provided. you may use additional sheets if more space is needed.
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For Examiner's Use only

(10) Combined Mathematics II		
Part	Question No	Marks Awarded
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
	Total	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
	Total	
Paper 1 total		
Percentage		

Paper I	
Paper II	
Total	
Final Marks	

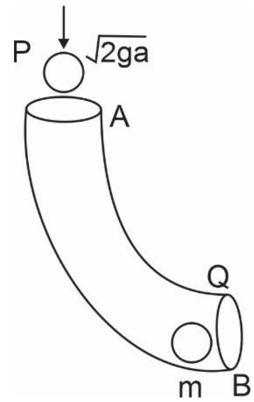
Final Marks

In Numbers	
In Words	

Marking Examiner	
Marks Checked by 1	
2	
Supervised by	

(Part A)

01) A thin smooth tube AB which is bent as an arc of radius a is fixed in a vertical plane so that its open ends A vertical and B horizontal.
A particle Q of mass m is placed on the lowest point B of the tube and another particle P of mass m is projected vertically downwards to the tube with an initial velocity of $\sqrt{2ga}$. Particle P moving along the tube collides and coalesces with the particle Q at B.
Find the velocity of the composite particle when it leaves the tube. (Here g is the gravitationl acceleration.)



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02) The motion of a particle projected under the gravity with a velocity u inclined at angle θ to the horizontal is perpendicular to the initial direction of projection after the time t. Show that $t = \frac{u \sin\theta}{g}$. (Here g is the gravitationl acceleration.)

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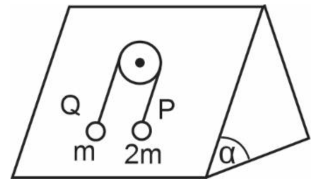
03) A smooth light pulley is fixed to a smooth plane inclined at an acute angle α to the horizontal as shown in the figure. Two particles of mass $2m$ and m are connected to the ends of a light inextensible string passing over a pulley and released gently. Show that the acceleration of each particle is $\frac{g\sin\alpha}{3}$ and the tension of the string is $\frac{4}{3}mg\sin\alpha$.



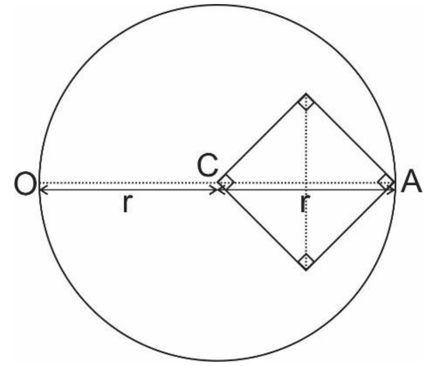
04) A car of mass $M\ kg$ moves with a constant velocity of $v\ ms^{-1}$ along a level road against a resistance of $R\ N$ with a constant power of $H\ kw$. Show that the resistance on the car $\mathrm{R} = \frac{\mathrm{H\ x\ 10^3}}{\mathrm{V}}\ N$.

Then, the car moves down along a road inclined at an angle α to the horizontal with a velocity of $\frac{\mathrm{v}}{2}\mathrm{ms^{-1}}$ and same power $H\ kw$ against the same resistance $R\ N$. Show that the acceleration of the car is $\left\{\frac{\mathrm{H\ x\ 10^3}}{\mathrm{Mv}} - \mathrm{g\sin\alpha}\right\}\mathrm{ms^{-2}}$. (Here g is the gravitationl acceleration.)

05) In the usual notation the position vectors of the points A, B and C are $\left(2\underline{i} + 4\underline{j}\right)$, $\left(4\underline{i} + 4\underline{j}\right)$ and $(\lambda\underline{i} + \mu\underline{j})$ respectively. Here λ and μ are real constants. When O is the origin, $OABC$ respresents a trapezium. Here OA and CB are parallel and $CB = \frac{1}{2}\,OA$. Find the values of λ and μ. If $A\hat{O}C = \theta$, show that $\cos\theta = \frac{7}{\sqrt{65}}$.

06) A particle P of mass m is freely suspended by means of a light inextensible string of length l fixed to a point O. A velocity of $\sqrt{3l\,g}$ is given to the particle in the direction perpendicular to OP (horizontal). When OP makes an acule angle $\cos^{-1}\left(\frac{3}{5}\right)$ with the downward vertical, find the velocity of the particle P . Show that the tension of string is $\frac{14\,mg}{5}$. (Here g is the gravitationl acceleration.)

07) As shown in the figure a square shape portion which is symmetrical about OA and the diagonal equals to the radius is removed from a circular lamina of radius r. Show that the center of gravity of the remainig part lies on the axis of symmetry OA at a distance $\frac{(4\pi-3)}{2\,(2\pi-1)}\,r$ from O.



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08) One end of an elastic string of natural length l and modulus of elasticity $2\,mg$ is tied to a fixed point O and a particle P of mass m is attached to the other end. Initially the particle P is placed at O and projected vertically down with a velocity of $\sqrt{2\,lg}$. Using the law of conservation of energy, find the maximum distance that can be moved by the particle from O.

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09) Given that $P(A \cup B) = \frac{9}{10}$, $P(A') = \frac{1}{3}$, $P(B) = \frac{1}{2}$. Find $(A \cap B)$ and $P\,(A' \cap B)$. Here A' is the complement of event A.

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10) A box contains n number of red identical balls and 4 blue identical balls. Two balls are drawn without any replacement. If the probability that both balls drawn out are being red is $\frac{1}{3}$, find the value of n.

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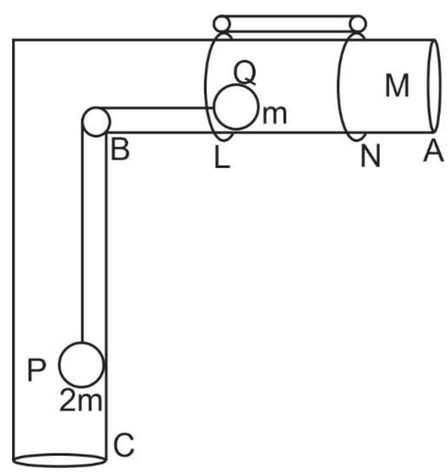
11) (a) A space shuttle starting form rest moves vertically upwards with a constant acceleration $\frac{g}{2}$.
After time T a part of the shuttle is released from the shuttle and it moves vertically downwards under the gravity so as to fall on the ground. When the released part is at its maximum height, engine of the space shuttle stops suddenly and falls vertically down under the gravity.
Draw velocity - time curves for the motions of the space shuttle and the released part of the shuttle and the remaining part of the shuttle in a same diagram until they reach to the ground from the initial moment.
Hence show that the velocity of the shuttle when the part is released is $\frac{gT}{2}$ and the maximum height of the released part is $\frac{3gT^2}{8}$.
Further show that the velocity of the shuttle is $\frac{3gT}{4}$ when the engine stops. And also show that the maximum height reached by the shuttle is $\frac{27gT^2}{32}$.
Show that the velocities of the released part and the shuttle from the initial moment when they fall on the ground are $\frac{\sqrt{3}}{2}\, gT$ and $\frac{3\sqrt{3}}{4}\, gT$ respectively.

(b) A destroyer D sails due east with uniform speed $u\ km\ h^{-1}$. Another ship S sails in the direction α north of east at a constant speed $v\ km\ h^{-1}$ $(v\cos\alpha > u)$. At a certain moment, the ships S is at a distance $a\ km$ south of D.
Draw the velocity triangles for the relative motions of S and D. Also draw the locus of the ship relative to D. Show that the shortest distance between the ship S and D is , $\frac{a\,(v\cos\alpha - u)}{\sqrt{v^2 + u^2 - 2uv\cos\alpha}}\ km$

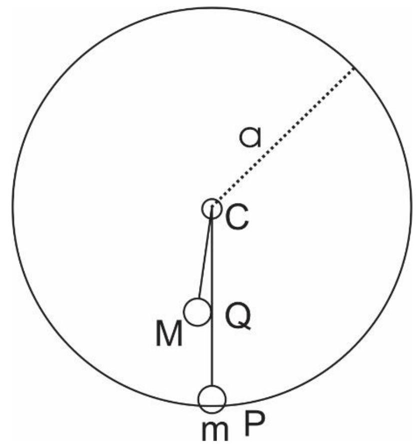
Further show that the time taken to reach this closest moment from the moment where S is at a distance $a\ km$ south of D is $\frac{av\sin\alpha}{v^2 + u^2 - 2uv\cos\alpha}$ hours.

12) (a) A thin smooth tube of mass M is bent at B so as to from a right angle. Part AB is horizontal and it is free to move through two smooth rings L and N. BC is vertical. Two particles P and Q of masses $2m$ and m are connected by a light inextensible string passing over a smooth pulley fixed at B. Initially particle Q is placed at a point in tube AB and particle P is hung vertically inside BC. Then the system is released gently with the string taut.



Write down the equations of motion for particle P along BC, for particle Q along AB and for the system along BA.
Show that the acceleration of the tube is $\frac{2mg}{3M+8m}$
Also show that the acceleration of each particle relative to the tube is, $\left(\frac{M+3m}{3M+8m}\right) 2g$ and the acceleration of the particle P relative to earth is, $\left(\frac{2g}{3M+8m}\right)\sqrt{M^2 + 10\,m^2 + 6Mm}$.

b) A smooth bead P of mass m is thread to a circular wire of radius a fixed in a vertical plane. The bead is free to move along the wire. One end of a light inextensible string passing through a smooth ring at the center C, is connected to the bead P and the other end of the string is connected to a particle Q of mass M. Initially the bead P is placed at the lowest point and projected horizontally with a velocity of $\sqrt{kga}$ $(k > 1)$ so that the bead is in a circular motion along the wire.



Show that the speed v of the bead P, when the string PC makes an acute angle θ with the downward vertical is given by $v^2 = kga - 2ga + 2ga\cos\theta$ and the reaction R on the bead P from the wire is given by $R = mg\left(k - 2 + 3\cos\theta - \frac{M}{m}\right)$.

Taking $k = 6$, if $m < M < 7m$ show that the reaction on the bead by the wire is disappeared at a certain moment.

13) One end of a light elastic string of natural length l is connected to a particle P of mass m and the other end is connected to a fixed point at O.
When the particle is suspended in equilibrium the extension of the string is l. Show that the modulus of elasticity is mg.
Then the particle is placed at O and released gently. Show that the velocity of the particle is $\sqrt{2gl}$ when it falls a distance l vertically downwards. When the length of the string $x(x > l)$ is from O, show that the equation of motion of the particle is given by $-\frac{9}{l}(x - 2l) = \ddot{x}$ with the usual notation.

Also assuming that the velocity of the particle is given by $\dot{x}^2 = w^2(A^2 - x^2)$; $A > 0$ $(Constant)$ find the value of A.

Show that the time taken by the particle to reach the point O again is, $2\sqrt{\frac{l}{g}}\left\{\sqrt{2} + \pi - \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)\right\}$

14) (a) $\underline{a}$ and $\underline{b}$ are, two non -zero , non-parallel vectors. Prove that $\alpha = 0$ and $\beta = 0$ is the necessary and sufficient condition for $\alpha\underline{a} + \beta\underline{b} = 0$. Here α and β are scalars.
In the parallelogram $OACB$, $\overrightarrow{OA} = \underline{a}$ and $\overrightarrow{OB} = \underline{b}$. D is a point on OA such that $OD:DA = 1:2$. BD and AC intersect at X. λ and μ are two scalars such that $OX = \lambda OC$ and $BX = \mu BD$. Find the values of λ and μ and show that $BX:XD = 3:1$ and $OX:XC = 1:3$.

(b) In the rectangle $ABCD$. $AB = a$, $AD = 2a$. M is the mid point of AD. Forces P, $2P$, $4P$, $6P$, $3\sqrt{2}P$ and $\sqrt{5}P$ act along the sides CB, DA, BA, CD, MB and DB in the direction of the order of the letters respectively. If the system is reduced to a couple and a single force acting at A, find the magnitude and the direction of the single force. Show that the magnitude of the couple is $6Pa$ and find the sense of it.

15) (a) A rhombus $ABCD$ is formed of four uniform rods AB, BC, CD, and AD each of length $2a$ and weight w, smoothly jointed at their ends. The rhombus is suspended from A and a light inextensible string is connected to the points $L$and M on the rods AB and BC respectively. Here $AL = CM = \frac{a}{2}$.

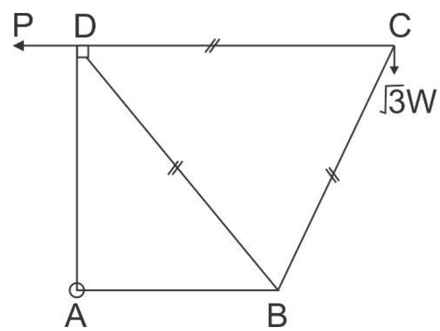
The string LM and AC are vertical and the system is in equilibrium in a vertical plane with the vertex A is above C. Given that $B\hat{A}D = B\hat{C}D = 60^0$.

(i) Find the reaction at C and show that its inclination to the horizontal is $tan^{-1}(2\sqrt{3})$.

(ii) Show that the tension of the string LM is $\frac{8w}{3}$.

(iii) Find the magnitude and the direction of the reaction at B.

(b) Five light rods AB, BC, CD, BD and AD are smoothly jointed at their ends to from the framework shown in the figure. Given that $BC = BD = CD = 2a.$ The framework is smoothly hinged at A and weight $\sqrt{3}w$ is hung at C. A horizontal force P applied at D, keeps the frame work in a vertical plane such that AB and DC horizontal and AD vertical.

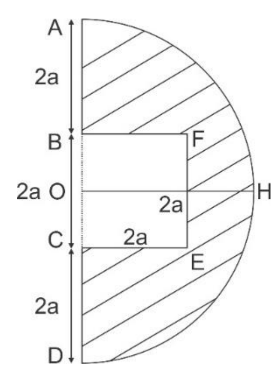


(i) Find the value of P.

(ii) Find the reaction at A and its inclination to the horizontal.

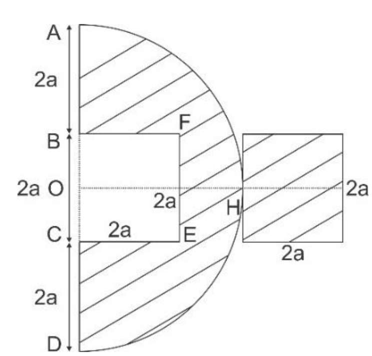
(iii) Draw stress diagrams for each joint on the same figure, using Bow's notation. Hence find the stresses of rods indicating whether they are tensions or thrusts.

16) Show that the center of mass of a semi circular lamina of radius a and center O is at a distance $\frac{4a}{3\pi}$ from O.



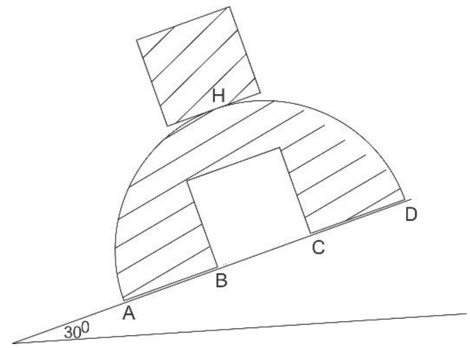
A square $BFEC$ of side $2a$ is removed symmetrically about OH from a uniform semi circular lamina AHD of radius $3a$. Show that the center of mass of the remaining part is at a distance $\frac{28\,a}{9\pi - 8}$ from O on the axis of symmetry.

Then the removed square is attached at H as shown in the figure.



Show that the center of mass of the this object is on the axis of symmetry at a distance $\frac{2a}{3\pi}$ from O.

If the new lamina rests on a rough plane inclined at an angle 30^0 to the horizontal and AB and CD are on the line of greatest slope, show that $\mu \geq \frac{1}{\sqrt{3}}$.
Here μ is the coefficient of friction between the inclined plane and the lamina.



17) (a) Let A and B be any two events of the sample space Ω . Define each of following events.

(i) A and B are mutually exclusive events.
(ii) A and B are exhaustive events.

(b) Given that A, B and C are three mutually exclusive and exhaustive events of the sample space Ω.
If $P(A) = 2a^2$, $P(B) = 2a$ and $P(C) = 8a - 1$, find the value of a.

(c) Let A and B are any two events of the sample space Ω. Show that,

(i) $P(A) = P(A \cap B) + P\,(A \cap B')$

(ii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
Here, B' is the complement of event B.
If $P(A') = \frac{1}{4}$, $P(B) = \frac{1}{2}$, $P(A \cap B') = \frac{2}{5}$,
Find $P(A \cap B)$, $P\,(A \cup B)$, $P\,(A' \cap B)$, $P\,(A' \cup B)$ and $P\,(\,A' \cup B')$

(d) A biased die which has $\frac{3}{5}$ of probability of getting head is tossed. If the head is obtained, then 2 balls are taken out randomly from a box A containing 3 red balls and 2 blue balls which are identical without replacement.
If the tail is obtained, then two balls are taken out randomly from a box B containing 2 red balls and one blue ball without replacement. Find the probabilities of,
(i) Obtaining 2 red balls.

(ii) Obtaining only one red ball when tail is obtained.