Confidential

NEW & OLD

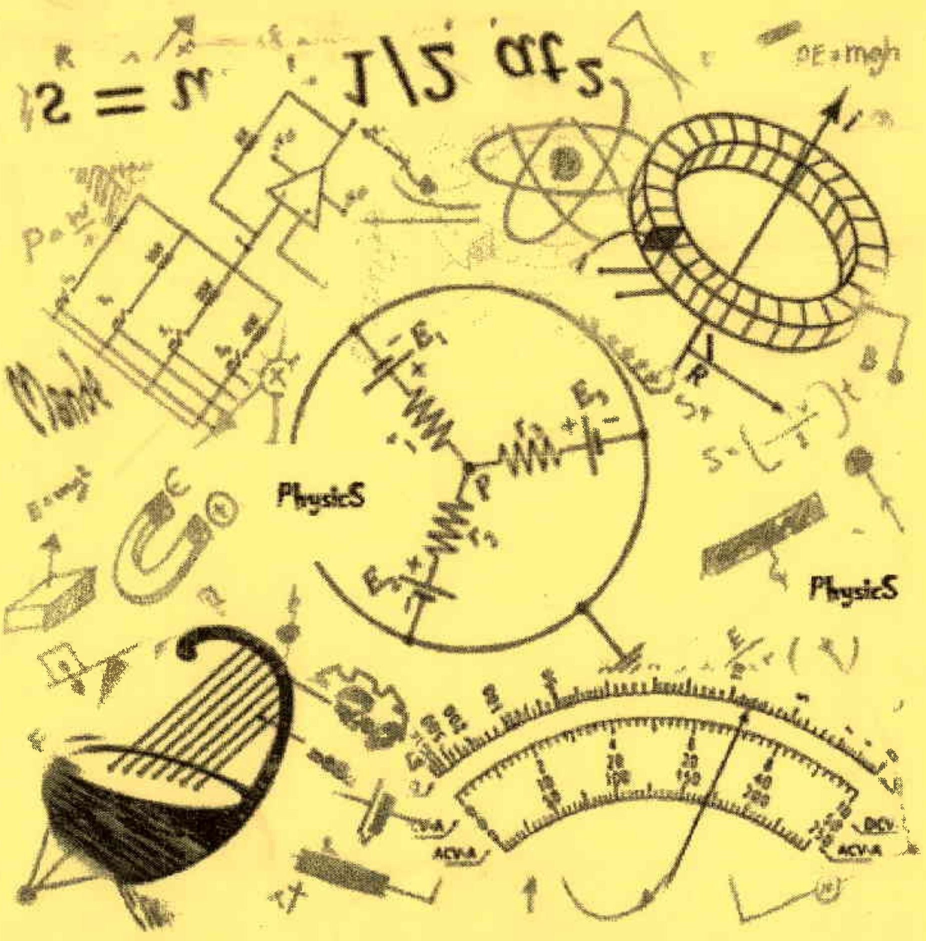
Department of Examinations - Sri Lanka

G.C.E. (A/L) Examination - 2019

01 - Physics

New & Old Syllabus

Marking Scheme



This document has been prepared for the use of Marking Examiners. Some changes would be made according to the views presented at the Chief Examiners' meeting.

Amendments to be included

ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව
இலங்கைப் பரீட்சைத் திணைக்களம்
අ.පො.ස. (උ.පෙළ) විභාගය/ க.பொ.த. (உயர் தர)ப் பரீட்சை - 2019

නව සහ පැරණි නිර්දේශ/ புதிய மற்றும் பழைய பாடத்திட்டம்

විෂයය අංකය பாட இலக்கம்	01	විෂයය பாடம்	භෞතික විද්‍යාව

ලකුණු දීමේ පටිපාටිය/புள்ளி வழங்கும் திட்டம்
I පත්‍රය/பத்திரம் I

ප්‍රශ්න අංකය வினா இல.	පිළිතුරු අංකය விடை இல.	ප්‍රශ්න අංකය வினா இல.	පිළිතුරු අංකය விடை இல.	ප්‍රශ්න අංකය வினா இல.	පිළිතුරු අංකය விடை இல.	ප්‍රශ්න අංකය வினா இல.	පිළිතුරු අංකය விடை இல.	ප්‍රශ්න අංකය வினா இல.	පිළිතුරු අංකය விடை இல.
01.	2	11.	4	21.	1	31.	4	41.	2
02.	4	12.	4	22.	2	32.	2	42.	2
03.	5	13.	3	23.	2	33.	2	43.	3
04.	5	14.	5	24.	5	34.	2	44.	2
05.	2	15.	2	25.	4	35.	4	45.	4
06.	3	16.	4	26.	3	36.	4	46.	4
07.	5	17.	1	27.	4	37.	5	47.	2
08.	4	18.	3	28.	5	38.	1	48.	4
09.	3	19.	5	29.	2	39.	5	49.	4
10.	1	20.	4	30.	3	40.	2	50.	3

✪ විශේෂ උපදෙස්/ விசேட அறிவுறுத்தல் :

එක් පිළිතුරකට/ ஒரு சரியான விடைக்கு 01 ලකුණු බැගින්/புள்ளி வீதம்

මුළු ලකුණු/மொத்தப் புள்ளிகள் $1 \times 50 = 50$

1. An experimental setup used in a school laboratory to determine the surface tension of a liquid is shown in figure (1).

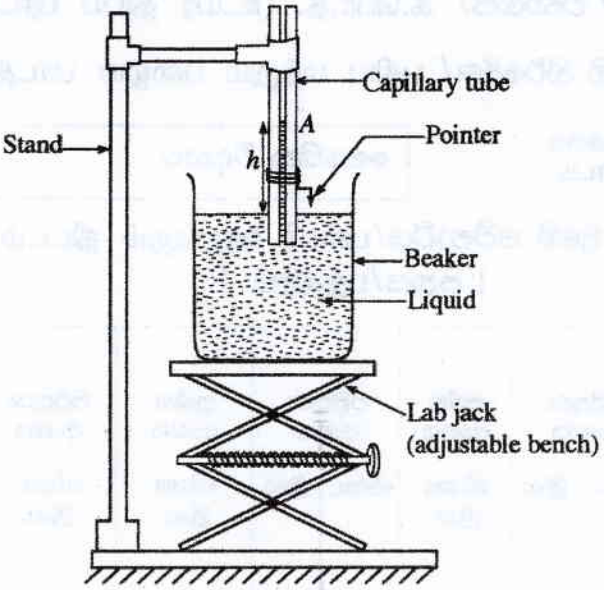


Figure (1)

(*a*) (i) Figure (2) shows the enlarged view of the vertical cross section of the capillary tube along the axis. Draw the meniscus of the liquid inside the capillary tube and indicate the surface tension T, and the contact angle θ between the liquid and the glass surface of the capillary tube in this figure.

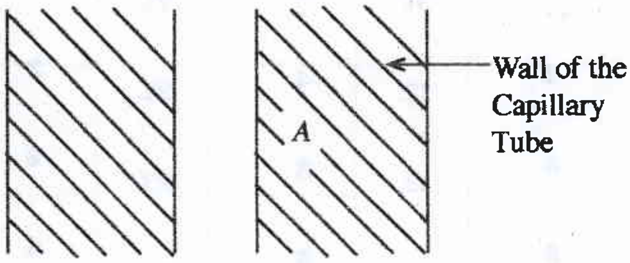
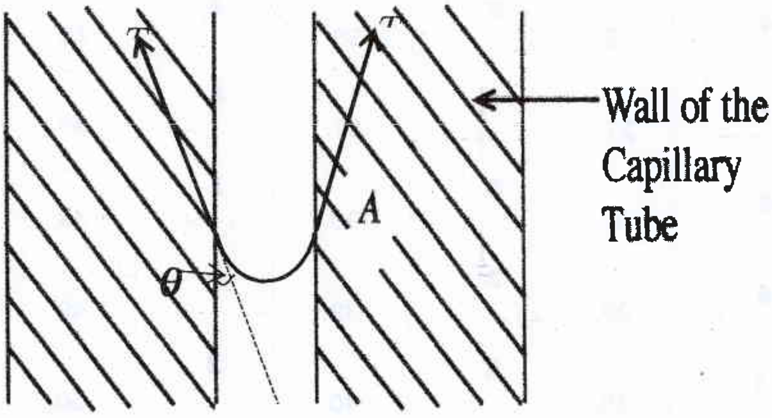


Figure (2)



Drawing the meniscus correctly ....(01)

Indicating one T with an arrow in correct direction ....(01)

Indicating the Contact angle θ ....(01)

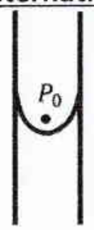
(ii) If the height of the liquid column in the capillary tube, the inner radius of the capillary tube, and the density of the liquid are h, r, and ρ, respectively, obtain an expression for $h\rho g$ in terms of T, r, and θ.

$$(2\pi r)T\cos\theta(= mg) = (\pi r^2)h\rho g$$ ....(01)

$$h\rho g = \frac{2T\cos\theta}{r}$$ ....(01)

(No Mark only for writing this equation)

Alternative Method



$$P_0 - \frac{2T\cos\theta}{r} + h\rho g = P_0$$ ....(01)

$$h\rho g = \frac{2T\cos\theta}{r}$$ ....(01)

(iii) Clearly writing the assumption made, show that the equation obtained in (ii) above can be reduced to $h = \dfrac{2T}{r\rho g}$.

The contact angle between the glass and the liquid should be very small or zero. ....(01)

for small $\theta \rightarrow \cos\theta \approx 1$ **OR** $h\rho g = {}^{2T}/_{r}$ ....(01)

(iv) In order to satisfy the assumption mentioned in (iii) above for a given liquid, write down the experimental procedure that should be followed in **the correct order**.

Wash/clean the capillary tube with a base first, then with an acid, and finally with pure water.(dry the tube) ....(02)

(Only for the correct answer with correct order)

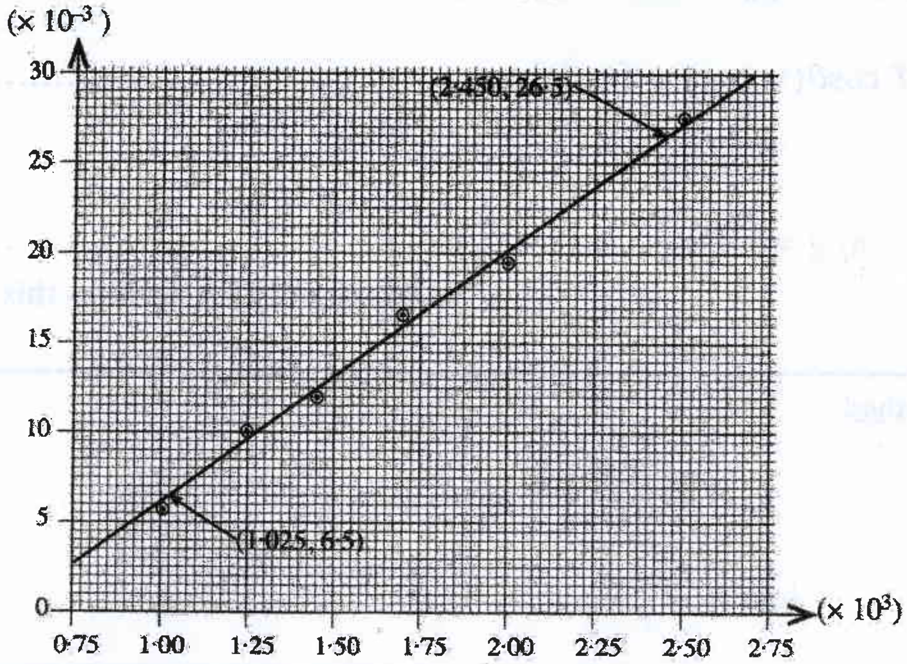
(v) Before taking the readings required to determine the height h, what is the adjustment to be made in the experimental setup shown in figure (1)?

Raising the lab jack until the pointer just touches the liquid surface.

..............................(02)

(1 Mark for only raising the lab jack/adjust the pointer until the tip touches the liquid surface)

(*b*) **The following graph shows the experimental data (in SI units) obtained using 6 capillary tubes with different radii to determine the surface tension of water.**



(i) Considering the equation in (*a*)(iii) above, identify and write down the independent variable (*x*) and the dependent variable (*y*) of the graph.

x: ....

y: ....

x: $^1/_r$ ....(01)

y: h ....(01)

(ii) Determine the surface tension of water using the graph and state the answer with SI units. (Density of water is 1000 kg m^{-3}.)

Gradient

$$m = \frac{(26.5-6.5)\times10^{-3}}{(2.450-1.025)\times10^{3}} = 1.404\times10^{-5}\ \mathrm{m}^2$$ ....(01)

$$m = {}^{2\mathrm{T}}/_{\rho g} \quad \textbf{OR} \quad \mathrm{T} = {}^{m\rho g}/_{2}$$ ....(01)

$$\therefore \mathrm{T} = \frac{1.404\times10^{-5}\times1000\times10}{2}$$ ....(01)

$$\mathrm{T} = 7.02\times10^{-2}\ \mathrm{N\,m^{-1}}\ \textbf{OR}\ \mathrm{kg\,s^{-2}}$$ ....(02)

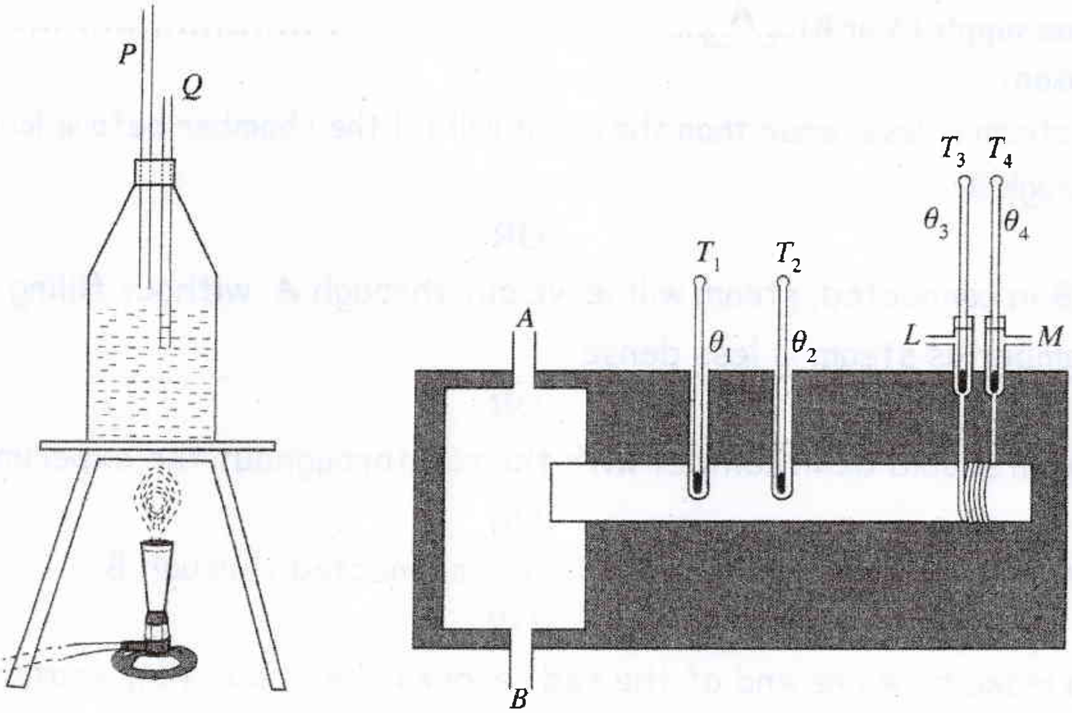
(01 Mark for the correct answer and 01 Mark for the correct unit. If only the unit is written without workout, then no marks)

(iii) What would happen to the capillary. rise if soap water is used instead of water? Briefly explain the answer.

∴ The capillary rise would be smaller compared to the water when soap water is used. ....(01)

The surface tension of water is reduced when soap is added OR the contact angle between glass and water becomes larger when soap is added. ....(01)

2. An incomplete diagram of an experimental setup to determine the thermal conductivity of a metal by the Searle's method is shown below.



(*a*) What are the purposes of inserting tubes *P* and *Q* into the steam generator?

P: ....

Q: ....

P: To supply steam ....(01)

Q: To control the pressure **OR** to maintain the pressure inside the steam generator at the atmospheric pressure ....(01)

(*b*) Proper connections of steam and water supply to Searle's apparatus are necessary to obtain the accurate result. Accordingly, select each connection and give reasons.

(i) Steam supply (*A* or *B*) : ....

Reason: ....

....

(ii) Water supply (*L* or *M*) : ....

Reason: ....

....

(i) Steam supply (A or B): $\boxed{A}$ ....(01)

Reason:

As steam is less dense than the air, it will fill the chamber before leaving through B

OR

If B in connected, steam will leave out through A, without filling the chamber as steam is less dense.

OR

Steam should be in contact with the rod throughout the experiment.

OR

Condensed water may block B, if it is connected through B

OR

To make sure one end of the rod reaches the steam temperature

..................(01)

(ii) Water supply (L or M): $\boxed{M}$ ....(01)

Reason:

To get large difference between the temperatures of the thermometers T_3 and T_4

OR

To ensure maximum heat absorption by water

OR

To achieve the steady state quickly ....(01)

(*c*) State **three** more measuring instruments needed in this experiment and briefly state the specific measurement taken by each of them.

Instrument	Measurement
(i)	....
(ii)	....
(iii)	....

Instrument	**Measurement**
Vernier Caliper	Diameter of the rod / Separation between T_1 and T_2 (in the rod)
Stopwatch	Time taken to collect water (at steady state)
Electronic/ 3 beam/ 4 beam balance	Mass of the water collected (at steady state)
Meter Ruler	Separation between T_1 and T_2 (in the rod)

For any three correct instruments with correct specific Measurement/s.. (03)

(*d*) The separation between the thermometers T_1 and T_2 is 8·0 cm. If the constant temperature readings of T_1 and T_2 are 73·8 °C and 59·2 °C, respectively, calculate the temperature gradient.

$$\text{Temperature gradient} = \frac{73.8-59.2}{8\times10^{-2}} = \frac{14.6}{8\times10^{-2}}$$ ...... **(01)**

$$= 182.5\ ^\circ\text{C m}^{-1} \textbf{ OR } 182.5\ \text{K m}^{-1}$$ ...... **(01)**

(*e*) Does this temperature gradient vary along the rod? Briefly explain the answer.

No ....**(01)**

Because the rod in Lagged (Insulated) ....**(01)**

(*f*) At thermal steady state, the difference in thermometer readings of T_3 and T_4 is 9·5 °C and the flow rate of water is 120 g per minute. Calculate the rate of heat absorption by water. (Specific heat capacity of water is 4200 J kg^{-1} K^{-1}.)

$$\text{Absorption Rate} = {}^{Q}/_{t} = \frac{ms\theta}{t} \rightarrow \frac{m}{t}\times s\times\theta = \frac{0.12}{60}\times 4200\times 9.5$$ ...**(01)**

$$= 79.77\ \text{W}\ (79.8\ \text{W})$$ ....**(01)**

(g) If the cross-sectional area of the rod is 12·0 cm^2, calculate the thermal conductivity of the metal and state the answer with SI unit.

$$Q/_t = K.A.\frac{\theta_1-\theta_2}{l} \quad \textbf{OR} \quad 79.8 = K \times 12 \times 10^{-4} \times 182.5$$ ....(01)

$$K = 364.4 \quad \mathrm{W\,m^{-1}K^{-1}}$$ ....(02)

(1 Mark for the correct answer and 1 Mark for the correct unit. If only the unit is written without workout, then no marks. No mark for the unit $\mathrm{W\,m^{-1}\,{}^{o}C^{-1}}$)

(h) Is it possible to use the Searle's method to find thermal conductivity of a poor conductor? Briefly explain the answer.

No. ....(01)

Heat flow through axial direction of the rods is not possible/ not sufficient

OR

Temperature difference/gradient between the thermometers T_1 and T_2 is not measurable

OR

Temperature difference between the thermometers T_3 and T_4 is not measurable. ....(01)

3. A standard spectrometer, a glass prism, and a monochromatic light source are used to determine the refractive index of the glass.
   (a) A few necessary adjustments are to be done to the spectrometer before starting to take measurements.
      (i) What is the adjustment that should be done to the eyepiece?

Eye piece should be adjusted to obtain a <u>clear veiw of the cross wires</u> ....(01)

(ii) Telescope is pointed to a distant object and it is adjusted until a clear image of the object is formed on the cross wires. What is the purpose of this adjustment?

To <u>receive parallel beam/rays of light</u>. ....(02)

(iii) What is the adjustment that should be done to the slit of the collimator?

The slit should be adjusted to be <u>narrow</u> and vertical, (and illuminated by a light source.) ....(01)

(iv) The telescope is brought in line with the collimator. Then the collimator is adjusted until a sharp image of the slit is formed on the cross wires. What is the purpose of this adjustment?

To obtain a parallel beam/rays of light from the collimator/to the telescope. ....(02)

(*b*) In order to level the prism table, the prism is placed as shown in figure (1) and the screws *P*, *Q*, and *R* are adjusted.

(i) When the telescope is at position T_1, the screw Q is adjusted to obtain a symmetric image of the slit on the cross wires. When the telescope is moved to the position T_2, which screw should be adjusted to get a symmetric image of the slit?

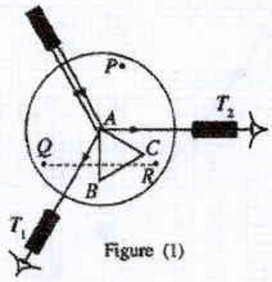


Figure (1)

Screw P ....(01)

(ii) A student stated that the levelling of the prism table could easily be done using a spirit level. Is this statement correct? Briefly explain the answer.

No. ....(01)

Prism table should be leveled parallel to the optical axis of the collimator and the telescope, (not to level parallel to the horizontal / the table.)

OR

Purpose of leveling the prism table is to make it parallel to the light beam of the collimator and the telescope, not to the horizontal.

OR

Leveling the prism table parallel to the horizontal will not make it parallel to the light beam of the collimator and the telescope.

........................(01)

(*c*) When the telescope is at positions T_1 and T_2, the readings of the spectrometer are 279° 58′ and 38° 02′, respectively. Note that the telescope passes the zero of the main scale when it is moved from T_1 to T_2. Calculate the prism angle *A*.

$$2A = 360° - T_1 + T_2 = 360° - 279°\,58' + 38°\,02'$$ ....(01)

$$= 118°\,04'$$

$$A = 59°\,02'$$ ....(01)

(*d*) To determine the angle of deviation of a light ray by the given glass prism, a student measured the incident and emergent angles i_1 and i_2, respectively, as shown in figure (2). The graph shows the variation of i_2 with i_1.

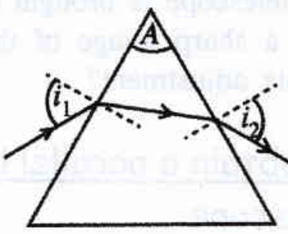
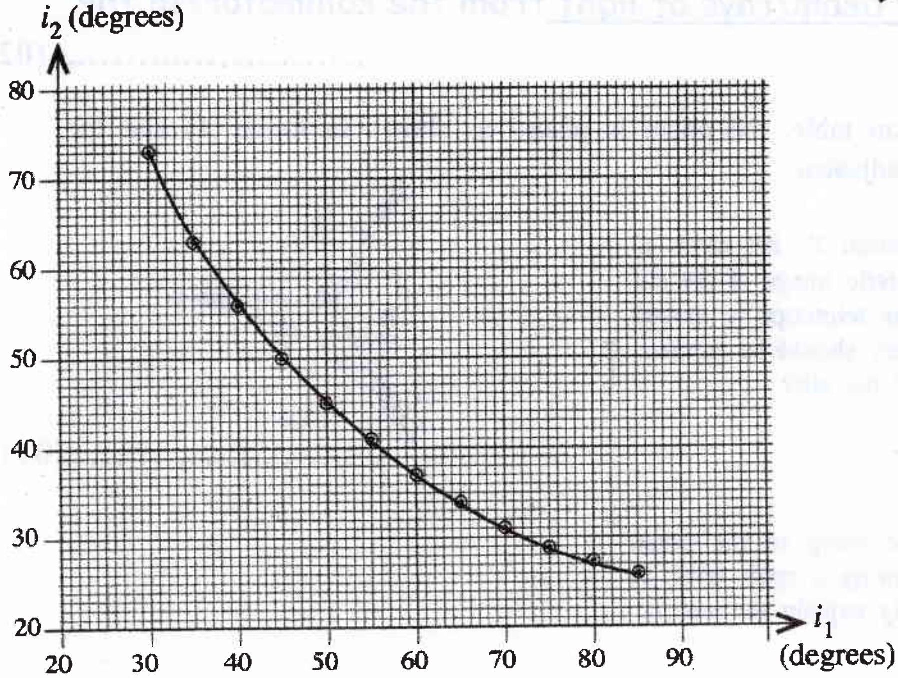


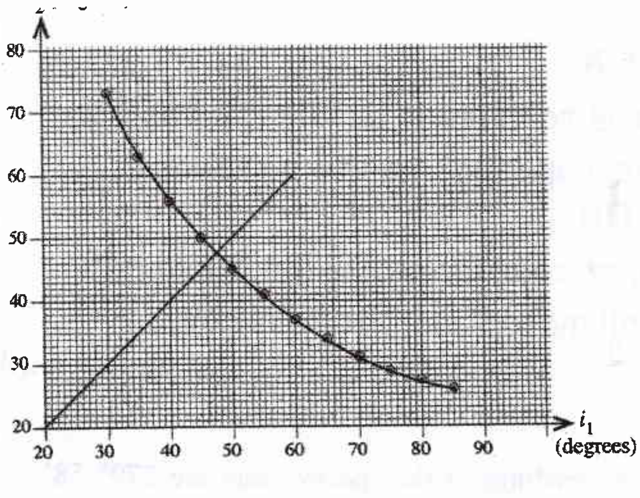
Figure (2)



(i) Write down an expression for the angle of deviation d, in terms of the prism angle A, and the angles i_1 and i_2.

$d = (i_1 + i_2) - A$ ....(02)

(ii) Determine the minimum angle of deviation D by using the graph.



From the graph $i_1 = i_2 = i$ **OR**
Correct line as indicated in the graph
......................(01)

$i = 47.5°$ **OR** $47° 30'$ **OR** $47°$ **OR** $48°$ ....(01)

Minimum angle of deviation $\Rightarrow D = 2i - A$ ....(01)

$= 2 \times (47.5°) - 59°02'$ ....(01)

$= 35° 58'$ ($34° 58'$ **OR** $36° 58'$) ....(01)

(iii) Calculate the refractive index of the glass that the prism is made of.

$$n = \frac{\sin\left(\frac{A+D}{2}\right)}{\sin\left(A/_2\right)} = \frac{\sin\left(\frac{59^\circ\,02' + 35^\circ\,58'}{2}\right)}{\sin\left(\frac{59^\circ\,02'}{2}\right)}$$ ....(01)

$= 1.49 \quad (1.48 - 1.51)$ ....(01)

Alternative method

$$n = \frac{\sin i}{\sin r} = \frac{\sin 47^\circ\,30'}{\sin 29^\circ\,31'}$$ ....(01)

$= 1.49 \quad (1.48 - 1.51)$ ....(01)

4. Figure (1) shows an experimental setup of a potentiometer with a 4m long wire, that can be used to determine the internal resistance r of a given cell with electromotive force (emf) E ($< E_0$).

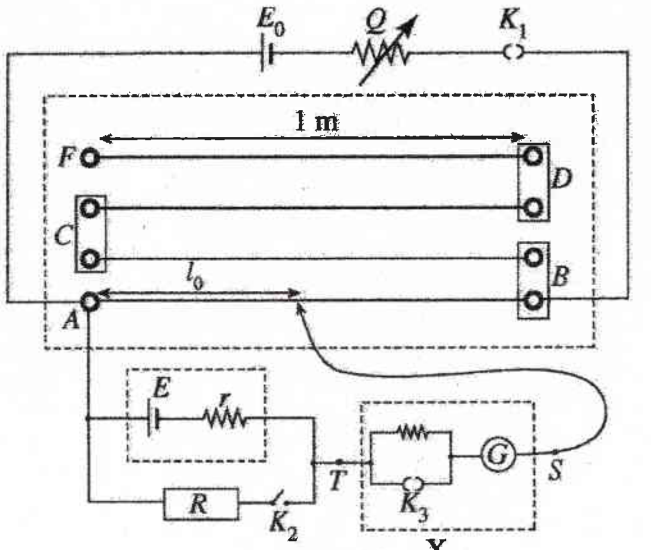


Figure (1)

(*a*) State two possible qualities of a potentiometer wire that affect the accuracy of measurements.

Non-uniformity / Uniformity of the potentiometer wire. ....(01)

Temperature dependence of the resistance of the potentiometer wire **OR**
Temperature coefficient of the resistance of the potentiometer wire **OR**
The resistance of the wire ....(01)

(*b*) Can the potentiometer shown in figure (1) be used as a voltmeter having an adjustable range? Give reasons for the answer.

Yes. ....(01)

Range can be adjusted by

<u>varying the value of Q</u>

OR

Increasing the length of the potentiometer wire

....................(01)

(*c*) A student observed a small deflection of the galvanometer even when there is no current passing through it. Is it advisable to use this galvanometer for this experiment? Give reasons for the answer.

Yes. ....(01)

Zero error of the galvanometer does not affect the experiment

OR

It is the deflection, not the correct reading of the galvanometer that matters in the experiment.

OR

Experiment can be continued by observing the deflection from the initial position. ....(01)

(*d*) When the switch K_2 is open, the balance length of the potentiometer wire is l_0. When K_2 is closed, the balance length is l. Obtain an expression for the internal resistance r of the given cell in terms of l, l_0, and R.

$$\left.\begin{array}{l} E = kl_0 \\ V = kl \end{array}\right\} \quad \textbf{OR} \quad \frac{V}{E} = \frac{l}{l_0}$$ ....(01)

$$V = E\left(\frac{R}{R+r}\right) \quad \textbf{OR} \quad \frac{V}{E} = \frac{R}{R+r}$$ ....(01)

$$\therefore \frac{R}{R+r} = \frac{l}{l_0}$$

$$r = R\left(\frac{l_0}{l} - 1\right)$$ ....(01)

(*e*) With the given potentiometer, the balance length can be measured with a maximum error of 1 mm. If $R = 8\,\Omega$, $l_0 = 72{\cdot}4$ cm and $l = 50{\cdot}1$ cm, calculate the maximum value that could be obtained for the internal resistance r.

$$l_0 = 72.4 + 0.1 \text{ cm} \quad \textbf{OR} \quad l = 50.1 - 0.1 \text{ cm}$$ ....(01)

$$r = 8 \times \left(\frac{72.4+0.1}{50.1-0.1} - 1\right) = 8 \times \left(\frac{72.5}{50.0} - 1\right)$$ ....(01)

$$r = 3.55\,\Omega \quad \textbf{OR} \quad 3.60\,\Omega$$ ....(01)

Alternative Method

$$\delta r = r\left\{\frac{\delta l_0}{l_o} + \frac{\delta_l}{l}\right\}$$

$$\text{here } r = 8\left(\frac{72.4}{50.1} - 1\right) = 3.56$$ ....(01)

$$\delta r = 3.56\left\{\frac{0.1}{72.4} + \frac{0.1}{50.1}\right\} = 0.01\,\Omega$$ ....(01)

$$r(= 3.56 + 0.1) = 3.57\,\Omega \quad (3.6\,\Omega)$$ ....(01)

(*f*) **Internal resistance *r* can be determined more accurately by a graphical method. Considering *R* as a variable resistor, rearrange the equation obtained in (*d*) to plot a suitable graph. Write down the independent (*x*) and dependent (*y*) variables of the graph.**

$$r = R\left(\frac{l_0}{l} - 1\right)$$

$$\frac{l_0}{l} = (r)\frac{1}{R} + 1 \qquad \textbf{OR} \qquad \frac{1}{l} = \left(\frac{r}{l_0}\right)\frac{1}{R} + \frac{1}{l_0}$$ ....(01)

x: $^1/_R$

y: $^{l_0}/_l$ **OR** $^1/_l$ ....(01)

(Award this mark only if the equation is correct)

(*g*) The potentiometer circuit shown in figure (1) can be modified by replacing the part of the circuit marked X in figure (1), by the circuit shown in figure (2). For this, the terminals *S'* and *T'* of the circuit shown in figure (2) are connected respectively to points *S* and *T* of the potentiometer circuit shown in figure (1).

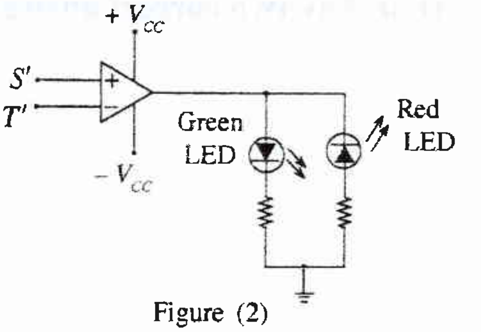


Figure (2)

(i) Assume that the balance point is located between *A* and *B* in the modified circuit. What is the colour of the Light Emitting Diode (LED) which is lit when the sliding key is placed at *A* and *B*?

At *A* : ....

At *B* : ....

At A: Green ....(01)

At B: Red ....(01)

(ii) Briefly explain how the balance point could be found using the modified circuit.

When the sliding key is kept at different points along the potentiometer wire, at the balance point both LEDs should be turned off

OR

When the sliding key is kept at different points along the potentiometer wire, at the balance point LEDs light ON and OFF alternately. ....(02)

(iii) State **two** advantages of this modified circuit in finding the balance point, when compared with the circuit shown in figure (1).

- Balance point can be determined with high accuracy (due to the very high sensitivity of the circuit)
- No current passes through the point S and T even when the potentiometer is not balanced.
- Cell discharges slowly.
- Rough balancing of the potentiometer can be avoided.

(For Any two correct answers, 1 Mark for each) ....(02)

5. (*a*) In electric power generators, the frequency of the output voltage depends on the number of magnetic poles P and the number of revolutions per minute N of the generator. The frequency f in Hz is given by $f = \dfrac{P \times N}{120}$.

A portable generator consisting of two magnetic poles typically works at 3000 revolutions per minute (rpm).

Find the following:

(i) The frequency of the output voltage of the generator

(ii) The rotational speed of the generator in radians per second (rad s^{-1}) (Take $\pi = 3$)

(*b*) A student has designed a model of a hydro-power plant by replacing the engine of the portable generator mentioned in (*a*) above, with a turbine that can be rotated by a water flow. He observed that the frequency of the output voltage varies with the consumption of electricity even at a constant water flow. To control the frequency variation of the output, he designed a controlling device to adjust the water flow to the turbine. Schematic diagram of the controlling device connected to a throttle valve is shown in figure (1).

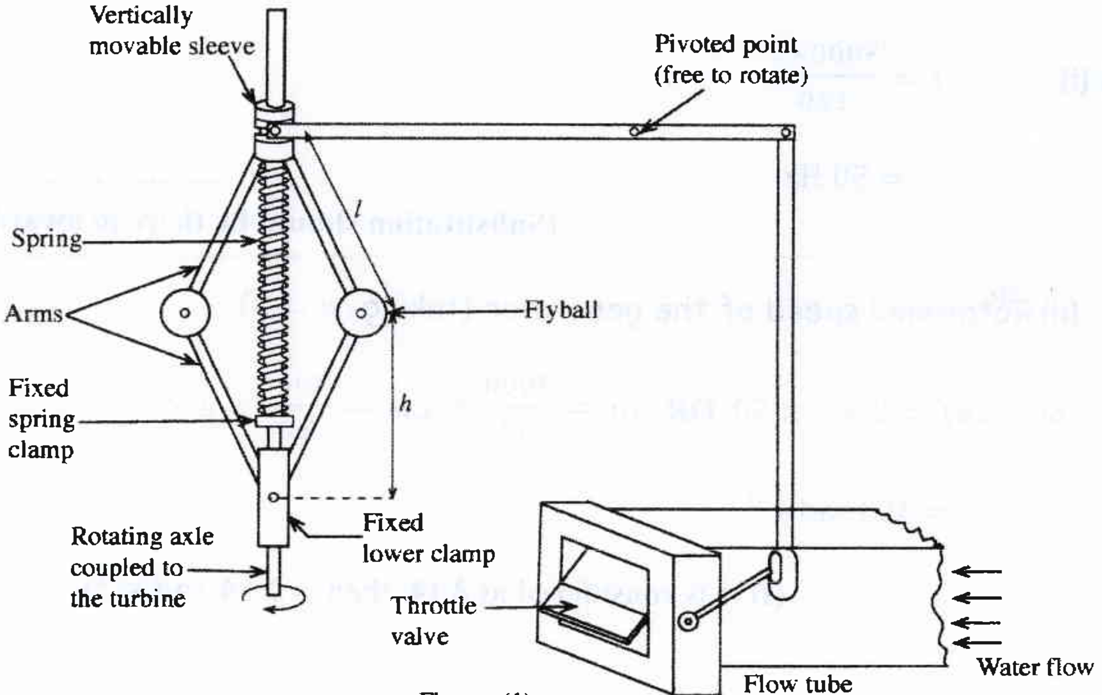


Figure (1)

Assume all the joints in this device are free to move without friction. During the rotation, flyballs move horizontally and it makes the sleeve move up and down along the rotating axle. This device is symmetric about the rotating axle. Opening and closing of the throttle valve is automatically controlled by the rotational speed of the turbine. All the other parts of the device can be assumed to be massless except the flyballs.

(i) Draw the free body force diagram for a flyball assuming each arm connected to it, is under tension. Consider the mass of a flyball to be m.

(ii) If the angular velocity of each flyball about the rotational axle is ω rad s^{-1}, show that the tensions in the upper and lower arms are respectively given by $\dfrac{ml}{2}\left(\omega^2 + \dfrac{g}{h}\right)$ and $\dfrac{ml}{2}\left(\omega^2 - \dfrac{g}{h}\right)$.

Here l is the length of each arm and h is the height to each flyball from the lower clamp.

(iii) When the frequency of the output voltage is 50 Hz, the value of h is 30 cm. Show that the contribution to the tension from the term $\dfrac{g}{h}$ can be neglected.

(iv) If m = 1 kg and l = 50 cm, calculate the tension in an upper arm.

(v) When the frequency of the output voltage is 50 Hz, the contraction of the spring is 20 cm. Determine the spring constant of the spring.

(c) When the frequency of the output voltage is 50 Hz, the throttle valve is set to block 50% of the flow. That is, the valve is making an angle of 45° with the axis of the flow tube as shown in figure (2). Assume that the closing of the throttle valve is proportional to the angle of the valve with the axis of the tube.

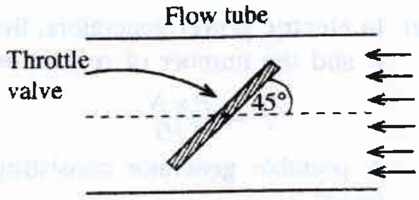


Figure (2)

The frequency of the output voltage depends on the consumption of electricity. When the consumption increases, the output frequency decreases and vice versa.

(i) According to the design, when the frequency of the output voltage becomes 25 Hz, the throttle valve will be fully opened. The valve will remain fully open even for frequencies lower than 25 Hz. Determine the following at the instant of fully opening the throttle valve. (Neglect the contribution from the term $\frac{g}{h}$)

(1) Tension of an upper arm

(2) Contraction of the spring

(ii) When the frequency of the output voltage increases, the throttle valve closes gradually to decrease the flow rate. If the flow is to be blocked by 75%, what should be the frequency of the output voltage?

(a) (i) $f = \dfrac{3000 \times 2}{120}$

$= 50$ Hz ....**(01)**

(Substitution should be there to award this mark)

(ii) Rotational speed of the generator (taking $\pi = 3$)

$\omega = 2\pi f = 2 \times 3 \times 50$ **OR** $\omega = \dfrac{3000}{60} \times 2\pi = \dfrac{3000}{60} \times 2 \times 3$ ....**(01)**

$= 300 \text{ rad s}^{-1}$ ....**(01)**

(If π is considered as 3.14, then = 314 rad s^{-1})

(b) (i)

....**(02)**

(01 Mark for mg being vertical and 01 Mark for tensions with any labeling, and when awarding marks indicating angles is not necessary, If there is clearly identifiable difference in angles, deduce 01 mark)

(ii) For the 1st figure (or the relevant diagram)

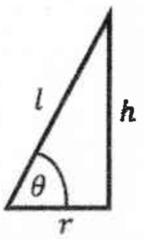
Applying Newton's 2nd law ($F = ma$) along → direction

$$(T_1 + T_2)\cos\theta = mr\omega^2 = m\frac{v^2}{r}$$ ....(02)

(01 Mark for LHS and 01 Mark for RHS, To award this mark, instead of r or any other symbol can be used or correct expression)

Considering the forces along ↑ direction for the equilibrium of the flyball

$$(T_1 - T_2)\sin\theta = mg$$ ....(01)



Since $\sin\theta = \frac{h}{l}$ **OR** $\cos\theta = \frac{r}{l}$ ....(01)

where r is the distance to the centre of the flyball from the central axle.

$$T_1 + T_2 = ml\omega^2 \text{..............(1)}$$ ....(01)

$$T_1 - T_2 = mg\frac{l}{h} \text{..............(2)}$$ ....(01)

$$(1) + (2) \Rightarrow \quad T_1 = \frac{ml}{2}\left[\omega^2 + \frac{g}{h}\right]$$

$$(1) - (2) \Rightarrow \quad T_2 = \frac{ml}{2}\left[\omega^2 - \frac{g}{h}\right]$$

(iii) When the generator operates at 50 Hz, the rotational speed

$\omega = 300 \text{ rad s}^{-1}$, and $h = 30$ cm.

∴ Therefore, $\omega^2 = (300)^2 = 90000 \text{ s}^{-2}$

$(\omega = 314 \text{ rad s}^{-1} \Rightarrow \omega^2 = (314)^2 = 98596 \text{ s}^{-2})$

$$\frac{g}{h} = \frac{10}{30\times10^{-2}} = 33.3 \text{ s}^{-2}$$ ....(01)

∴ $\frac{g}{h} \ll \omega^2$ **(For the comparison of two correct values)** ....(01)

Therefore, the term g/h can be neglected when determining the tensions T_1 and T_2.

(iv) Tension in an upper arm

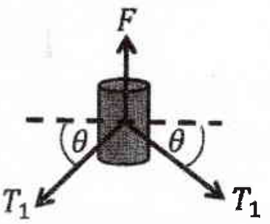
$$T_1 = \frac{ml}{2}\left[\omega^2 + \frac{g}{h}\right] \approx \frac{ml\omega^2}{2}$$

$$= \frac{1 \times 50 \times 10^{-2} \times (300)^2}{2}$$ ....(01)

$$= 22500 \text{ N}$$ ....(01)

$(\omega = 314 \text{ rad s}^{-1} \Rightarrow \quad T_1 = 24649 \text{ N})$

(v) When the sleeve is in equilibrium, the spring force on the sleeve is balanced by the tensions in two upper arms as below.



When the compression of the spring (say x) is 20 cm, the spring force

$$F = kx$$ ....(01)

$$= 2T_1 \sin\theta = 2T_1 \frac{h}{l}$$ ....(01)

where k is the spring constant.

(To award this mark above free body force diagram can also be considered)

$$k \times 20 \times 10^{-2} = 2 \times 22500 \times \frac{30\times10^{-2}}{50\times10^{-2}}$$ ....(01)

$$k = 1.35 \times 10^5 \text{ Nm}^{-1}$$ ....(01)

$(T_1 = 24649 \text{ N} \Rightarrow \quad k = 1.48 \times 10^5 \text{ Nm}^{-1})$

(c) **(i) (1)** When the frequency is $f = 25$ Hz, the rotational speed of the generator is

$$\omega = {}^{300}/_{2} = 150 \text{ rad s}^{-1}$$ ....(01)

$(\omega = {}^{314}/_{2} = 157 \text{ rad s}^{-1})$

Tension in the upper arm

$$T_1 = \frac{ml\omega^2}{2}$$

$$= \frac{1 \times 50\times10^{-2} \times(150)^2}{2}$$ ....**(01)**

$$= 5625 \text{ N}$$ ....**(01)**

$$(\ \omega = 157 \text{ rad s}^{-1} \quad \Rightarrow \quad T_1 = 6162 \text{ N})$$

(2) When the sleeve moves up by a distance (say d), the throttle valve opens. Then, the contraction (say e) of the spring becomes,

$$e = x - d = 20 - d$$ ....**(01)**

The height (h) to the flyball from the fixed lower clamp becomes,

$$h = 30 + {}^{d}/_{2}$$ ....**(01)**

Now for the equilibrium of the sleeve

$$F = ke = 2T_1 \sin\theta = 2T_1\ \frac{h}{l}$$

$$1.35 \times 10^5\ \times\ (20 - d) \times 10^{-2}\ = 2 \times 5625 \times \frac{(30+{}^{d}/_{2}) \times10^{-2}}{50\times10^{-2}}$$ ....**(01)**

(for the substitution)

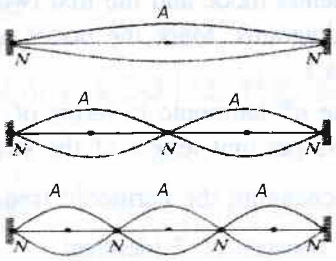
$$d = 13.84 \text{ cm} \ \ (13.8 \text{ cm})$$ ....**(01)**

$$(T_1 = 6162.25 \text{ N and } k = 1.48 \times 10^5 \text{ Nm}^{-1} \quad \Rightarrow \quad d = 13.85 \text{ cm} \ \ (13.9 \text{ cm})$$

Therefore, the contraction of the spring $= 20 - 13.84$ cm

$= 6.16\ cm$ (6.2 cm)............**(01)**

(a) (i)



..........................(01)

..........................(01)

..........................(01)

(Atleast one of the diagrams should have 'A' & 'N', if not deduct 01 Mark. No need to look for amplitude of the wave. For different lengths of the string deduct 01 Mark)

(ii) $l = n\dfrac{\lambda_n}{2}$ --------(A) ....(01)

$v = f_n \lambda_n$ ---------(B) ....(01)

$v = \sqrt{\dfrac{T}{m}}$ ---------(C) ....(01)

$\Rightarrow f_n = \dfrac{\sqrt{\dfrac{T}{m}}}{\dfrac{2l}{n}}$ ....(01)

$\Rightarrow f_n = \dfrac{n}{2l}\sqrt{\dfrac{T}{m}}$ ....(01)

(iii) Varying the (vibrating) length of the string ....(01)

Varying the tension of the string ....(01)

(b) (i) Fundemental frequencies $n = 1,\ f_1 = \dfrac{1}{2l}\sqrt{\dfrac{T}{m}}$

Since T & m are constant, $f_1 \times l = constant$ ....(01)

$260\text{ Hz} \propto \dfrac{1}{l_1}$-------------(X) ....(01)

Let f_2 & f_3 be the fundemental frequencies of the musical notes 'F' and 'B'.

$f_2 \propto \dfrac{1}{0.7l_1}$-------------(Y) ....(01)

$f_3 \propto \dfrac{1}{0.53l_1}$-----------(Z) ....(01)

$(Y)/(X) \Rightarrow \dfrac{f_2}{260} = \dfrac{1}{0.70}$

$f_2 = 371.43\text{ Hz}\ \ (371\text{ Hz})$ ....(01)

$$(Z)/(X) \Rightarrow \frac{f_3}{260} = \frac{1}{0.53}$$

$f_3 = 490.57$ Hz (491 Hz) ....(01)

(ii) $f \propto \sqrt{T}$ **OR** $f^2 \propto T$

$$\Rightarrow \frac{T'}{T} = \left[\frac{1.01f}{f}\right]^2$$ ....(01)

$$\Rightarrow \frac{T'}{T} = [1.01]^2 = 1.02,$$

$\frac{T'-T}{T}\% = 2\%$ ....(01)

Alternative Method

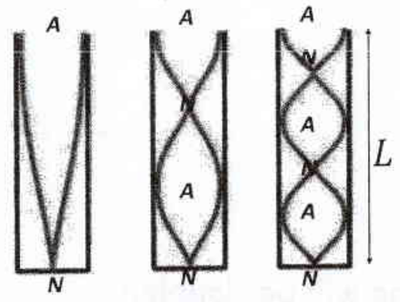
$f \propto \sqrt{T}$ **OR** $f^2 \propto T$

$$\Rightarrow \frac{\Delta f}{f} = \frac{1}{2}\frac{\Delta T}{T}$$ ....(01)

$$\Rightarrow \frac{\Delta T}{T} = 2\frac{\Delta f}{f}$$

$\frac{T'-T}{T}\% = 2\%$ ....(01)

(c) (i)



($\frac{\lambda}{4}$ $\frac{3\lambda}{4}$ $\frac{5\lambda}{4}$)

(01 × 3) ....**(03)**

(Atleast one of the diagrams should have 'A' & 'N', if not deduct 01 Mark. Deduct 1 Mark if lengths of the tubes are different)

(ii) $L=\frac{\lambda}{4}$ ....(01)

$L=\frac{v}{4f}=\frac{340}{4f}=\frac{85}{f}\times 100$ ....(01)

Required length of the pipe which produces the musical notes 'C' of frequency 260 Hz

$$=\frac{85}{260}\times 100$$

= 32.69 cm (32.7 cm) ....(01)

Required length of the pipe which produces the musical notes 'B' of frequency 491 Hz

$$=\frac{85}{491}\times 100$$

= 17.31 cm (17.3 cm) ....(01)

(iii) $(L\times f = constant)$

32.7 x 260 = L x 255 ....(01)

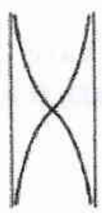
$$L=\frac{260}{255}\times 32.7$$

= 33.33 cm (33.3 cm) ....(01)

0.64 cm (0.6 cm) towards the open end. ....(01)

(iv) Fundamental frequency produced by the pipe will be doubled.

..........................(01)



$(f=\frac{v}{4L} \quad f'=\frac{v}{2L})$

..........................(01)

(For the correct diagram)

7. When an object is falling through a viscous medium, it is subjected to the buoyant force and the drag force. The buoyant force pushes the object upward while the drag force acts against the motion of the object with respect to the medium.

(a) The drag force for a solid spherical object falling in a liquid medium can be expressed by the Stokes' Law.

(i) Write down the Stokes' formula for a solid spherical object and name the parameters.

(ii) Write down **two** assumptions that are used in deriving the Stokes' formula.

(b) Consider an air bubble rising gradually upward in a viscous fluid. Stokes' Law can be applied to determine the time taken by an air bubble to reach the surface of the fluid. Neglecting the effect of the pressure change with height, the instantaneous velocity $V(t)$ of an air bubble in a viscous medium at a given time t can be given by $V(t) = V_T\left(1 - e^{-\frac{t}{\tau}}\right)$, where V_T and τ are the terminal velocity and the relaxation time of the motion of the air bubble, respectively.

(i) If the relaxation time for the motion of an air bubble in a viscous medium is $4\,\mu s$, calculate the time it takes for the instantaneous velocity to be 50% of V_T from the rest (Take $\ln 0{\cdot}5 = -0{\cdot}7$)

(ii) Calculate the time taken by the air bubble to increase the instantaneous velocity from 50% to 90% of V_T. (Take $\ln 0{\cdot}1 = -2{\cdot}3$)

(iii) Considering the answers obtained in (b)(i) and (b)(ii) above, plot the variation of the instantaneous velocity of the air bubble as a function of time. Clearly indicate V_T on the graph.

(c) Consider an air bubble rising from the bottom of an oil tank which is filled upto 10 m height.

(i) Obtain an expression for the resultant force acting on the air bubble in terms of η, ρ_o, ρ_a, a, and v, where η is the coefficient of viscosity of oil, ρ_o is the density of the oil, ρ_a is the density of air, a is the radius of the air bubble, and v is the velocity of the air bubble.

(ii) It is given that $\eta = 7{\cdot}5 \times 10^{-2}$ Pa s, $\rho_o = 900$ kg m^{-3}, $\rho_a = 1{\cdot}225$ kg m^{-3}, and the average radius of an air bubble $a = 0{\cdot}1$ mm. Neglecting the weight of the air bubble, and the effect due to the variation of pressure with height, calculate the terminal velocity of the air bubble.

(iii) Calculate the radius of the air bubble just below the surface of the oil, if the internal pressure of the bubble is 100·33 kPa, atmospheric pressure is 100 kPa, and the surface tension of oil is $2{\cdot}0 \times 10^{-2}$ N m^{-1}.

(iv) Considering the change in radius of the air bubble with height, sketch the variation of its instantaneous velocity with time.

(a) (i) $F = 6\pi\eta a v$ ....(02)

η - Coefficient of viscosity

a - <u>Radius</u> of the sphere

v - <u>Velocity</u> of the sphere

(01×3)..................................(03)

(ii) Flow is streamline with respect to the object

Surface of the object is smooth

No interaction with other objects/ Infinite large area around the object.

The temperature of the fluid is constant

Made of homogeneous material

Fluid must be at rest

(For any two assumptions with 01 Mark each) (01 × 2)........................(02)

(a) **(i)** $V(t) = V_T(1 - e^{-t/\tau})$

$50\,\%\, V_T = V_T(1 - e^{-t/\tau}) \Rightarrow 1 - e^{-t/\tau} = 0.5$ ....**(01)**

$\Rightarrow e^{-t/\tau} = 0.5 \Rightarrow -{}^{t}/_{\tau} = \ln 0.5 = -0.7$ ....**(01)**

$t = 0.7 \times \tau = 0.7 \times 4 \times 10^{-6}\ \mathrm{s}$ ....**(No marks allocated)**

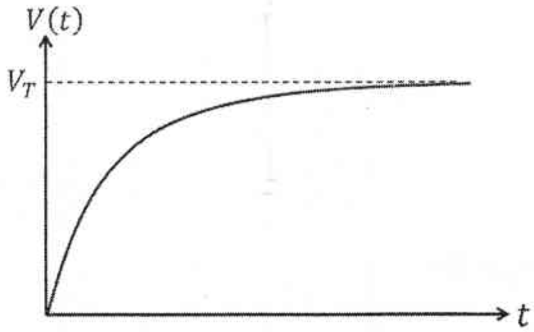
$= 2.8 \times 10^{-6}\ \mathrm{s}$

(ii) $90\,\%\, V_T = V_T(1 - e^{-t/\tau}) \Rightarrow 1 - e^{-t/\tau} = 0.9$ ....**(01)**

$e^{-t/\tau} = 0.1 \Rightarrow -{}^{t}/_{\tau} = \ln 0.1 = -2.3$ ....**(01)**

$t = 2.3 \times \tau = 2.3 \times 4 \times 10^{-6}\ \mathrm{s} = 9.2 \times 10^{-6}\ \mathrm{s}$ ....**(No marks allocated)**

(iii)



................................**(03)**

(01 Marks for the shape of the graph, and 01 Mark for marking the axis, 01 mark for indicating V_T, Award marks for drawing the graph by calculating points)

(c) (i) Forces acting on the air bubble are buoyant force (upthrust) ↑, drag force ↓, and weight of the air bubble ↓.

The resulting force on the air bubble along ↑ direction

$F_R = V\rho_o g - 6\pi\eta a v - V\rho_a g$ ....(03)

(For each correct term with correct sign: 01 Mark)

$= \frac{4}{3}\pi a^3 \rho_o g - 6\pi\eta a v - \frac{4}{3}\pi a^3 \rho_a g$ ....(01)

(ii) When the terminal velocity is achieved, $F_R = 0$ ....(01)

Neglecting the weight (i.e. $\frac{4}{3}\pi a^3 \rho_a g$) of the air bubble and the effect due to the variation of pressure with height (i.e. no change in volume)

$6\pi\eta a v_T = \frac{4}{3}\pi a^3 \rho_o g \quad \Rightarrow \quad v_T = \frac{2}{9}\frac{\rho_o g}{\eta} a^2$ ....(02)

$v_T = \frac{2}{9} \times \frac{(900)\times 10}{7.5\times 10^{-2}} \times (0.1 \times 10^{-3})^2$ ....(01)

$= 2.67 \times 10^{-4}\ \mathrm{m\,s^{-1}}\ (2.7 \times 10^{-4}\ \mathrm{m\,s^{-1}})$ ....(01)

(Award All the mark for the calculation even if the weight of the air bubble is considered)

(iii) The difference in pressure of the air bubble inside and outside

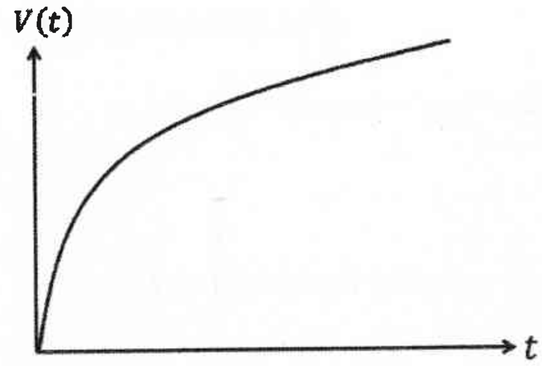
$\Delta P = P_{\text{inside}} - P_{\text{outside}} = {}^{2T}/_{r}$ ....(02)

(01 Mark each for each side of the equation)

$(100.33 - 100) \times 10^3 = {}^{2 \times (2 \times 10^{-2})}/_{r}$ ....(01)

$r = 1.21 \times 10^{-4}\ \mathrm{m}\ (1.2 \times 10^{-4}\ \mathrm{m})$ ....(01)

(iv) The terminal speed $v_T \propto a^2$, therefore v_T increases as the radius of the air bubble a increases. But due to the pressure variation with height, the volume of the air bubble increases and therefore a increases. Due to this continuous variation of a, the air bubble accelerates without achieving the terminal speed.



....**(03)**

(1 mark for axis labeling, 1 mark for behavior of initial rising, and the 1 mark is for later, continuous slow rise)

8. (*a*) (i) A current I flows through a thin wire of very small length Δl. Show that the magnetic flux density ΔB at a point with a perpendicular distance d away from this wire, is given by $\frac{\mu_0 I \Delta l}{4\pi d^2}$.

(ii) A current I flows through a flat circular coil of radius R with N number of turns as shown in figure (1). Obtain an expression for the magnitude of the magnetic flux density B at the centre of the coil.

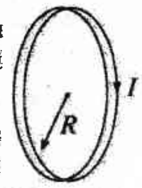


Figure (1)

(iii) Two such coils are placed coaxially with a separation R as shown in figure 2(a). The current I flows through both coils in the same direction. Figure 2(b) shows the vertical cross section of the coils through the common axis.

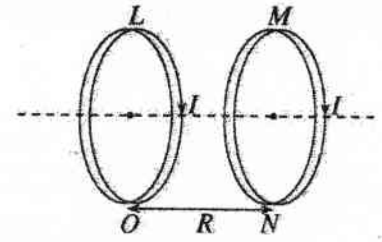


Figure 2(a)

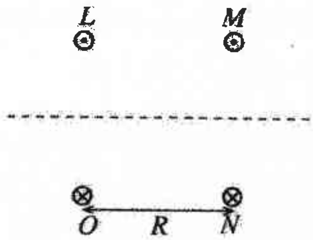


Figure 2(b)

Copy the figure 2(b) onto the answer script and draw the magnetic field lines to illustrate the magnetic field due to both coils.

(*b*) The apparatus shown in figure (3) can be used to determine the charge to mass ratio $\left(\frac{e}{m_e}\right)$ of an electron. The vacuum tube has a filament cathode C, electrodes A_1 and A_2, and a vertical fluorescent screen S with grid lines. The path of the electron beam can be seen on the fluorescent screen.

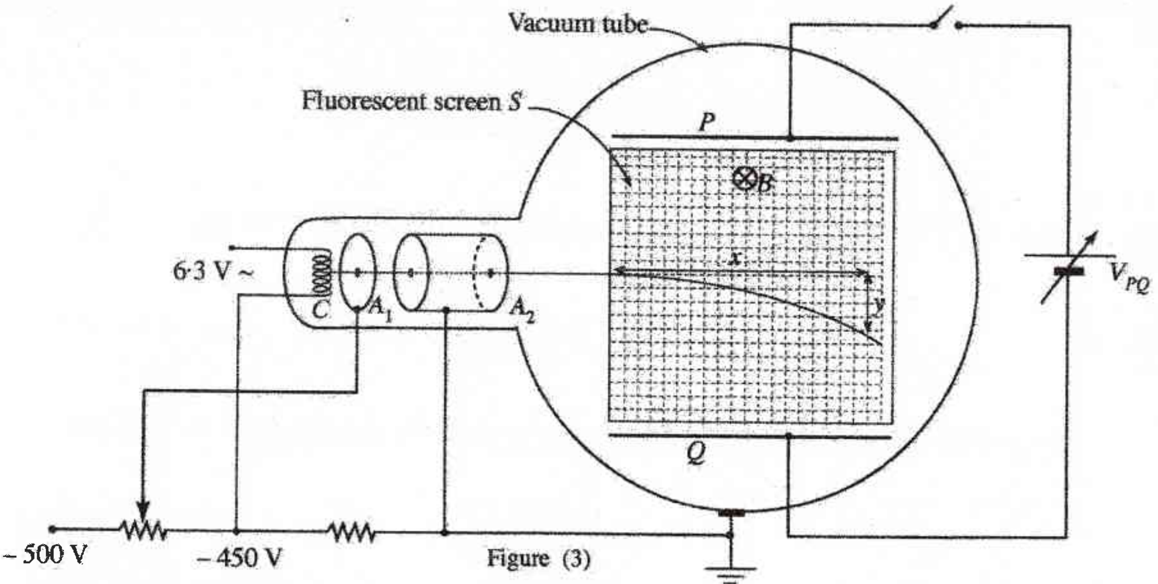


Figure (3)

(i) The function of the electrode A_1 is to control the intensity of the electron beam. What is the function of the electrode A_2?

(ii) If a negative voltage $(-V)$ is applied to electrode A_1, obtain an expression for the speed of an electron travelling through the electrode A_2. (Charge of an electron is $-e$ and mass of an electron is m_e.)

(iii) The spherical part of the tube is placed between two flat circular coils carrying the same current as shown in figure (4). Thereby a uniform magnetic field B is applied perpendicularly to the screen S. This makes the electrons move in a circular path.

If the radius of the path of the electron beam is r, obtain an expression for the ratio $\left(\frac{e}{m_e}\right)$ of the electron.

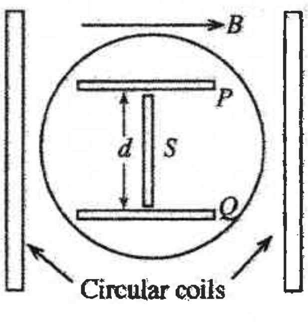


Figure (4)

(c) A dc voltage can be applied between two parallel metal plates P and Q as shown in figure (3). The plates P and Q are separated by a distance d as shown in figure (4). While the magnetic field B is applied, the potential difference between the plates V_{PQ} can be adjusted until there is no deflection of the electron beam. This process can be utilized as an alternative way to determine the speed of the electrons.

(i) Draw the electric and magnetic forces acting on an electron within the plates P and Q, after the above adjustment is done.

(ii) Obtain an expression for the speed of the electrons in terms of d, B and V_{PQ}.

(iii) When $B = 1$ mT and $V_{PQ} = 0$, the radius of the path of the electrons is 6 cm. When $V_{PQ} = 840$ V, there is no deflection of the electron beam. The separation between the plates P and Q is 8 cm.
Calculate
(1) the speed of an electron, and
(2) the charge to mass ratio $\left(\frac{e}{m_e}\right)$ of an electron.

(a) (i) From Biot-Savart Law $\Delta B = \frac{\mu_0 I \Delta l}{4\pi d^2}\sin\theta$ ....(01)

$$\Delta B = \frac{\mu_0 I \Delta l}{4\pi d^2}\sin\left(\frac{\pi}{2}\right)$$ ....(01)

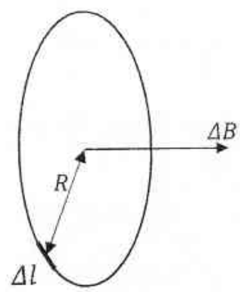
(Identifying $\theta = \frac{\pi}{2}$ OR 90^o)

$$\Delta B = \frac{\mu_0 I \Delta l}{4\pi d^2}$$

(ii) Magnetic flux density at the <u>centre</u> of the coil due to <u>Δl</u>,

$$\Delta B = \frac{\mu_0 I \Delta l}{4\pi R^2}$$ ....(01)

Magnetic flux density at the <u>centre</u> due to <u>whole coil</u>, $B = \sum \Delta B$



$$B = \sum \frac{\mu_0}{4\pi}\frac{I\Delta l}{R^2}$$ **OR** $$B = \frac{\mu_0}{4\pi}\frac{I}{R^2}\sum \Delta l$$

OR

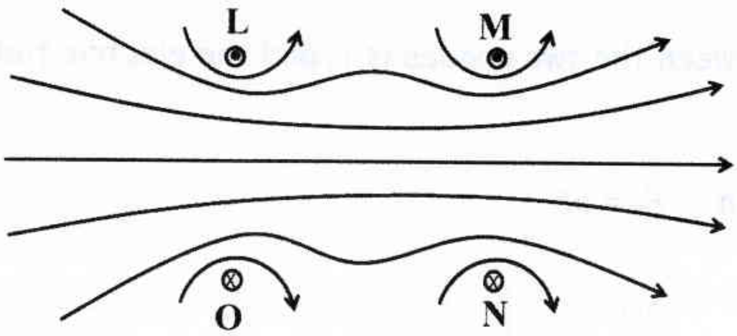
$$B = \frac{\mu_0 I}{4\pi R^2}(\Delta l_1 + \Delta l_2 + \Delta l_3 + \cdots\cdots\cdots \Delta l_n)$$ ....(01)

$$B = \frac{\mu_0}{4\pi}\frac{I}{R^2}(2\pi R N)$$ ....(02)

(1 Mark for $2\pi R$ and 1 Mark for multiplying by N)

$$B = \frac{\mu_0 I N}{2R}$$ ....(01)

(iii)



At least 2 nearly parallel lines close to the centre of the coils ....**(01)**

Arrow/s in a central line/s of force in correct direction ....**(01)**

Additional Symmetric lines with arrow/s of force on the same coil ....**(01)**

(b)(i) Accelerating electrons (towards A_2) **OR** Producing a collimated beam of high velocity electrons ....**(02)**

(ii) Kinetic Energy + Potential Energy at A_1 = K.E. + P.E. at A_2

OR

Considering the conservation of energy

OR

Any correct alternative reasoning ....**(01)**

$$0 + (-e)(-V) = \frac{1}{2}m_e v^2 + 0$$ ....**(02)**

(1 Mark each for writing each side of the equation)
(If all the terms in this equation are written correctly, all 03 marks can be awarded without any reasoning)

$$v^2 = \frac{2eV}{m_e}$$

$$v = \sqrt{\frac{2eV}{m_e}}$$ ....**(01)**

Alternative Method

If the distance between the two anodes is l, and the electric field between the two anodes is E,

Force on an electron $F_e = eE$

$$m_e a = e\left(\frac{V}{l}\right)$$ ....(01)

$$\therefore a = \frac{eV}{lm_e}$$ ....(01)

Using $v^2 = u^2 + 2as$

$$v^2 = 0 + 2\left(\frac{eV}{lm_e}\right)l$$ ....(01)

$$v = \sqrt{\frac{2eV}{m_e}}$$ ....(01)

(iii) For circular motion of an electron;

Centripetal force = force on an electron due to magnetic field

$$\frac{m_e v^2}{r} = Bev$$ ....(02)

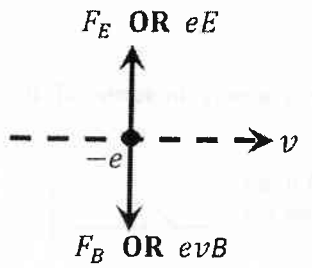
(1 Mark each for writing each side of the equation)

$$v = \frac{Ber}{m_e}$$

$$\therefore \frac{Ber}{m_e} = \sqrt{\frac{2eV}{m_e}} \quad \textbf{OR} \quad \left(\frac{Ber}{m_e}\right)^2 = \frac{2eV}{m_e}$$ ....(01)

$$\frac{e}{m_e} = \frac{2V}{B^2 r^2}$$ ....(01)

(c) (i)



........................(01)

........................(01)

(E is the Electric Field Intensity between the plates, P and Q)

(ii) For no deflection of electrons; $F_B = F_E$

$$Bev = eE$$ ....(01)

$$Bev = e\left(\frac{V_{PQ}}{d}\right)$$ ....(01)

$$v = \frac{V_{PQ}}{Bd}$$ ....(01)

(iii) (1) $v = \frac{V_{PQ}}{Bd}$

$$= \frac{840}{(1\times10^{-3})\times(8\times10^{-2})}$$ ....(01)

$$v = 1.05 \times 10^{7}\ \mathrm{m\ s^{-1}}$$ ....(01)

(2) For circular motion of an electron

$$Bev = \frac{m_e v^2}{r}$$

$$\frac{e}{m_e} = \frac{v}{Br}$$ ....(01)

$$= \frac{1.05\times10^{7}}{(1\times10^{-3})\times(6\times10^{-2})}$$ ....(01)

(For the correct substitution)

$$= 1.75 \times 10^{11}\ \mathrm{C\ kg^{-1}}$$ ....(01)

9. Answer either part (A) or part (B) only.

Part (A)

(*a*) The electromotive force (emf) of an electric source is defined as the work done by the source on a unit charge. Using this definition;

(i) determine the units of emf.

(ii) obtain an expression for the power generated by a source in terms of its emf E and the current I flowing through it.

(*b*) A source of emf E and internal resistance r is connected to an external resistor with resistance R. Obtain an expression for the total energy dissipated in the circuit in time t, in terms of E, r, R, and t.

(*c*) Consider an electrochemical battery of a car that powers the starter motor and the headlamps as shown in the circuit of figure (1). Rated power of each headlamp is 60 W. The internal resistance of the battery is 0·03 Ω. Consider that the ammeter behaves as an ideal ammeter.

When only the headlamps are turned on (S_1 is closed) without starting the car (S_2 is open), the voltmeter shows a value of 12·0 V.

(i) What is the reading of the ammeter?
(ii) What is the resistance of a headlamp?
(iii) Calculate the emf of the battery.

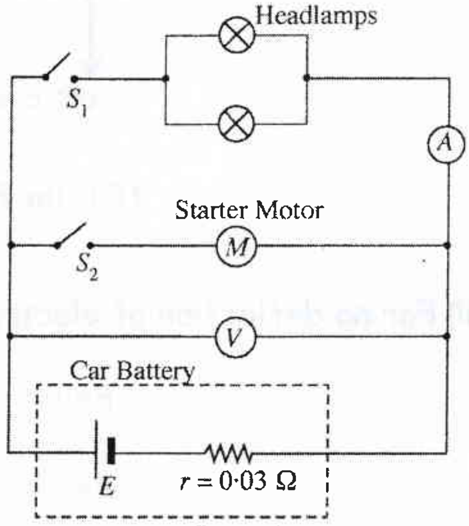


Figure (1)

(*d*) When the starter motor is just turned on (S_2 is just closed) while the headlamps are ON, the ammeter shows a value of 8·0 A. Calculate,

(i) the current through the starter motor, and
(ii) the resistance of the starter motor.

(*e*) When the armature of the starter motor is rotating while the headlamps are ON, the current through the starter motor is 34·2 A and the voltmeter reading is 11·0 V.

Calculate,

(i) the back emf, and
(ii) the efficiency of the starter motor, at this instant.

(*f*) Sketch the variation of the back emf E_b of the motor with the current flowing through it.

(*g*) The battery discharged considerably because the driver parked the car without turning off the headlamps on a certain night. As a result, emf of the battery dropped to 10·8 V and its internal resistance increased to 0·24 Ω. The current through the starter motor was not sufficient to rotate it due to the discharge of the battery. Find the current through the starter motor at this instance.

(*h*) In the situation mentioned in (*g*) above, the driver used an external battery with an emf 12·3 V and an internal resistance 0·02 Ω to jump start the car. For this, the external battery was connected to the discharged battery using two jumper cables, each having a resistance of 0·015 Ω and the car was then started.

(i) Draw the circuit diagram showing the connections to the external battery with the discharged battery, when jump starting the car.

(ii) Calculate the **maximum** current through the starter motor when starting the engine.

9. (Part A)

a) Electromotive Force (EMF) = Work Done/Charge

(i) $E = \frac{W}{q}$

Units   J C^{-1} ....**(02)**

(No marks for other units)

(ii) Work done $W = Eq$ ....(01)

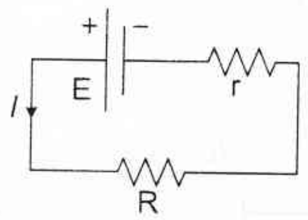
Power generated by the source

$P = \frac{W}{t} = E\frac{q}{t}$ ....(01)

$P = EI$ ....(01)

(Student must use the given definition.)

b)



Total energy dissipated in the circuit in time t $= EIt$ ....(01)

$E = I(R + r)$ **OR** $I = \frac{E}{R+r}$ ....(01)

$\therefore$ Total energy dissipated in the circuit in time t is $E\left(\frac{E}{R+r}\right)t = \frac{E^2}{(R+r)}t$

..................................(01)

Alternative Method

Total energy dissipated in the circuit in time t $= I^2(R + r)t$ ....(01)

$E = I(R + r) \Rightarrow I = \frac{E}{R+r}$ ....(01)

$\therefore$ Total energy dissipated in the circuit in time t is

$\left(\frac{E}{R+r}\right)^2 (R + r)t = \frac{E^2}{(R+r)}t$ ....(01)

(c)(i)Apply $P = VI$ for a headlamp

$60 = 12 \times I$ **OR** $I = 5\ A$ ....(01)

Reading of the ammeter $= 2I = 10$ A ....(02)

(ii) To find the resistance of a headlamp use one of the equations below

$$P = I^2R \textbf{ OR } P = \frac{V^2}{R} \textbf{ OR } V = IR$$

$$P = I^2R \quad \textbf{OR} \quad 60 = 25R$$ ....(01)

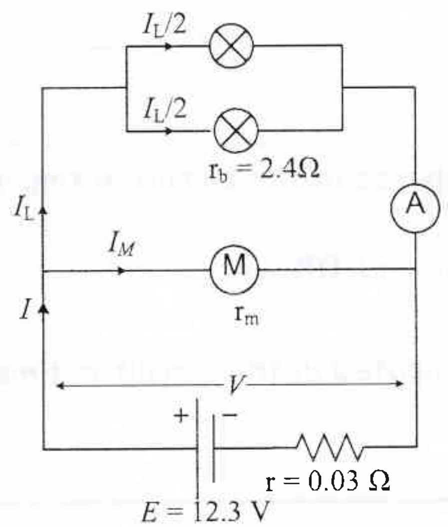
$$R = 2.4\,\Omega$$ ....(01)

(iii) For battery

$$E = V + Ir = 12 + (10 \times 0.03)$$ ....(01)

$$= 12.3\,V$$ ....(01)

(d) $I_L = 8\,A$



(i) $I = I_L + I_M \rightarrow (1)$

$V = E - Ir \rightarrow (2)$

$V = \frac{I_L}{2} r_b \rightarrow (3)$

$(3) \Rightarrow V = 4 \times 2.4 = 9.6\,V$ ....(01)

$(2) \Rightarrow I = \frac{12.3-9.6}{0.03} = 90\,A$ ....(01)

$(1) \Rightarrow I_M = 90 - 8 = 82\,A$ ....(01)

(ii) $V = I_M r_m \Rightarrow r_m = \frac{9.6}{82}$ ....(01)

$$= 0.117\,\Omega = 0.12\,\Omega$$ ....(01)

(e) (i) $V^{'} = 11.0\,V,\ \ I_M^{'} = 34.2\,A$

$$V^{'} = E_{back} + I_M^{'} r_m \quad \textbf{OR} \quad E_{back} = 11 - 34.2 \times 0.12$$

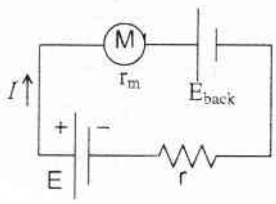
$$E_{back} = 6.90\,V$$ ....**(No marks allocated)**

(ii) $Efficiency\ of\ the\ stater\ motor = \frac{Useful\ output\ power}{Input\ Power} \times 100\ \%$

$$\eta = \frac{E_{back} \times I_M^{'}}{V^{'} \times I_M^{'}} \times 100 = \frac{6.896}{11} \times 100$$

$$= 62.7\%$$ ....**(No marks allocated)**

(f)

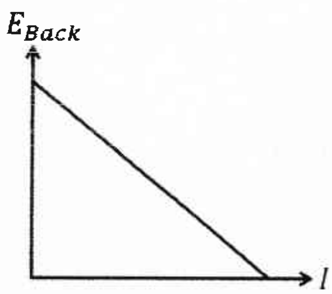


$$V - Ir = Ir_m + E_b$$

$$E_b = -I(r + r_m) + V$$

$$E_b = -r_t I + V$$

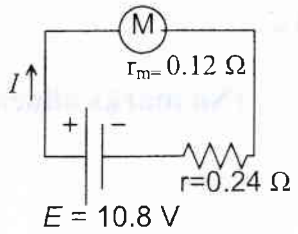
$$y = -mx + C$$



....**(01)**

(Expect shape with correct axis only)

(g) Case I: Head lamps are OFF.

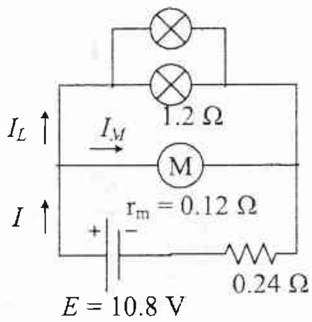


$10.8 = (0.24 + 0.12)\, I$ ....(01)

(For the correct Substitution)

$I = 30$ A **OR** 26 A ....(01)

Case II: The headlamps are kept ON



$$10.8 - (I_L + I_M)0.24 = I_M 0.12$$

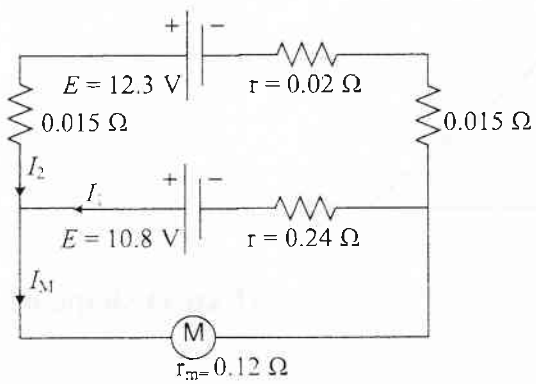
$$10.8 - (I_L + I_M)0.24 = I_L 1.2$$ ....(01)

(For any correct equation)

Solving above two equations $I_M = 28.12$ A ....(01)

(h)

(i)



..........(02)

(Positive terminal of discharged battery to positive terminal of external battery should be connected)

(ii) $I_M = I_1 + I_2 \rightarrow (1)$ ....**(01)**

$10.8 = 0.12(I_1 + I_2) + 0.24I_1$

$36I_1 + 12I_2 = 1080 \rightarrow (2)$ ....**(01)**

$12.3 = 0.12(I_1 + I_2) + 0.02I_2 + 0.03I_2$

$12I_1 + 17I_2 = 1230 \rightarrow (3)$ ....**(01)**

$(3) \times 3 - (2) => 39I_2 = 2610$

$I_2 = \frac{2610}{39} = 66.9 \approx 67 \text{ A}$ ....**(01)**

$(2) => I_1 = \frac{1080-12\times(67)}{36} = 7.66 \approx 8.0 \text{ A}$

$(1) => 67 + 8 \approx 75 \text{ A}$ ....**(01)**

Part (B)

(*a*) (i) Why Field Effect Transistors (FET) are called unipolar devices? What are the charge carriers contributing to the operation of FETs?

(ii) State why FETs are also known as voltage-controlled devices.

(iii) Calculate the drain current I_D and the Gate-Source voltage V_{GS} for the circuit shown in figure (1), assuming $V_D = 5$ V.

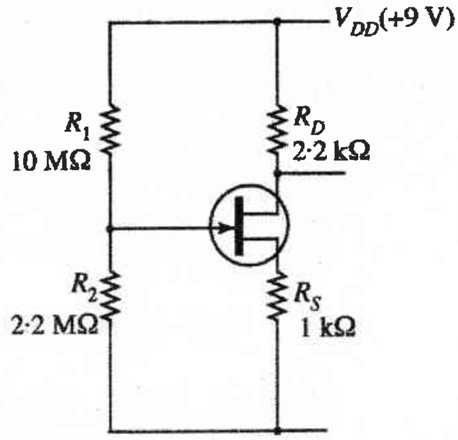


Figure (1)

(*b*) In the Op–amp circuit shown in figure (2), each electromechanical switch S_i (i = 0, 1, 2, 3) is operated by applying an electrical signal D_i (i = 0, 1, 2, 3) which can be 'High' (5 V) or 'Low' (0 V). When D_i is 'High' the respective switch S_i will be closed and otherwise, it will be open.

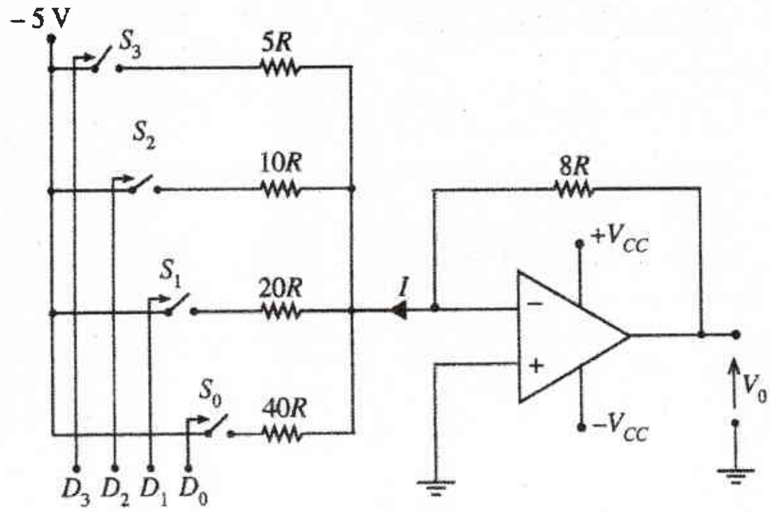


Figure (2)

(i) When D_2 is 'High', find the current through the resistor $10R$ in terms of R.

(ii) If a set of voltages (5 V, 0 V, 5 V, 5 V) is applied simultaneously to operate the switches S_3, S_2, S_1, S_0, respectively, calculate the current I indicated in figure (2) in terms of R.

(iii) Calculate the output voltage V_0 when a set of voltages (5 V, 5 V, 5 V, 5 V) is applied simultaneously to operate the switches S_3, S_2, S_1, S_0, respectively.

(*c*) A cash operated snack dispenser will provide a pack of '*Marie*' or '*Chocolate Cream*' biscuits und the following conditions.

- The correct amount of cash is inserted (*I*)
- '*Marie*' (*M*) or '*Chocolate Cream*' (*C*) is selected
- If '*Marie*' is selected, 'Availability of *Marie*' in the dispenser (*X*)
- If '*Chocolate Cream*' is selected, 'Availability of *Chocolate Cream*' in the dispenser (*Y*)

(i) Obtain the logic expression for the conditions under which a pack of biscuits may be obtained.

(ii) Show how this may be implemented using logic gates.

09.Part B

(a) (i) Because they operate only with one type of charge carriers ....(01)

Type of charge carriers either electrons or holes ....(01)

(No Marks for electrons and holes)

(ii) Voltage between two of the terminals (Gate and Source) controls the current through the device ....(01)

(iii) $I_D = \frac{V_{DD}-V_D}{R_D} = \frac{9-5}{2\cdot2\times10^3}$ ....(01)

$= 1.82\ mA$ ....(01)

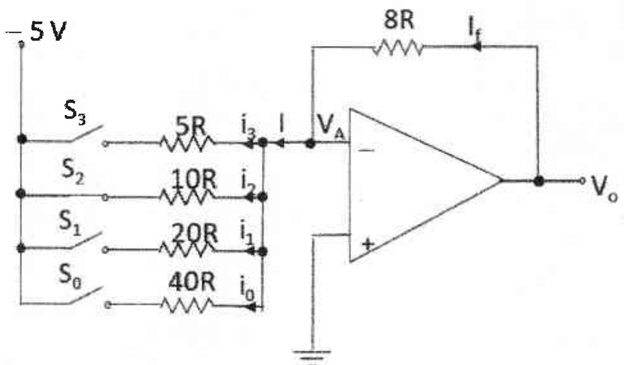
$V_S = I_D R_S = (1.82\times10^{-3})\times1\times10^3 = 1\cdot82\ V$ ....(01)

$V_G = \left(\frac{R_2}{R_1+R_2}\right)V_{DD} = \frac{2\cdot2\times10^6}{12\cdot2\times10^6}\times9 = 1\cdot62\ V$ ....(01)

$V_{GS} = V_G - V_S = 1\cdot62 - 1\cdot82$ ....(01)

$= -0\cdot2\ V$ ....(01)

(b) (i)



$i_2 = \frac{V_A-(-5)}{10R} = \frac{0-(-5)}{10R}$ ....(01)

$= \frac{1}{2R}$ ....(01)

ii) $I = i_3 + i_2 + i_1 + i_0$ ....(01)

$$= \frac{0-(-5)}{5R} + \frac{0}{10R} + \frac{0-(-5)}{20R} + \frac{0-(-5)}{40R}$$ ....(01)

$$= \frac{1}{R} + 0 + \frac{1}{4R} + \frac{1}{8R}$$

$$= \frac{11}{8R}$$ ....(01)

Alternative Method

Finding equivalent resistance

$$\frac{1}{R'} = \frac{1}{5R} + \frac{1}{20R} + \frac{1}{40R}$$ ....(01)

$$\frac{1}{R'} = \frac{11}{40R}$$ ....(01)

$$I = \frac{0-(-5)}{40R/11} = \frac{11}{8R}$$ ....(01)

iii) All the switches are closed

$$I = i_3 + i_2 + i_1 + i_0$$

$$I = \frac{5}{5R} + \frac{5}{10R} + \frac{5}{20R} + \frac{5}{40R}$$

$$I = \frac{1}{R} + \frac{1}{2R} + \frac{1}{4R} + \frac{1}{8R}$$

Also $I_f = I$ ....(01)

$$I_f = \frac{V_0 - V_A}{8R}$$ ....(01)

$$= \frac{V_0 - 0}{8R}$$ ....(01)

$$\therefore \frac{V_0}{8R} = \frac{15}{8R}$$ ....(01)

$$V_o = 15\,\mathrm{V}$$ ....(01)

(Alternative method)

Since all switches are closed, equivalent resistance of the input side

$$\frac{1}{R'} = \frac{1}{5R} + \frac{1}{10R} + \frac{1}{20R} + \frac{1}{40R}$$ ....(01)

$$\frac{1}{R'} = \frac{15}{40R}$$

$$\therefore \; R' = \frac{40R}{15}$$ ....(01)

Voltage gain of an inverting amplifier = $\frac{V_o}{V_{in}} = -\frac{R_f}{R_{in}}$ ....(01)

$$\therefore \; V_o = -\frac{8R\times 15}{40R} \times -5$$ ....(01)

$$V_o = 15\text{ V}$$ ....(01)

(c) (i) B = I [(MX) + (CY)] ....(04)

(01 mark each for correct I, MX, CY and + terms)

Alternative Method 1

B = IMX + ICY ....(04)

(01 mark each for correct IMX and ICY terms, and 02 marks for "+" term)

Alternative Method 2

I	M	C	X	Y	B
1	0	1	0	0	0
1	0	1	0	1	1
1	0	1	1	0	0
1	0	1	1	1	1
1	1	0	0	0	0
1	1	0	0	1	0
1	1	0	1	0	1
1	1	0	1	1	1

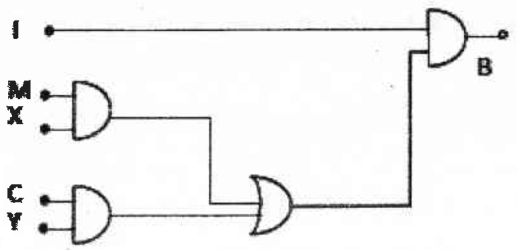
......................(02)

(Awarding for 4 rows when $B = 1$ with 1 mark each for correct 2 row values)

$B = I\bar{M}C\bar{X}Y + I\bar{M}CXY + IM\bar{C}X\bar{Y} + IM\bar{C}XY$ ....(02)

(01 mark for each for correct 2 product terms. For the correct expression without truth table, 1 Mark each for correct product term.)

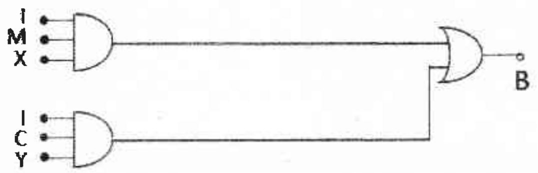
(ii)



..................(07)

(02 marks each for 1st two AND gates with correct inputs, 02 marks for OR gate with correct inputs, 01 mark for final AND gate with correct "I" input)

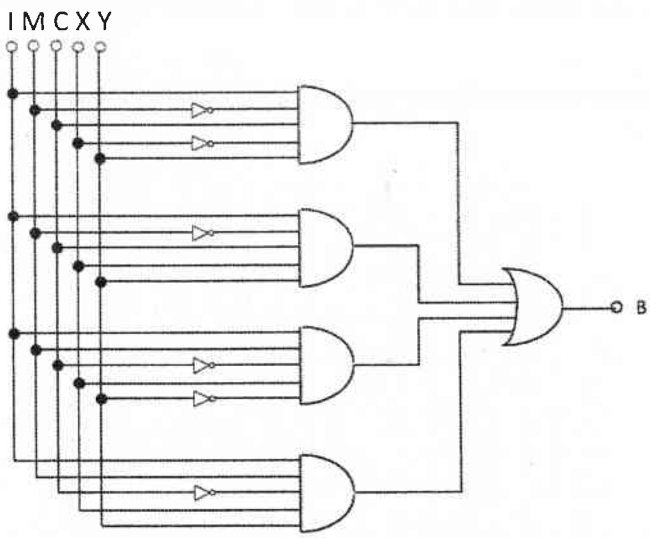
Alternative Method 1



....................(07)

(03 marks each for 1st two AND gates with correct inputs, 01 mark for final OR gate with correct inputs)

Alternative Method 2



....................(07)

(02 marks for top AND gate, 01 mark each for remaining three AND gates with correct inputs, 02 marks for final OR gate with correct inputs)

10. Answer either part (A) or part (B) only.

Part (A)

(*a*) (i) State the Boyle's law and the Charles' law.

(ii) Derive the ideal gas equation using the above laws.

(*b*) A deflated tyre of volume V and initial pressure P_0, at room temperature T_R is connected to a compressed nitrogen (N_2) gas tank via a valve. The tyre initially contains only N_2 gas. After inflating the tyre with N_2 gas, its final pressure is P and it contains a total of n number of N_2 moles. Assume that there is no change in volume of the tyre.

(i) Assuming that the N_2 gas inside the tyre behaves like an ideal gas, show that the number of moles of N_2 gas pumped into the tyre is $n\left(1-\frac{P_0}{P}\right)$.

(ii) Obtain an expression for the work done to inflate the tyre with N_2 gas.

(iii) Assuming that the pumping process of N_2 gas is adiabatic, show that the change in the temperature of the N_2 gas inside the tyre is $\frac{2}{5}\left(1-\frac{P_0}{P}\right)T_R$. The change in internal energy of an ideal gas is given by $\Delta U = n\,C_V\,\Delta T$, where C_V is the molar heat capacity at constant volume and ΔT is the change in temperature. The molar heat capacity at constant volume of a diatomic ideal gas is $\frac{5R}{2}$, where R is the universal gas constant.

(iv) This change in temperature, increases the pressure temporarily to a higher value. Show that this change in pressure is $\frac{2}{5}(P-P_0)$.

(*c*) Gauge pressure is the pressure measured relative to atmospheric pressure. Gauge pressure of a tyre is usually expressed in psi (pound per square inch) units. ($1\,\text{atm} \simeq 100\,\text{kPa}$ and $1\,\text{psi} \simeq 7\,\text{kPa}$)

A deflated tyre at 20 psi pressure is pumped further with N_2 gas to a pressure of 30 psi at room temperature (27° C).

(i) Calculate the change in temperature of N_2 gas in the tyre.

(ii) Calculate the maximum pressure in the tyre due to this change in temperature.

(iii) Usually this temporary increase in pressure is not observable when pumping N_2 gas further to a deflated tyre. Give **two** possible reasons for not observing the increase in pressure.

10. Part (A)

(a) (i) Boyle's law

The pressure of a given mass of a gas is inversely proportional to its Volume provided that the temperature remains constant

OR

$P \propto \frac{1}{V}$ for a given mass of the gas at constant temperature, where V & P are volume and pressure of the gas, respectively.

OR

$PV = constant$ for a given mass of the gas at constant temperature, where V & P are volume and pressure of the gas, respectively.

...............**(02)**

Charles' law

The volume of a given mass of the gas is directly proportional to its absolute temperature, provided that the pressure remains constant.

OR

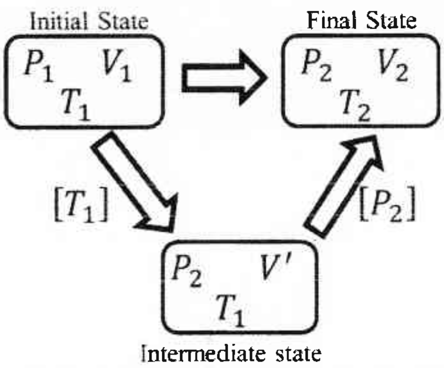
$V \propto T$ for a given mass of the gas under a constant pressure, where V & T are volume and absolute temperature of the gas, respectively.

OR

$\frac{V}{T}$ = constant for a given mass of the gas under a constant pressure, where V & T are volume and absolute temperature of the gas, respectively.

........................(02)

(ii) Consider one mole of a gas going through the following two stage processes with the initial and final values of the volume, pressure and absolute temperature are (V_1, P_1, T_1) and (V_2, P_2, T_2), respectively.



Applying Boyle's law for the constant temperature process

$P_1V_1 = P_2V'$ ----------------------------(A) ....(01)

Applying Charles' law for the constant pressure process

$\frac{V'}{T_1} = \frac{V_2}{T_2}$ ----------------------------(B) ....(01)

(A) & (B) $\Rightarrow$

$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \Rightarrow \frac{PV}{T} = \text{constant},$ .. ..(01)

For one mole of gas the constant is known as the universal gas constant R.

$\frac{PV}{T} = R$ for one mole of gas. ....(01)

If there are n mole of gas, $\frac{PV}{T} = nR$ ....(01)

$PV = nRT$

Alternative method

Q10 (A) (ii)

Consider one mole of gas of volume V under pressure P at absolute temperature T.

According to Boyle's law, for one mole of gas at absolute temperature T

PV = constant ------------------(A) ....(01)

According to Charles' law, for one mole of gas at pressure P

$\frac{V}{T}$ = Constant -----------------(B) ....(01)

(A) & (B) $\Rightarrow \frac{PV}{T}$ = constant ....(01)

For one mole of gas, the constant is known as the universal gas constant 'R'. $\frac{PV}{T} = R$ for one mole of gas. ....(01)

If there are n mole of gas, $\frac{PV}{T} =$ nR ....(01)

$PV = nRT$

(b) (i) Let n_0 be the number of moles of the air in the type at pressure P_0 at T_R

$n_0 = \frac{P_0 V}{RT_R}$ ....(02)

$n = \frac{PV}{RT_R}$ ....(02)

Number of moles of the air from the tank to the tyre

$n' = n - n_0 = \frac{PV}{RT_R} - \frac{P_0 V}{RT_R}$ ....(01)

$= \frac{V(P-P_0)}{RT_R}$

$= n\left(\frac{P-P_0}{P}\right)$ ....(01)

$= n\left(1-\frac{P_0}{P}\right)$

(ii) Let V' be the volume of these n' moles of air in the tank under pressure P_C at temperature T_R

$$V' = \frac{n'RT_R}{P_C} = \left(1-\frac{P_0}{P}\right)\frac{nRT_R}{P_C}$$

As N_2 gas flows from the tank into the tyre through the valve, the tank does work at constant pressure P_C given $= P_C V'$ ....(01)

(Award this mark for identifying the work as $P\,\Delta V$)

$= nRT_R\left(1-\frac{P_0}{P}\right)$ ....**(No marks allocated)**

(iii) $\Delta Q = \Delta U + \Delta W$ ....(01)

Adiabatic process $\Delta Q = 0 \Rightarrow \quad -\Delta U = \Delta W$ ....(01)

$-\Delta U = \Delta W = -nRT_R\left(1-\frac{P_0}{P}\right)$ (work done on the system)

Given $\Delta U = nC_V\,\Delta T$, $C_V = 5R/2$

$\Rightarrow \Delta T = \frac{\Delta U}{nC_V}$ ....(01)

$= \frac{\left(nRT_R\left(1-\frac{P_0}{P}\right)\right)}{n\,5/2\,R}$

$= \frac{2}{5}\left(1-\frac{P_0}{P}\right)T_R$ ....**(No marks allocated)**

(iv) Pressure rises to $\frac{nR}{V}(T_R + \Delta T)$ ....(01)

$= \frac{nRT_R}{V} + \frac{nR\Delta T}{V} = P + P\left[\frac{2}{5}\left(1-\frac{P_0}{P}\right)\right]$......................(01)

Change in pressure $\Delta P = \frac{2}{5}(P - P_0)$

Alternative Method

$$\frac{\Delta P}{\Delta T} = \frac{P}{T_R}$$ ....(01)

$$\Delta P = \frac{P}{T_R} \times \frac{2}{5}\left(1 - \frac{P_0}{P}\right) T_R$$ ....(01)

Change in pressure $\Delta P = \frac{2}{5}(P - P_0)$

(c) (i) $1 \text{ psi} = 7 \text{ kPa}$

$$P_0 = (20 \times 7 + 100) = 240 \text{ kPa}$$ ....(01)

$$P = (30 \times 7 + 100) = 310 \text{ kPa}$$ ....(01)

$$\Delta T = \frac{2}{5}\left(1 - \frac{p_0}{p}\right) T_R = \frac{2}{5}\left(1 - \frac{240}{310}\right) \text{x } 300$$ ....(01)

$\Delta T = 27 \text{ K OR } 27\,^0\text{C}$ ....(01)

(ii) $\Delta P = \frac{2}{5}(310 - 240)$ ....(01)

$= 28 \text{ kPa}$ **OR** 4 psi ....(01)

Maximum pressure in the tyre due to change in pressure,

$P_{max} = (310 + 28)$ **OR** $(30 + 4)$

$= 338 \text{ kPa}$ **OR** $= 34 \text{ psi}$ ....(01)

(iii) 1. Usual pumping process is not adiabatic. ....(01)

2. Normal air cannot be considered as an ideal gas. ....(01)

Part (B)

Read the following passage and answer the questions.

Radioactivity is a spontaneous decay process by which an unstable nucleus becomes a stable nucleus by emitting radiation. Decay rate is directly proportional to the number of radioactive atoms present at that instant but independent of external physical conditions.

Radioactive iodine ^{131}I is used in nuclear medicine to treat patients with thyroid cancer. The half-life time of ^{131}I is 8 days. It decays to stable ^{131}Xe initially by emitting a β^- particle and then by emitting a γ-photon. The maximum tissue penetration length of this β^- is 2 mm. Usually ^{131}I is administered to patients as sodium iodide ($Na^{131}I$) in the form of a capsule. Once administered, it is absorbed into the blood stream and concentrated in the thyroid gland. Radiation emitted from ^{131}I kills most of the cancer cells in the thyroid gland.

Since the patient becomes a potential source of radiation, precautions must be taken to minimize the radiation exposure to others around. The amount of radiation emitted by the patient is proportional to the activity of the dose administered. In medical practice, the common unit used for activity is Curie (Ci) which is not an SI unit. One Curie is equal to 37×10^9 disintegrations per second.

A radioactive material inside the body, diminishes not only by radioactive decay but also by biological clearance. This clearance is purely a biological process and follows an exponential variation, characterized by the decay constant λ_b. Hence the effective decay constant λ_e, due to both radioactive decay and biological clearance can be stated as $\lambda_e = \lambda_p + \lambda_b$, where λ_p is the decay constant corresponding to physical radioactive decay. The effective half-life time, which is used for radiation protection measures, is calculated from the effective decay constant.

(*a*) (i) State two differences between the emissions of β^- and γ.

(ii) Rewrite the following decay equation replacing *a*, *b*, and *c* with correct numbers.

$$^{131}_{53}I \longrightarrow {}^{131}_{a}Xe + {}^{b}_{c}\beta^-$$

(*b*) A fresh sample of $Na^{131}I$, having an activity of 100 mCi is received by a hospital. The sample is stored in a lead container at room temperature.

(i) What is the SI unit used for activity?

(ii) Write down an expression for the decay constant λ in terms of half-life time T.

(iii) Calculate the activity of the above sample after 4 days and express the answer in SI units. (Take $\ln 2 = 0{\cdot}7$ and $e^{-0{\cdot}35} = 0{\cdot}7$)

(iv) Hence, express the change in activity as a percentage.

(v) Is it possible to reduce the activity of the $Na^{131}I$ sample if it is stored at 0 °C instead of storing at room temperature? Explain the answer.

(*c*) A small amount of $Na^{131}I$ sample having an activity of 100 mCi is administered to a thyroid patient.

(i) When dealing with such a patient, for which mode of emission, the radiation protection measures should be taken? Explain the answer.

(ii) Show that the effective half-life time T_e of ^{131}I in thyroid gland can be given by $\frac{1}{T_e} = \frac{1}{T_p} + \frac{1}{T_b}$, where T_p and T_b are the half-life times due to radioactive decay and biological clearance, respectively.

(iii) If the biological half-life time of ^{131}I in thyroid gland is 24 days, calculate the effective half-life time of ^{131}I (in days).

(iv) Calculate the percentage change in the activity after 4 days of administration of ^{131}I. (Take $e^{-0{\cdot}46} = 0{\cdot}63$)

(v) According to radiation protection regulations, ^{131}I treated patients can be discharged from the hospital when the activity is below or equal to 50 mCi. If this regulation is followed, how long the above ^{131}I treated patient has to be kept in isolation in the hospital before discharging?

(a) **(i)**

β^-	γ
is a particle emission	is a photon/ an electromagnetic radiation.
β^- emission changes the proton number/ atomic number)	no change in proton number/ atomic number

(02 Marks for each difference)..........................**(04)**
(No Marks for properties of β^- and γ)

(ii) $^{131}_{53}\mathrm{I} \rightarrow {}^{131}_{54}Xe + {}^{0}_{-1}\beta^-$

$a = 54, \quad b = 0, \text{and } c = -1$ **(01× 3)**....................**(03)**

(b) (i) Bq (Becquerel) ....**(02)**

(ii) $\lambda = \frac{\ln 2}{T}$ **OR** $\lambda = \frac{0.693}{T}$ **OR** $\lambda = \frac{0.7}{T}$ ....**(02)**

(iii) $A_4 = A_0 e^{-\lambda t}$

$= 100 \times e^{-\frac{0.693}{8} \times 4} = 100 \times e^{-0.35}$

$= 70 \text{ mCi}$

$= 70 \times 37 \times 10^6 \text{ Bq}$

$= 2.59 \times 10^9 \text{ Bq}$ ....**(No marks allocated)**

(iv) Change $= \frac{(100-70)\text{mCi}}{100 \text{ mCi}} \times 100\%$

$= 30\ \%$ ....**(No marks allocated)**

(v) No ....**(02)**

Radioactivity is independent of external physical conditions.
..........................**(02)**

(c) **(i)** γ radiation ....(02)

β^- will not come out of the body as the maximum penetration length is 2 mm.

OR

γ radiation has longer penetration length/Power ....(02)

(ii) $\lambda_e = \lambda_p + \lambda_b$

Since $\lambda = \frac{0.693}{T}$

$$\frac{0.693}{T_e} = \frac{0.693}{T_p} + \frac{0.693}{T_b}$$ ....(02)

Therefore, $\frac{1}{T_e} = \frac{1}{T_p} + \frac{1}{T_b}$

(iii) $\frac{1}{T_e} = \frac{1}{8} + \frac{1}{24}$ ....(02)

$T_e = 6$ days ....(02)

(iv) $A_4 = A_0 e^{-\lambda t}$

$= 100 \times e^{-\frac{0.693}{6}\times 4} = 100 \times e^{-0.46}$ (for substitution)

$= 63$ mCi

Change $= \frac{(100-63)}{100} \times 100\%$

$= 37\ \%$ ....**(No marks allocated)**

(v) 6 days ....(02)

Because the effective half lifetime is 6 days. ....(03)

10. Answer either part (A) or part (B) only.

Part (A)

(a) (i) State the Boyle's law and the Charles' law.

(ii) Derive the ideal gas equation using the above laws.

(b) A deflated tyre of volume V and initial pressure P_0, at room temperature T_R is connected to a compressed nitrogen (N_2) gas tank via a valve. The tyre initially contains only N_2 gas. After inflating the tyre with N_2 gas, its final pressure is P and it contains a total of n number of N_2 moles. Assume that there is no change in volume of the tyre.

(i) Assuming that the N_2 gas inside the tyre behaves like an ideal gas, show that the number of moles of N_2 gas pumped into the tyre is $n\left(1-\frac{P_0}{P}\right)$.

(ii) Obtain an expression for the work done to inflate the tyre with N_2 gas.

(iii) Assuming that the pumping process of N_2 gas is adiabatic, show that the change in the temperature of the N_2 gas inside the tyre is $\frac{2}{5}\left(1-\frac{P_0}{P}\right)T_R$. The change in internal energy of an ideal gas is given by $\Delta U = n\,C_V\,\Delta T$, where C_V is the molar heat capacity at constant volume and ΔT is the change in temperature. The molar heat capacity at constant volume of a diatomic ideal gas is $\frac{5R}{2}$, where R is the universal gas constant.

(iv) This change in temperature, increases the pressure temporarily to a higher value. Show that this change in pressure is $\frac{2}{5}(P-P_0)$.

(c) Gauge pressure is the pressure measured relative to atmospheric pressure. Gauge pressure of a tyre is usually expressed in psi (pound per square inch) units. ($1\,\text{atm} \simeq 100\,\text{kPa}$ and $1\,\text{psi} \simeq 7\,\text{kPa}$)

A deflated tyre at 20 psi pressure is pumped further with N_2 gas to a pressure of 30 psi at room temperature (27° C).

(i) Calculate the change in temperature of N_2 gas in the tyre.

(ii) Calculate the maximum pressure in the tyre due to this change in temperature.

(iii) Usually this temporary increase in pressure is not observable when pumping N_2 gas further to a deflated tyre. Give **two** possible reasons for not observing the increase in pressure.

10. Part (A)

(a) (i) Boyle's law

The pressure of a given mass of a gas is inversely proportional to its Volume provided that the temperature remains constant

OR

$P \propto \frac{1}{V}$ for a given mass of the gas at constant temperature, where V & P are volume and pressure of the gas, respectively.

OR

$PV = constant$ for a given mass of the gas at constant temperature, where V & P are volume and pressure of the gas, respectively.

................**(02)**

Charles' law

The volume of a given mass of the gas is directly proportional to its absolute temperature, provided that the pressure remains constant.

OR

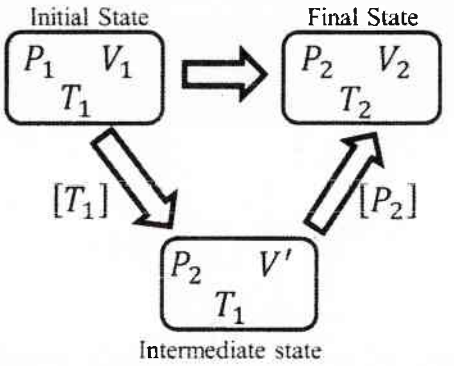
$V \propto T$ for a given mass of the gas under a constant pressure, where V & T are volume and absolute temperature of the gas, respectively.

OR

$\frac{V}{T}$ = constant for a given mass of the gas under a constant pressure, where V & T are volume and absolute temperature of the gas, respectively.

........................**(02)**

(ii) Consider one mole of a gas going through the following two stage processes with the initial and final values of the volume, pressure and absolute temperature are (V_1, P_1, T_1) and (V_2, P_2, T_2), respectively.



Applying Boyle's law for the constant temperature process

$P_1V_1 = P_2V'$ ----------------------------(A) ....**(01)**

Applying Charles' law for the constant pressure process

$\frac{V'}{T_1} = \frac{V_2}{T_2}$ ----------------------------(B) ....**(01)**

(A) & (B) $\Rightarrow$

$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \Rightarrow \frac{PV}{T} = \text{constant},$ ....**(01)**

For one mole of gas the constant is known as the universal gas constant R.

$\frac{PV}{T} = R$ for one mole of gas. ....(01)

If there are n mole of gas, $\frac{PV}{T} = nR$ ....(01)

$PV = nRT$

Alternative method

Q10 (A) (ii)

Consider one mole of gas of volume V under pressure P at absolute temperature T.

According to Boyle's law, for one mole of gas at absolute temperature T

$PV =$ constant ------------------(A) ....(01)

According to Charles' law, for one mole of gas at pressure P

$\frac{V}{T} =$ Constant -----------------(B) ....(01)

(A) & (B) $\Rightarrow \frac{PV}{T} =$ constant ....(01)

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If there are n mole of gas, $\frac{PV}{T} =$ nR ....(01)

$PV = nRT$

(b) (i) Let n_0 be the number of moles of the air in the type at pressure P_0 at T_R

$$n_0 = \frac{P_0V}{RT_R}$$ ....(02)

$$n = \frac{PV}{RT_R}$$ ....(02)

Number of moles of the air from the tank to the tyre

$$n' = n - n_0 = \frac{PV}{RT_R} - \frac{P_0V}{RT_R}$$ ....(01)

$$= \frac{V(P-P_0)}{RT_R}$$

$$= n\left(\frac{P-P_0}{P}\right)$$ ....(01)

$$= n\left(1-\frac{P_0}{P}\right)$$

(ii) Let V' be the volume of these n' moles of air in the tank under pressure P_C at temperature T_R

$$V' = \frac{n'RT_R}{P_C} = \left(1-\frac{P_0}{P}\right)\frac{nRT_R}{P_C}$$

As N_2 gas flows from the tank into the tyre through the valve, the tank does work at constant pressure P_C given $= P_C V'$ ....(01)

(Award this mark for identifying the work as $P\,\Delta V$)

$$= nRT_R\left(1-\frac{P_0}{P}\right)$$ ....**(No marks allocated)**

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Given $\Delta U = nC_V\,\Delta T$, $C_V = 5R/2$

$$\Rightarrow \Delta T = \frac{\Delta U}{nC_V}$$ ....(01)

$$= \frac{\left(nRT_R\left(1-\frac{P_0}{P}\right)\right)}{n\,5/2\,R}$$

$$= \frac{2}{5}\left(1-\frac{P_0}{P}\right)T_R$$ ....**(No marks allocated)**

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Change in pressure $\Delta P = \frac{2}{5}(P-P_0)$

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$$\frac{\Delta P}{\Delta T} = \frac{P}{T_R}$$ ....(01)

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Change $= \frac{(100-63)}{100} \times 100\%$

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