Provincial Department of Education - NWP

10 E I

Second Term Test - Grade 12 - 2020

Index No : ....

Combined Mathematics I

Three hours only

Instructions:

- *This question paper consists of two parts.*
  Part A (Question 1 - 10) and **Part B** (Question 11 - 17)
- **Part A**
  Answer all questions. Write your answers to each question in the space provided. you may use additional sheets if more space is needed.
- **Part B**
  Answer five questions only. Write your answers on the sheets provided.
- *At the end of the time allocated, tie the answers of the two parts together so that* **Part A** *is on top of* **part B** *before handing them over to the supervisor.*
- *You are permitted to remove only* **Part B** *of the question paper from the Examination Hall.*

For Examiner's Use only

(10) Combined Mathematics I		
Part	Question No	Marks Awarded
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
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	10	
	Total	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
	Total	
Paper 1 total		
Percentage		

Paper I	
Paper II	
Total	
Final Marks	

Final Marks

In Numbers	
In Words	

Marking Examiner		
Marks Checked by	1	
	2	
Supervised by		

Combined Mathematics 12 - I (Part - A)

Answer all the questions in Part A and only for five questions in Part B.

01) Show that the roots of the equation $(1-k)x^2 + x + k = 0$ are real and negative if, $0 < k < 1$.

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02) Find all real values of x satisfying the inequality $3 - |x+1| < x^2$

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03) Solve , $3^{2x+1} - 3^{x+4} + 3^3 = 3^x$

04) Show that , $\lim_{x \to 0} \frac{\sin(\pi \cos^2 x)}{x^2} = \pi$

05) Solve the simultaneous equations of , $log_3\ x + log_3\ y = 3$ and $log_y\ x = 2$

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06) Let there is a common linear factor for both polynomials $x^3 + ax^2 + b$ and $ax^3 + bx^2 + x - a$. Show that the above common linear factor is a factor of the polynomial $(b - a^2)x^2 + x - a(1 + b)$ too.

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07) Let, $f(x) = \frac{1}{\sqrt{x+2}}$; $x \geq -2$ and $g(x) = 2x + 1$

(i) Find the domain of the function $\frac{f}{g}$

(ii) Obtain the value of $\left(\frac{f}{g}\right)(0)$

08) Show that $a > \frac{11}{9}$, if both roots of the equation $x^2 - 6ax + 2 - 2a + 9a^2 = 0$ are greater than three.

09) If, $\sec\theta + \tan\theta = P,$ deduce that $\tan\theta = \frac{P^2-1}{2P}$. Here P is a non-zero real constant.

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10) Solve, $sin^{-1}\left(\frac{5}{x}\right) + sin^{-1}\left(\frac{12}{x}\right) = \frac{\pi}{2}$.

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Combined Mathematics 12 - I (Part - B)

Answer only five questions.

11) a) Let $f(x) = x^4 + Px^2 + r$. Find p and r if $f(1) = -9$ and $f(0) = -8$. Find the values of the real constants a, b and c, if $f(x)$ can be expressed in the form $(ax^2 + b)^2 + c$. Here $a > 0$. Hence find the real roots of $f(x) = 0$.

b) Find the range of values of P, for the expression $(p-1)x^2 - 4x + p - 1$ to be positive for all real values of x.

c) If, α, β are the roots of $ax^2 + bx + c = 0$, find the roots of $cx^2 - 2bx + 4a = 0$ in terms of α, β.

d) Separate $\frac{x^2-1}{x^2\,(2x+1)}$ in to partial fractions.

12) a) Sketch the graphs of $y = 2|x+1| - 3$ and $y = x + 2\,|x-1|$ in the same diagram. Hence solve the equation $x + 2\,|x-1| = 2\,|x+1| - 3$.
Find the set of value of x, satisfying the inequality $x + 2\,|x-1| > 2\,|x+1| - 3$.

b) Let $a = \log_{2n} n$, $b = \log_{3n} 2n$ and $C = \log_{4n} 3n$, for a positive real number n.
Prove that $1 + abc = 2bc$.

c) If $a^x = b^y = c^z = d^w$, obtain that $x\left(\frac{1}{y} + \frac{1}{z} + \frac{1}{w}\right) = \log_a \mathrm{bcd}$.

13) a) $f : \mathcal{R} \to \mathcal{R}$ is defined as follows,

$$f(x) = \begin{cases} -x^4 + 4\,; & x < 1 \\ -2x\,; & x \geq 1 \end{cases}$$

(i) Draw the rough sketch of f(x).

(ii) Evaluate $\lim\limits_{x \to 1^-} f(x)$ and $\lim\limits_{x \to 1^+} f(x)$

(iii) Is the function continuous at x = 1? Justify the answer.

(iv) Find $\lim\limits_{x \to 1} f(x)$, if exists.

b) Prove that $\lim\limits_{\theta \to o} \frac{\sin\theta}{\theta} = 1$

Evaluate the following limits.

(i) $\lim\limits_{x \to 3} \frac{\sqrt{2x-1} - \sqrt{5}}{\sin(x-3)}$

(ii) $\lim\limits_{x \to 0} \frac{\left(\sqrt{4+x^2} - 2\right)(1-\cos 2x)}{x^4}$

14) a) Write the conditions which should be satisfied for the roots of the equation $ax^2 + bx + c = 0$ to be real and positive. Here $a, b, c \in \mathcal{R}$ and $a \neq 0$.
When those conditions are satisfied show also that the roots of the equation $a^2x^2 + a(3b - 2c)x + (2b - c)(b - c) + ac = 0$ are real and positive. If the roots of the second quadratic equation are α and β, using a suitable linear transformation, obtain the quadratic equation with roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

b) (i) Find k, if the distance between the two points $(k, 2)$ and $(3,4)$ is 8 units.
(ii) Find the coordinates of the remaining vertex, if $(1,0)$, $(6, 1)$ and $(5, 6)$ are the vertices of a square separately.
(iii) In the triangle ABC, let $A = (1,3)$ and $B = (5, 3)$.
Find the coordinates of C, if the coordinates of the centroid of the triangle ABC is $\left(\frac{10}{3}, 4\right)$.

15) a) State and prove the factor theorem.
If the polynomial $f(x) = x^4 + ax^3 + bx + c$ is divisible by $(x - 1)(x + 1)(x - 2)$, find a, b, and c and find the remaining factors.
Also find the solutions of $2f(x + 1) = x^2 + x - 2$.

b) Separate the rational function $\frac{x^2}{(x-a)(x-b)}$ in to partial fractions in terms of a and b. Hence obtain the partial fractions of $\frac{4x^2}{4x^2-1}$.

c) Evaluate $\lim_{x \to \infty} \left(\sqrt{x^2 + ax + a^2} - \sqrt{x^2 + a^2}\right)$

16) a) Using the expression for $\sin(A + B)$, show that $\sin\left(\frac{5\pi}{12}\right) = \frac{\sqrt{6} + \sqrt{2}}{4}$.
Hence deduce that, $\cos\left(\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2}$.

b) If $\alpha + \beta - \gamma = \pi$, Prove that $sin^2 \alpha + sin^2 \beta - sin^2 \gamma = 2 \sin\alpha \sin\beta \cos\gamma$.

c) Find the general solutions of the equation $2 cos^2 x + \sqrt{3} \sin x + 1 = 0$.

d) Obtain that $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \frac{\pi}{4}$

17) State the sine rule for a triangle ABC.

a) Prove that, $\frac{a^2 + b^2}{a^2 + c^2} = \frac{1 + \cos(A - B)\cos C}{1 + \cos(A - C)\cos B}$

b) In the triangle ABC, the mid point of the side BC is D. According to the standard notation, show that $AD = \frac{\sqrt{2b^2 + 2c^2 - a^2}}{2}$,

If $B\hat{A}D = \beta$, show that $\sin\beta = \frac{a \sin B}{\sqrt{2b^2 + 2c^2 - a^2}}$

If $A\hat{D}C = \theta$, show that $\sin\theta = \frac{2b \sin C}{\sqrt{2b^2 + 2c^2 - a^2}}$

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Provincial Department of Education - NWP

10 | E | II

Second Term Test - Grade 12 - 2020

Index No : ....

Combined Mathematics II

Three hours only

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(Part - A)

1) The resultant of two forces of magnitude P and $2P$ is $\sqrt{3}\,P$. Find the angle between the two forces. Also find the angle between the resultant force and the first force.

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2) A particle of mass 5kg is on a fixed smooth plane of inclination 30^0 to the horizontal. Find the magnitude of the force which should be applied parallel to the inclined plane and find the reaction between the inclined plane and the particle.

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3) The position vectors of the points A, B and C relative to a fixed point O are $\underline{a}$, $\underline{b}$ and $\underline{c}$ respectively. If $3\underline{a} + 5\underline{b} = 8\underline{c}$, show that the points A, B and C are collinear. Find the ratio $AC : CB$.

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4) A particle of weight 6N is suspended by two light strings and it is in equilibrium. If the tensions of the strings are 3N and $3\sqrt{3}\,N$, find the angles which the two strings made with the vertical.

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5) A particle P is projected vertically upwards with a velocity $2u$ from a point O in a plane. At the same instant from the point O, a particle Q is projected vertically downwards with velocity u. Both particles move under gravity. Draw the velocity time graphs for the motion of both particles P and Q in the same diagram and show that the velocity of the particle Q is $3u$. When the particle P reaches its maximum height.

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6) A particle is projected vertically upwards from a point O, on the ground floor. If the time taken for the particle to travel upwards and downwards passing a point, which is at a vertical height h, is t_1 and t_2 respectively. Show that $h = \frac{1}{2}gt_1t_2$.

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7) $\underline{a}$ and $\underline{b}$ are two unit vectors and the angle between the two vectors is θ. Show that $\sin\frac{\theta}{2} = \frac{1}{2}\left|\underline{a} - \underline{b}\right|$.

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8) The two ends of a non-uniform rod of mass $4kg$ and length $24m$ are A and B. The rod is kept in equilibrium horizontally on two pegs which lie at a distance $8m$. The distance to the nearest peg from A is $4m$. When a weight of $6N$ is hung at B, the reactions on the rod by the pegs are equal. Find the reaction on the rods. Also find the distance to the center of gravity of the rod from A.

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9) The forces of $-4\underline{i} + 3\underline{j}$, $6\underline{i} - 7\underline{j}$ and $-2\underline{i} + 4\underline{j}$ act at the points $2\underline{i} - \underline{j}$, $-3\underline{i} + 4\underline{j}$ and $4\underline{j}$ respectively. Show that the system of forces reduces to a couple and find the moment of the couple.

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10) Two motor cars move with uniform velocities v and u towards each other in a straight line path. When the distance between the motor cars is d , both cars apply brakes at the same time knowing that the two card get collide together. Two cars obtained retardations of f_1 and f_2 respectively, as a result of applying brakes and just prevented the collision. Draw the velocity time graph for the motion. Show that $d = \frac{u^2}{2f_2} + \frac{v^2}{2f_1}$.

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Combined Mathematics 12 - II (Part B)

Answer five questions only.

11) a) A train passes a station A at $40\ kmh^{-1}$ and maintain this speed for $7km$ and is then uniformly retarded, stopping at B, which is 8.5 km from A. A second train starts from A at the instant the first train passes A and being accelerated for part of the journey and uniformly retarded for the rest, stops at B at the same time as the first train.

(i) Find the total time for the journey.

(ii) What is the greatest speed obtained by the second train?

b) A motor car X starts from rest at $t = 0$, moves with uniform acceleration. At $t = T$, another motor car Y starts at the same point with velocity u and moves with an uniform retardation of $2f$. If the motor cars meet each other,
show that , $2fT(u + fT) = u^2$.

12) a) Using the law of addition of vectors, show that the sum and difference of two vectors $\underline{a}$ and $\underline{b}$ gives by the diagonals of the parallelogram. If the sum of two unit vectors is an unit vector, show that the magnitude of their difference is $\sqrt{3}$.

c) The position vectors of the points A, B and C relative to the origin O are $4\underline{i} + 2\underline{j}$, $\underline{i} + \underline{j}$ and $(k+1)\underline{i} + 6\underline{j}$ respectively. Find the value of k $(k < 0)$ such that $A\hat{B}C = 45^0$.

c) In the parallelogram $OACB$, D and E are two points on BC and AC such that $BD{:}DC = 1{:}2$ and $AE : EC = 2{:}1$. F is the intersection point of OD and BE . The position vectors of A and B reative to O are $\underline{a}$ and $\underline{b}$ respectively. Show that $\overrightarrow{OF} = \frac{3}{10}\left(\underline{a} + 3\underline{b}\right)$. If OB and CF intersect at P, find the ratio $OP{:}PB$.

13) a) OAB is an equilateral triangle of length of a side $2a$ C is the mid point of OA. The forces 4P,P and P act along the sides OB, BA and AO inthe directionofthe order of the letters respectively. Taking OA and OY (Parallel to CB) as X and Y axes respectively, express each force in the form $x\underline{i} + y\underline{j}$. Here $\underline{i}$ and $\underline{j}$ are unit vectors along OX, OY respectively. Show that the system of forces canbe reduced toa single force $3P$.
Also show that the above single force can be reduced to a couple of moment $2\sqrt{3}\,ap$ and to a like parallel force acting along the centre of the triangle.

b) The points O, A, B and C are (0,0) (2,0) (2,1) , and (0,1) respectively. The forces P, Q, R at along the sides OA, AB, BC respectively. If the resultant force of this system of forces lies on x + 2y = 7 , find

i) The resultant force interms of P.

ii) The moment of the couple such that the resultant lies on the line $x + 2y = 9$.

14) a) The position vectors of the points A and B relative to a fixed point O are $\underline{a}$ and $\underline{b}$ respectively. The points C and D lie on the OB and OA such that $OC:CB = 5:2$ and $OD:DA = 3:2$. The lines AC and BD interest at E . Show that $\overrightarrow{OE} = \underline{b} + \lambda\left[\frac{3}{5}\,\underline{a} - \underline{b}\right]$. Here λ is a constant. Obtain a similar expression for $\overrightarrow{OE}$. Here find the position vector of the point E in terms of $\underline{a}$ and $\underline{b}$.

b) The position vectors of the points A, B, C relative to a point O are $\underline{a}$, $\underline{b}$ and $\underline{c}$ respectively. The point P lies on the line BC and it is given that $\overrightarrow{PC} = \frac{1}{10}\,\overrightarrow{BC}$.

(i) Find $\overrightarrow{OP}$ in terms of $\underline{b}$ and $\underline{c}$.

(ii) If it is given that AP and BC are perpendicular to each other show that,

$$(9\underline{c} + \underline{b}).(\underline{c} - \underline{b}) = 10\underline{a}\,.(\underline{c} - \underline{b})$$

c) Hence or otherwise prove that $(3\underline{c} - \underline{b})\,.(3\,\underline{c} + \underline{b}) = 0$, if OA, OB and OC are perpendicular to each other.

15) a) The mid points of the sides AB , BC , CD and DA of the rectangle are respectively P, Q, R and S respectively. Here $AB = 6a$ and $BC = 2\sqrt{3}\,a$. Six forces of magnitudes $15N$, $\lambda\,N$, $5N$, $10\,N$, $\mu\,N$ and $30\sqrt{3}$ N act along the sides PQ, QR, RS, SP, AD and CD respectively in the diection incicated by the order of the letters. Show that,

(i) This system of forces cannot be in equilibrium.

(ii) If this system of forces reduces to a couple, then $\lambda = -40$ and $\mu = 20$

(iii) $\lambda = -40$ and $\mu = 30$, if the system reduces to a force of $10\,N$, acting along the direction AD.

b) A string of length $2m$ is attached to two pints A and B $1m$ apart inthe same level. A smooth ring of weight $10N$ is suspended by the string and it is kept in equilibrium vertically below B, by applying a horizontal force P on the ring. Find the tension in the string and the magnitude of the force P.

16) a) In the rectangle $ABCD$, $AB = 4cm$ and $BC = 3m$. The forces of magnitudes 8,7,3,2,8,7 Newtons act along the sides $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CD}$, $\overrightarrow{DA}$, $\overrightarrow{AC}$ and $\overrightarrow{DB}$ respectively. Find the horizontal and vertical coponents of the resultant R. Hence find the distance from A, to the point which the line of action of the resultant cut AB.
Now a couple of forces of $9\,Nm$ is added to the system in the sense $ABCD$. Show that the line of action of the new resultant cut the side AB at a distance $2m$ from A.

b) State the lami's theorem for the equilibrium of three coplanar forces acting at a point. A string $ABCD$ attached to the two fixed points A and D on the same horizontal level supports two weights W_1 and W_2 at B and C respectively. In the equilibrium position, B is above C and the parts AB, BC and CD of the string inclined at acute angles α, β, γ to the verticle respecively.

Show that $\frac{W_1}{W_2} = \frac{\sin\gamma \ \ \sin(\beta - \alpha)}{\sin\alpha \ \ \sin(\beta + \gamma)}$.

17) a) A particle starts its motion with initial velocity u and moves with uniform acceleration a for a time t and obtains a final velocity v at a displacement of s. Using the velocity time graph for the motion of the particle, derive the equations of motion $v = u + at$, $S = \left(\frac{u+v}{2}\right)t$, $s = ut + \frac{1}{2}at^2$ and $v^2 = u^2 + 2as$.

b) A particle is projected vertically upwards with a velocity $u\ ms^{-1}$ and after t seconds another particle is projected vertically upwards from the same point with the same velocity. Prove that,

(i) they meet after a time $\left(\frac{t}{2} + \frac{u}{g}\right)$ from the first projection.

(ii) Particles meet at a height $\frac{4u^2 - g^2t^2}{8g}$.

c) From a tap drops of water fall within equal time intervals. When one drop of water falls, earlier drop has travelled a distance of $\frac{1}{4}\ m$ when the distance between the two drops has increased to $\frac{3}{4}\ m$, find the distance travelled downwards by the first drop. (g = 10 ms^{-2})