



Grade - 13 (2020)

Combined Maths I

Marking Scheme

1) For  $n=1$   
 L.H.S =  $\frac{1}{1 \cdot 2} = \frac{1}{2}$  R.H.S =  $\frac{1}{1+1} = \frac{1}{2}$   
 L.H.S = R.H.S  
 The result is true for  $n=1$  (5)

Take any  $P \in \mathbb{Z}^+$  and assume that the result is true for  $n=P$   

$$\sum_{r=1}^P \frac{1}{r(r+1)} = \frac{P}{P+1}$$
 (5)

Now  $n=P+1$   

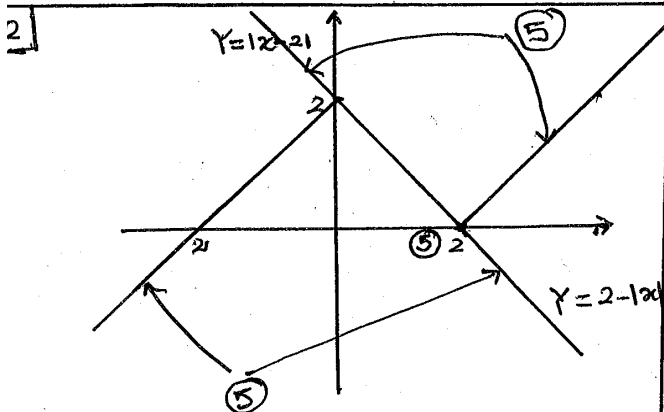
$$\sum_{r=1}^{P+1} \frac{1}{r(r+1)} = \frac{P}{P+1} + \frac{1}{(P+1)(P+2)}$$
 (5)

$$= \frac{P(P+2) + 1}{(P+1)(P+2)}$$

$$= \frac{(P+1)^2}{(P+1)(P+2)} = \frac{P+1}{P+2}$$
 (5)

hence if the result is true for  $n=P$  then it is also true for  $n=P+1$  hence by the principle of mathematical induction the result is true for all  $n \in \mathbb{Z}^+$  (5)

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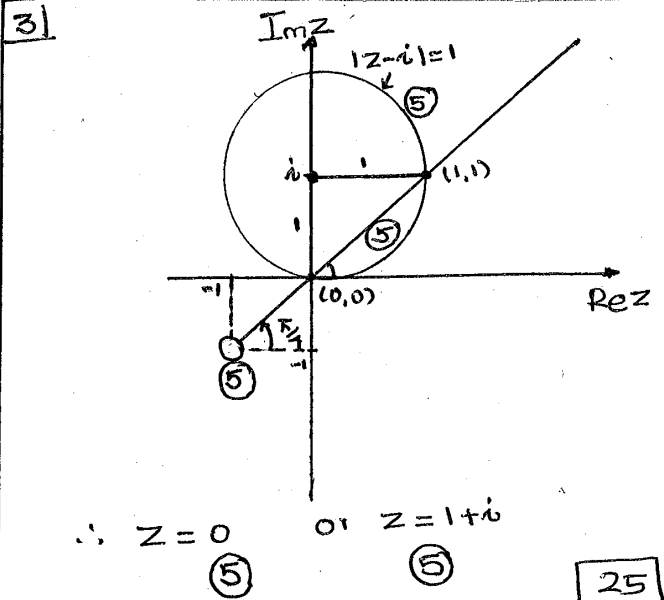


$$|x-2| = 2-|x| \iff 0 \leq x \leq 2$$
 (5)

$$|x-2| + |x| = 2 \iff 0 \leq x \leq 2$$
 (5)

$\therefore$  The solutions of  $x$  are  $0 \leq x \leq 2$  (5)

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4) 
$$\lim_{x \rightarrow 2} \frac{\cos \frac{\pi x}{4}}{\sqrt{x+3} - \sqrt{5}}$$
 (5)

$$= \lim_{x \rightarrow 2} \frac{\sin(\frac{\pi}{2} - \frac{\pi x}{4})}{(x+3) - 5} (\sqrt{x+3} + \sqrt{5})$$
 (5)
$$= \lim_{x \rightarrow 2} \frac{\sin \frac{\pi}{4} (2-x)}{(x-2) (\sqrt{x+3} + \sqrt{5})}$$
 (5)
$$= -\frac{\pi}{4} \lim_{x \rightarrow 2} \frac{\sin \frac{\pi}{4} (x-2)}{\frac{\pi}{4} (x-2)} \cdot \lim_{x \rightarrow 2} (\sqrt{x+3} + \sqrt{5})$$
 (5)
$$= -\frac{\pi}{4} \cdot 1 \cdot 2\sqrt{5}$$
 (5)
$$= -\frac{\sqrt{5}\pi}{2}$$
 (5)

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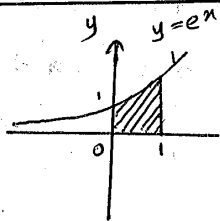
5.  $x = 2(1 - \cos\theta)$   
 $\frac{dx}{d\theta} = 2\sin\theta = 4\sin\frac{\theta}{2}\cos\frac{\theta}{2}$   
 $y = 2(\theta + \sin\theta)$   
 $\frac{dy}{d\theta} = 2(1 + \cos\theta) = 4\cos^2\frac{\theta}{2}$   
 $\frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx} = \frac{4\cos^2\frac{\theta}{2}}{4\sin\frac{\theta}{2}\cos\frac{\theta}{2}} = \cot\frac{\theta}{2}$   
 Point  $(4, 2\pi)$  on C  $\Rightarrow \theta = \pi$   
 $\left(\frac{dy}{dx}\right)_{\theta=\pi} = 0$   
 Equation of the tangent is  
 $y = 2\pi$

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6.  $\frac{d}{dx} \{x\sqrt{1-x^2}\}$   
 $= x \frac{1(-2x)}{2\sqrt{1-x^2}} + \sqrt{1-x^2}$   
 $= \frac{1-2x^2}{\sqrt{1-x^2}}$   
 $\int \frac{1+x^2}{\sqrt{1-x^2}} dx = -\frac{1}{2} \int \frac{1-2x^2}{\sqrt{1-x^2}} dx + \int \frac{\frac{3}{2}}{\sqrt{1-x^2}} dx$   
 $= -\frac{1}{2} x\sqrt{1-x^2} + \frac{3}{2} \sin^{-1}x + C$

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7. The volume generated  
 $= \int_0^1 \pi (e^x)^2 dx$   
 $= \int_0^1 \pi e^{2x} dx$   
 $= \pi \left[ \frac{e^{2x}}{2} \right]_0^1$   
 $= \frac{\pi}{2} (e^2 - 1)$



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8) The required equation can be written  
 $(3x - 4y + 1) + \lambda(5x + y - 1) = 0$   
 $(3+5\lambda)x + (\lambda-4)y + (1-\lambda) = 0$   
 $\lambda$  - parameter  
 But  $\left| \frac{\lambda-1}{\lambda-4} \right| = \left| \frac{\lambda-1}{3+5\lambda} \right|$   
 $\lambda-1=0$  or  $3+5\lambda = \pm(\lambda-4) \Rightarrow \lambda = \frac{-7}{4}$  or  $\frac{1}{6}$   
 The equations  
 $23x + 23y - 11 = 0$   
 $23x - 23y + 5 = 0$   
 $8x - 3y = 0$

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9)  $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$   
 $(0,0) \Rightarrow c = 0$   
 $(0,2) \Rightarrow 0 + 4 + 0 + 4f + c = 0 \Rightarrow f = -1$   
 $S \equiv x^2 + y^2 + 2gx - 2y = 0$   
 $S' \equiv x^2 + y^2 - 2x + 4y - 6 = 0$   
 Centre  $O' \equiv (1, -2)$   
 equation of common chord is  
 $S - S' = 0$   
 $(2g+2)x - 6y + 6 = 0$   
 $(1, -2) \Rightarrow 2(g+1) + 12 + 6 = 0 \Rightarrow g = -10$   
 $S \equiv x^2 + y^2 - 20x - 2y = 0$

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10)  $\tan^{-1}\left(\frac{x}{x-1}\right) + \tan^{-1}\left(\frac{x+2}{x+3}\right) = \tan^{-1}\left(\frac{2}{3}\right)$   
 $\downarrow \quad \downarrow \quad \downarrow$   
 $A \quad B \quad C$   
 $A+B=C \Rightarrow \tan(A+B) = \tan C$   
 $\frac{\tan A + \tan B}{1 - \tan A \tan B} = \tan C$   
 $\frac{\frac{x}{x-1} + \frac{x+2}{x+3}}{1 - \frac{x(x+2)}{(x-1)(x+3)}} = \frac{2}{3}$   
 $\frac{2x^2 + 4x - 2}{-3} = \frac{2}{3}$   
 $x^2 + 2x - 1 = -1$   
 $x = 0$  or  $x = -2$

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11 (a)

Suppose that 1 is a root of  $(p+1)x^2 + 8x + 2(p+1) = 0$ .

By substituting  $x=1$ , we must have  $(p+1)^2 + 8 + 2(p+1) = 0$  (5)

This is impossible, as  $p > -1$  implies that  $(p+1)^2 + 8 + 2(p+1) > 0$  (5)

$\therefore 1$  is not a root of (5)

$$(p+1)x^2 + 8x + 2(p+1) = 0$$

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The discriminant

$$\Delta = 8^2 - 4(p+1)^2(p+1) \quad (10)$$

$$= 8 \{ 8 - (p+1)^3 \} \geq 0 \quad (\because -1 \leq p \leq 1) \quad (5)$$

$\therefore \alpha$  and  $\beta$  are both real (5)

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$$\alpha + \beta = -\frac{8}{(p+1)^2}, \text{ and } \alpha\beta = \frac{2}{p+1} \quad (5)$$

$$\begin{aligned} \frac{1}{(\alpha-1)(\beta-1)} &= \frac{1}{\alpha\beta - (\alpha+\beta) + 1} \quad (5) \\ &= \frac{1}{\frac{2}{p+1} + \frac{8}{(p+1)^2} + 1} \\ &= \frac{(p+1)^2}{p^2 + 4p + 11} \quad (5) \end{aligned}$$

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$$\frac{\alpha}{\alpha-1} + \frac{\beta}{\beta-1} = \frac{2\alpha\beta - (\alpha+\beta)}{(\alpha-1)(\beta-1)} \quad (5)$$

$$\begin{aligned} &= \left( \frac{4}{p+1} + \frac{8}{(p+1)^2} \right) \cdot \frac{(p+1)^2}{p^2 + 4p + 11} \\ &= \frac{4(p+3)}{p^2 + 4p + 11} \quad (5) \end{aligned}$$

(5)

$$\frac{\alpha}{\alpha-1} \cdot \frac{\beta}{\beta-1}$$

$$= \frac{\alpha\beta}{(\alpha-1)(\beta-1)} = \frac{2}{p+1} \cdot \frac{(p+1)^2}{p^2 + 4p + 11} \quad (5)$$

$$= \frac{2(p+1)}{p^2 + 4p + 11} \quad (5)$$

Hence, the required quadratic equation is given by

$$x^2 - \frac{4(p+3)}{p^2 + 4p + 11}x + \frac{2(p+1)}{p^2 + 4p + 11} = 0 \quad (5)$$

$$(p^2 + 4p + 11)x^2 - 4(p+3)x + 2(p+1) = 0 \quad (30)$$

$$\frac{\alpha}{\alpha-1} + \frac{\beta}{\beta-1} = \frac{4(p+3)}{(p+2)^2 + 7} > 0 \quad (\because p > -1) \quad (5)$$

$$\frac{\alpha}{\alpha-1} \cdot \frac{\beta}{\beta-1} = \frac{2(p+1)}{(p+2)^2 + 7} > 0 \quad (\because p > -1) \quad (5)$$

Hence, both of these roots are positive (10)

(b)  $ax^n + b = (x^2 - 1)\phi(x) + x + 2$  (10)

$$x=1 \Rightarrow a+b=3 \quad (5)$$

$$x=-1 \Rightarrow -a+b=1 \quad (5)$$

$$a=1, b=2 \quad (5)$$

$$x^7 + 2 \equiv (x^2 - 1)\phi_1(x) + x + 2 \quad (5)$$

$$n=7 \Rightarrow x^7 + 2 \equiv (x^2 - 1)\phi_1(x) + x + 2 \quad (5)$$

$$n=5 \Rightarrow x^5 + 2 \equiv (x^2 - 1)\phi_2(x) + x + 2 \quad (5)$$

$$n=3 \Rightarrow x^3 + 2 \equiv (x^2 - 1)\phi_3(x) + x + 2 \quad (5)$$

$$\textcircled{1} + \textcircled{2} + \textcircled{3} \Rightarrow$$

$$x^7 + x^5 + x^3 + 6 \equiv (x^2 - 1)[\phi_1(x) + \phi_2(x) + \phi_3(x)] + 3x + 6$$

$$\therefore \text{Remainder} = 3x + 6 \quad (5)$$

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$$1 - \frac{1}{\alpha} = \frac{\alpha-1}{\alpha}, \quad 1 - \frac{1}{\beta} = \frac{\beta-1}{\beta} \quad (5)$$

Replacing  $x$  by  $\frac{1}{x}$  in (10), we get

$$(p^2 + 4p + 11)\left(\frac{1}{x}\right)^2 - 4(p+3)\frac{1}{x} + 2(p+1) = 0$$

$$2(p+1)x^2 - 4(p+3)x + p^2 + 4p + 11 = 0 \quad (5)$$

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(a)

$$(i) \quad {}^9C_3 = \frac{9!}{6! \times 3!} \quad (10)$$

$$= 84 \quad (5)$$

$$(ii) \quad {}^{18}C_3 - {}^9C_3 \quad (10)$$

$$= 816 - 84$$

$$= 732 \quad (5)$$

$$(iii) \quad {}^6C_1 {}^6C_1 {}^6C_1 = 216 \quad (5)$$

$$(iv) \quad {}^3C_1 {}^2C_1 {}^1C_1 \times 3 = 18 \quad (5)$$

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(b)

$$U_r = \frac{r+3}{r(r+1)(r+2)}, \quad V_r = \frac{2r+3}{r(r+1)}$$

$$V_r - V_{r+1}$$

$$= \frac{2r+3}{r(r+1)} - \frac{2r+5}{(r+1)(r+2)} \quad (5)$$

$$= \frac{(2r+3)(r+2) - (2r+5)r}{r(r+1)(r+2)} \quad (5)$$

$$= \frac{2r^2 + 7r + 6 - 2r^2 - 5r}{r(r+1)(r+2)}$$

$$= \frac{2(r+3)}{r(r+1)(r+2)} \quad (5)$$

$$= 2U_r \quad (5)$$

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$$2U_r = V_r - V_{r+1}$$

$$r=1; \quad 2U_1 = V_1 - V_2 \quad (5)$$

$$r=2; \quad 2U_2 = V_2 - V_3 \quad (5)$$

$$\vdots$$

$$r=n-1; \quad 2U_{n-1} = V_{n-1} - V_n \quad (5)$$

$$r=n; \quad 2U_n = V_n - V_{n+1}$$

$$\underline{2 \sum_{r=1}^n U_r = V_1 - V_{n+1}} \quad (5)$$

$$= \frac{5}{2} - \frac{2n+5}{(n+1)(n+2)} \quad (5)$$

$$\Rightarrow \sum_{r=1}^n U_r = \frac{5}{4} - \frac{2n+5}{2(n+1)(n+2)} \quad (5)$$

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$$\lim_{n \rightarrow \infty} \sum_{r=1}^n U_r = \lim_{n \rightarrow \infty} \left\{ \frac{5}{4} - \frac{2n+5}{2(n+1)(n+2)} \right\} \quad (5)$$

$$= \lim_{n \rightarrow \infty} \left\{ \frac{5}{4} - \frac{\frac{2}{n} + \frac{5}{n^2}}{2\left(1 + \frac{1}{n}\right)\left(1 + \frac{2}{n}\right)} \right\} \quad (5)$$

$$= \frac{5}{4} - 0$$

$$= \frac{5}{4} \quad (5)$$

$\therefore \sum_{r=1}^{\infty} U_r$  is convergent and the

$$\text{sum is } \frac{5}{4} \quad (5)$$

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$$\sum_{r=3}^{\infty} 3U_r$$

$$= 3 \sum_{r=1}^{\infty} U_r - 3U_1 - 3U_2 \quad (5)$$

$$= 3 \times \frac{5}{4} - 3\left(\frac{2}{3}\right) - 3\left(\frac{5}{24}\right) \quad (5)$$

$$= \frac{9}{8} // \quad (5)$$

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13  
 $z \in \mathbb{C} \quad z = x + iy \quad x, y \in \mathbb{R}$

a)  $|z| = \sqrt{x^2 + y^2}$   $\bar{z} = x - iy$  (5)

1.  $z \cdot \bar{z} = (x + iy)(x - iy) = x^2 - (iy)^2$  (5)  
 $= x^2 + y^2 = |z|^2$  (5)

$z + \bar{z} = x + iy + x - iy = 2x = 2\text{Re}z$  (5)

$z - \bar{z} = x + iy - x + iy = 2iy = 2i\text{Im}z$  (5)

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b)  $z = x + iy = \sqrt{x^2 + y^2} \left( \frac{x}{\sqrt{x^2 + y^2}} + i \frac{y}{\sqrt{x^2 + y^2}} \right)$  (5)  
 $= |z| (\cos\theta + i \sin\theta)$  (5)

$|z| = \sqrt{x^2 + y^2}$   $\text{Arg}(z) = \theta$  (5)

$\cos\theta = \frac{x}{\sqrt{x^2 + y^2}}$   $\sin\theta = \frac{y}{\sqrt{x^2 + y^2}}$

$z_1 = r_1 (\cos\theta_1 + i \sin\theta_1) \quad |z_1| = r_1$   
 $\text{Arg}(z_1) = \theta_1$

$z_2 = r_2 (\cos\theta_2 + i \sin\theta_2) \quad |z_2| = r_2$   
 $\text{Arg}(z_2) = \theta_2$

$z_1 \cdot z_2 = r_1 \cdot r_2 (\cos\theta_1 + i \sin\theta_1) (\cos\theta_2 + i \sin\theta_2)$  (5)  
 $= r_1 r_2 ((\cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2) + i (\sin\theta_1 \cos\theta_2 + \cos\theta_1 \sin\theta_2))$  (5)

$= r_1 r_2 (\cos(\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2))$  (5)

$|z_1 \cdot z_2| = r_1 r_2 = |z_1| |z_2|$  (5)

$\text{arg}(z_1 z_2) = \theta_1 + \theta_2 = \text{arg}(z_1) + \text{arg}(z_2)$  (5)

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c)  $z_1 = \frac{4}{1 - i\sqrt{3}}$   $z_2 = \frac{2}{1 + i}$

$z_1 = \frac{4(1 + i\sqrt{3})}{(1 - i\sqrt{3})(1 + i\sqrt{3})}$  (5)  $z_2 = \frac{2(1 - i)}{1 - i^2}$  (5)

$z_1 = (1 + i\sqrt{3})$   $z_2 = 1 - i$

$z_1 = 2 \left( \frac{1}{2} + i \frac{\sqrt{3}}{2} \right)$  (5)  $z_2 = \sqrt{2} \left( \frac{1}{\sqrt{2}} - i \frac{1}{\sqrt{2}} \right)$  (5)

$z_1 = 2 (\cos \frac{\pi}{3} + i \sin \frac{\pi}{3})$  (5)

$|z_1| = 2$   $z_2 = \sqrt{2} (\cos \frac{\pi}{4} + i \sin \frac{\pi}{4})$  (5)

$\text{Arg}(z_1) = \frac{\pi}{3}$   $|z_2| = \sqrt{2}$  (5)  
 $\text{Arg}(z_2) = \frac{\pi}{4}$

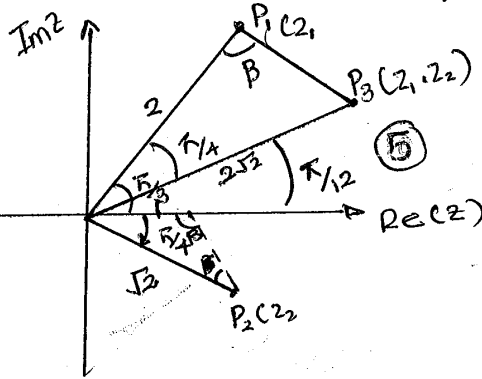
$|z_1 \cdot z_2| = |z_1| \cdot |z_2| = 2\sqrt{2}$  (5)

$\text{arg}(z_1 z_2) = \text{arg}(z_1) + \text{arg}(z_2)$   
 $= \frac{\pi}{3} + \frac{\pi}{4} = \frac{7\pi}{12}$  (5)

$\text{Arg}(z_1 z_2) = \frac{7\pi}{12}$

$z_1 \cdot z_2 = |z_1 z_2| (\cos(\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2))$

$= 2\sqrt{2} (\cos \frac{7\pi}{12} + i \sin \frac{7\pi}{12})$  (5)



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c)  $z_1 + iz_2 = 2^3 (\cos 3 \times \frac{\pi}{3} + i \sin 3 \times \frac{\pi}{3})$   
 $+ 2 \times (\sqrt{2})^4 (\cos \frac{\pi}{4} + i \sin \frac{\pi}{4})$  (5)

$= 2^3 [\cos \pi + i \sin \pi + \cos \frac{\pi}{4} + i \sin \frac{\pi}{4}]$  (5)

$= 2^3 \times 2 \cdot \cos \frac{\pi}{4}$  (5)

$= 16 \downarrow$  (5)

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14.  $f(x) = \frac{x+1}{(x+3)^2}$   
 $f'(x) = \frac{(x+3)^2(1) - (x+1)2(x+3)}{(x+3)^4}$  (10)  
 $= \frac{(x+3)[x+3-2x-2]}{(x+3)^4}$   
 $= -\frac{x-1}{(x+3)^3}$

$f''(x) = -\left\{ \frac{(x+3)^3(1) - (x-1)3(x+3)^2}{(x+3)^6} \right\}$  (10)  
 $= -\left\{ \frac{x+3-3x+3}{(x+3)^4} \right\}$   
 $= \frac{2(x-3)}{(x+3)^4}$  [20]

When  $x=0$ ,  $y = \frac{1}{9}$

When  $y=0$ ,  $x = -1$

$\lim_{x \rightarrow -3} f(x) = \infty$

Vertical asymptote:  $x = -3$  (5)

$\lim_{x \rightarrow \pm\infty} \frac{x+1}{(x+3)^2} = \lim_{x \rightarrow \pm\infty} \frac{\frac{1}{x} + \frac{1}{x^2}}{(1 + \frac{3}{x})^2} = 0$

Horizontal asymptote:  $y = 0$  (5)

When  $f'(x) = 0$ ,  $x = 1$  (5)

|             |              |             |
|-------------|--------------|-------------|
| $x < -3$    | $-3 < x < 1$ | $x > 1$     |
| $f'(x) < 0$ | $f'(x) > 0$  | $f'(x) < 0$ |
| decreasing  | increasing   | decreasing  |

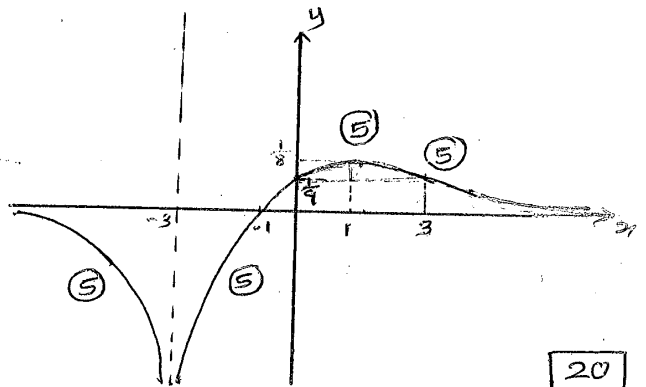
(1,  $\frac{1}{8}$ ) is a local maximum (5)

$f''(x) = 0 \Leftrightarrow x = 3$  (5)

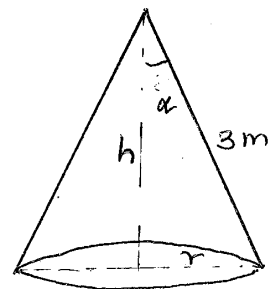
|                  |                  |                |
|------------------|------------------|----------------|
| $x < -3$         | $-3 < x < 3$     | $x > 3$        |
| $f''(x) < 0$     | $f''(x) < 0$     | $f''(x) > 0$   |
| concave down (5) | concave down (5) | concave up (5) |

(3,  $\frac{1}{9}$ ) is a point of inflection (5)

[60]



[20]



$h^2 + r^2 = 3^2$  (5)

$\Rightarrow r^2 = 9 - h^2$

Volume  $V = \frac{1}{3}\pi r^2 h$  (5)

$= \frac{1}{3}\pi h(9 - h^2)$  (5)

$\frac{dV}{dh} = \frac{1}{3}\pi(9 - 3h^2)$  (10)  
 $= -\pi(h^2 - 3)$

$\frac{dV}{dh} = 0 \Leftrightarrow h = \sqrt{3}$  [ $h > 0$ ] (5)

For  $0 < h < \sqrt{3}$ ,  $\frac{dV}{dh} > 0$  and  $h > \sqrt{3}$ ,  $\frac{dV}{dh} < 0$  (5)

$\therefore V$  is maximum when  $h = \sqrt{3}$  (5)

$h = \sqrt{3} \Rightarrow r = \sqrt{6}$

$\tan \alpha = \frac{r}{h} = \frac{\sqrt{6}}{\sqrt{3}} = \sqrt{2}$

$\therefore V$  is maximum when  $\alpha = \tan^{-1}(\sqrt{2})$  (5)

[50]

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$$\text{I) } \int \frac{d \ln(x^2+1)}{dx} = \frac{1}{(x^2+1)} \times 2x = \frac{2x}{(x^2+1)}$$

Integrating both side w.r.t x

$$\int \frac{2x}{x^2+1} = \ln(x^2+1) + C$$

$$\int \frac{x}{x^2+1} = \frac{\ln(x^2+1)}{2} + C$$

C - Arbitrary constant

$$\text{II) } \frac{x^3+4x^2-4x+4}{(x^2+1)(x^2-4)} = \frac{Ax+B}{(x^2+1)} + \frac{C}{(x-2)} + \frac{D}{(x+2)}$$

$$x^3+4x^2-4x+4 = (Ax+B)(x^2-4) + C(x^2+1) + D(x^2-4)$$

$$x=2 \quad 20C = 20 \Rightarrow C=1$$

$$x=-2 \quad -20D = 20 \Rightarrow D=-1$$

$$\text{com } 2C - 2D - 4B = 4 \Rightarrow B=0$$

$$x^3 \quad A+C+D = 1 \Rightarrow A=1$$

$$\frac{x^3+4x^2-4x+4}{(x^2+1)(x^2-4)} = \frac{x}{x^2+1} + \frac{1}{x-2} + \frac{-1}{x+2}$$

Integrating both side w.r.t x

$$\int \frac{x^3+4x^2-4x+4}{(x^2+1)(x^2-4)} dx = \int \frac{x dx}{x^2+1} + \int \frac{dx}{x-2} - \int \frac{dx}{x+2}$$

$$= \frac{\ln(x^2+1)}{2} + \ln|x-2| - \ln|x+2| + K$$

K - arbitrary constant

$$\text{b) } \int x \sin^2 x dx = \int \frac{x(1-\cos 2x)}{2} dx$$

$$= \int \frac{x}{2} dx - \int \frac{x \cos 2x}{2} dx$$

$$= \frac{x^2}{4} - \frac{x \sin 2x}{4} + \int \frac{\sin 2x}{4} dx + C$$

$$= \frac{x^2}{4} - \frac{x \sin 2x}{4} - \frac{\cos 2x}{8} + C$$

C - arbitrary constant

$$\int x \cos^2 x dx + \int x \sin^2 x dx = \int x (\sin^2 x + \cos^2 x)$$

$$\Rightarrow \int x \cos^2 x dx = \int x dx - \int x \sin^2 x dx$$

$$\Rightarrow \int x \cos^2 x dx = \frac{x^2}{2} - \left( \frac{x^2}{4} - \frac{x \sin 2x}{4} - \frac{\cos 2x}{8} \right) + C$$

$$= \frac{x^2}{4} + \frac{x \sin 2x}{4} + \frac{\cos 2x}{8} + C$$

$$\text{C) } x = 1 + 3 \sin^2 \theta$$

$$x: 1 \rightarrow 4 \quad x=1 \Leftrightarrow \sin^2 \theta = 0$$

$$\theta = 0$$

$$\theta: 0 \rightarrow \frac{\pi}{2} \quad x=4 \Leftrightarrow \sin^2 \theta = 1$$

$$\theta = \frac{\pi}{2}$$

$$dx = 6 \sin \theta \cos \theta d\theta \quad (0 \leq \theta \leq \frac{\pi}{2})$$

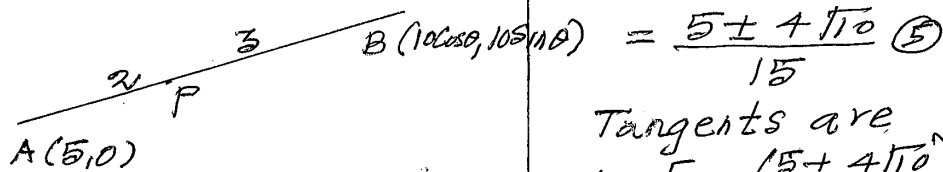
$$\int_1^4 \frac{dx}{\sqrt{(x-1)(4-x)}} = \int_0^{\frac{\pi}{2}} \frac{6 \sin \theta \cos \theta d\theta}{\sqrt{3 \sin^2 \theta \cdot 3 \cos^2 \theta}}$$

$$= \int_0^{\frac{\pi}{2}} \frac{6 \sin \theta \cos \theta d\theta}{3 \sin \theta \cos \theta} = 2 \int_0^{\frac{\pi}{2}} d\theta$$

$$= 2 \theta \Big|_0^{\frac{\pi}{2}}$$

$$= 2 \left( \frac{\pi}{2} - 0 \right) = \pi$$

Q16]



$$P = \left( \frac{20\cos\theta + 5}{5}, \frac{20\sin\theta + 0}{5} \right) \quad (10)$$

$$P = (4\cos\theta + 3, 4\sin\theta)$$

$$P = (\bar{x}, \bar{y}) \text{ (say)}$$

$$\bar{x} = 4\cos\theta + 3 \quad \bar{y} = 4\sin\theta \quad (5)$$

$$\text{but } \cos^2\theta + \sin^2\theta = 1$$

$$\left( \frac{\bar{x}-3}{4} \right)^2 + \left( \frac{\bar{y}}{4} \right)^2 = 1 \quad (5)$$

$$\bar{x}^2 + \bar{y}^2 - 6\bar{x} - 7 = 0 \quad (5)$$

put  $\bar{x} \rightarrow x, \bar{y} \rightarrow y$

$$S \equiv x^2 + y^2 - 6x - 7 = 0 \quad (5)$$

$$(x-3)^2 + (y-0)^2 = 4^2$$

The path is circle  
centre  $C = (3, 0)$

$$\text{radius} = 4 \quad (5)$$

$$Q \equiv (2, 5)$$

$$CQ = \sqrt{(3-2)^2 + (0-5)^2} \quad (5)$$

$$= \sqrt{26} > 4 \quad (5)$$

The point Q lies outside the circle

The equation of tangent is given by

$$y - 5 = m(x - 2) \quad (5)$$

$$mx - y + (5 - 2m) = 0$$

$$4 = \frac{|m \times 3 - 0 + 5 - 2m|}{\sqrt{m^2 + (-1)^2}} \quad (10)$$

$$16(m^2 + 1) = (m + 5)^2$$

$$15m^2 - 10m + 9 = 0 \quad (5)$$

$$m = \frac{10 \pm \sqrt{100 - 4(15)(-9)}}{2 \times 15} \quad (10)$$

$$= \frac{5 \pm 4\sqrt{10}}{15} \quad (5)$$

Tangents are

$$y - 5 = \left( \frac{5 \pm 4\sqrt{10}}{15} \right) (x - 2) \quad (10) \quad (25)$$

CD

$$2x + 5y - 3(2+x) - 7 = 0$$

$$x - 5y + 13 = 0 \quad (10)$$

S' can be write

$$x^2 + y^2 - 6x - 7 + \lambda(x - 5y + 13) = 0 \quad (5)$$

$$(1, 2) \Rightarrow$$

$$1 + 4 - 6 - 7 + \lambda(1 - 10 + 13) = 0$$

$$\lambda = 2 \quad (5)$$

$$S' \equiv x^2 + y^2 - 4x - 10y + 19 = 0 \quad (5) \quad (25)$$

$$S'' \equiv x^2 + y^2 + 2g''x + 2f''y + c'' = 0 \text{ (say)}$$

$$(1, 6) \Rightarrow 0 + 36 + 12f'' + c'' = 0 \quad (5)$$

$$c'' = -36 - 12f'' \quad (1)$$

The circles  $S' = 0, S'' = 0$  are intersect orthogonal

$$2\{g''(-2) + (f'')(-5)\} = c'' + 19 \quad (10)$$

$$-4g'' - 10f'' = -36 - 12f'' + 19 \quad (\text{by (1)})$$

$$4g'' - 2f'' - 17 = 0 \quad (5)$$

$$\text{put } -g'' = x, -f'' = y \quad (5)$$

$$4x - 2y + 17 = 0 \quad (5)$$

30



Q17]

a)  $\sin(A+B) = \sin A \cos B + \cos A \sin B$  (1)

$\sin(A-B) = \sin(A+(-B))$  (2)

$= \sin A \cos(-B) + \cos A \sin(-B)$

$= \sin A \cos B - \cos A \sin B$  (3)

(1)+(2)  $\Rightarrow$

$\sin(A+B) + \sin(A-B) = 2 \sin A \cos B$  (4)

put  $B \rightarrow A$

$\sin(2A) + \sin(A-A) = 2 \sin A \cos A$  (5)

$\sin 2A = 2 \sin A \cos A$  (6)

$\sin \theta \{ 8 \cos \theta \cos 2\theta \cos 3\theta - 1 \}$

$= 8 \sin \theta \cos \theta \cos 2\theta \cos 3\theta - \sin \theta$  (7)

$= 4 \sin 2\theta \cos 2\theta \cos 3\theta - \sin \theta$  (8)

$= 2 \sin 4\theta \cos 3\theta - \sin \theta$  (9)

$= \sin 7\theta + \sin \theta - \sin \theta$  (10)

$= \sin 7\theta$

$\cos \theta \cos 2\theta \cos 3\theta = \frac{1}{4}$

$\frac{1}{8} \left\{ \frac{\sin 7\theta}{\sin \theta} + 1 \right\} = \frac{1}{4}$  (11)

$\sin 7\theta = \sin \theta$

$7\theta = n\pi + (-1)^n \theta$  (12);  $n \in \mathbb{Z}$

$n=0 \Rightarrow \theta = 0$

$n=1 \Rightarrow \theta = \frac{\pi}{8}$

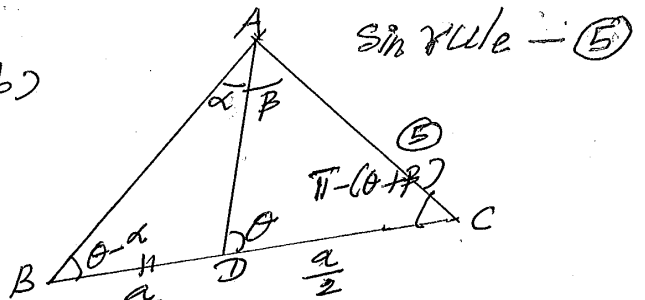
$n=2 \Rightarrow \theta = \frac{\pi}{4}$

$n=3 \Rightarrow \theta = \frac{3\pi}{8}$

$n=4 \Rightarrow \theta = \frac{2\pi}{3} > \frac{\pi}{2}$

Sol<sup>n</sup>:  $\left\{ \frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8} \right\}$  (13)

b)



for the triangle ABD

$\frac{BD}{\sin \alpha} = \frac{AD}{\sin(\theta-\alpha)}$  (14)

$\frac{\frac{a}{2}}{\sin \alpha} = \frac{AD}{\sin(\theta-\alpha)}$

$\frac{a}{2 \sin \alpha} = \frac{AD}{\sin(\theta-\alpha)}$  (15)

In  $\triangle ADC$   
 $\frac{AD}{\sin(\pi-(\theta+\beta))} = \frac{a}{2 \sin \beta}$  (16)

$\frac{AD}{\sin(\theta+\beta)} = \frac{a}{2 \sin \beta}$  (17)

(15)/(17)  $\Rightarrow \frac{a \sin(\theta+\beta)}{2 \sin \alpha} = \frac{a \sin \beta}{2 \sin \beta}$  (18)

$\sin \beta \{ \sin \alpha \cos \alpha - \cos \alpha \sin \alpha \} = \sin \alpha$  (19)

divide by  $\sin \alpha \sin \beta \sin \alpha$  both sides

$\cot \alpha - \cot \theta = \cot \beta + \cot \theta$  (20)

$\cot \alpha - \cot \beta = 2 \cot \theta$  (21)

c)  $2 \tan^{-1}(\frac{1}{5}) + \tan^{-1}(\frac{6}{5}) = \frac{\pi}{2}$  (22)

$\alpha = \tan^{-1}(\frac{1}{5})$   $\beta = \tan^{-1}(\frac{6}{5})$  (23)

$\Leftrightarrow 2\alpha + \beta = \frac{\pi}{2}$  (24)

$\Leftrightarrow 2\alpha = \frac{\pi}{2} - \beta$  (25)

$\Leftrightarrow \tan 2\alpha = \tan(\frac{\pi}{2} - \beta)$  (26)  
( $2\alpha, (\frac{\pi}{2} - \beta)$  are acute)

$\tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha}$  (27)

$= \frac{2(\frac{1}{5})}{1 - \frac{1}{25}}$   $\tan(\frac{\pi}{2} - \beta) = \cot \beta$  (28)

$= \frac{10}{24}$   $= \frac{5}{6}$   $\therefore 2\alpha + \beta = \frac{\pi}{2}$  (29)

$2 \tan^{-1}(\frac{1}{5}) + \tan^{-1}(\frac{6}{5}) = \frac{\pi}{2}$

$\tan^{-1}(\frac{1}{5}) = \frac{\pi}{4} - \frac{1}{2} \tan^{-1}(\frac{6}{5})$  (30)

$\sin^{-1}(\frac{1}{\sqrt{26}}) = \frac{\pi}{4} - \frac{1}{2} \tan^{-1}(\frac{6}{5})$  (31)  
 $\frac{1}{\sqrt{26}} = \sin \left\{ \frac{\pi}{4} - \frac{1}{2} \tan^{-1}(\frac{6}{5}) \right\}$  (32)



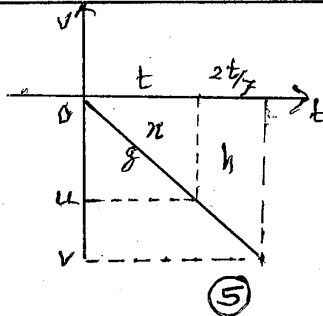
Grade - 13 (2020)

Combined Maths II

Marking Scheme

①

$$\begin{aligned}
 t &= \frac{u}{g} \\
 t + 2t &= \frac{u}{g} \\
 h &= \frac{1}{2}(u+v) \cdot \frac{2t}{g} \quad (5) \\
 &= \frac{t}{g} [gt + \frac{9gt}{g}] \\
 &= \frac{16gt^2}{49} \\
 t &= \frac{7}{4} \sqrt{\frac{h}{g}} \quad (5) \\
 AC &= h + x \\
 &= h + \frac{1}{2}gt^2 \quad (5) \\
 &= h + \frac{1}{2}g \left(\frac{49}{16} \frac{h}{g}\right) \\
 &= \frac{81h}{32} \quad (5)
 \end{aligned}$$



25

③

$$\begin{aligned}
 &\rightarrow u \leftarrow 3u \\
 &P \quad 2m \quad Km \quad Q \quad R \quad I \\
 &\leftarrow \frac{u}{2} \quad \rightarrow v \\
 \text{for } P &\rightarrow I = \Delta mu \\
 &-I = 2m \left(-\frac{u}{2}\right) - 2mu \\
 &I = 3mu \\
 \text{for } Q &\rightarrow I = kmv - km(-3u) \\
 3mu &= kmv + 3kmv \quad (5) \\
 v &= \frac{3u}{k} (1-k) \quad (5) \\
 v > 0 &\Rightarrow \frac{1}{k}(1-k) > 0 \quad k > 0 \\
 &\quad k < 1 \quad + k > 0 \\
 &\Rightarrow 0 < k < 1. \quad (5) \\
 \text{N.R.L } v + \frac{u}{2} &= e(4u + 3u) \quad (5) \\
 k = \frac{1}{2} &\Rightarrow v = 3u \\
 \therefore e &= \frac{7}{8} \quad (5)
 \end{aligned}$$

25

②

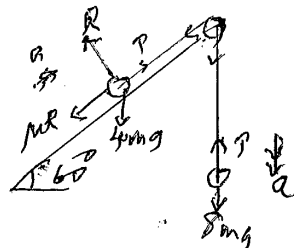
$$\begin{aligned}
 O \rightarrow P \text{ s.u.t} + \frac{1}{2}at^2 \\
 \rightarrow x = u \cos \theta \cdot t \quad (1) \\
 \uparrow y = u \sin \theta \cdot t - \frac{1}{2}gt^2 \quad (2) \\
 \text{H.O.E } y &= x \tan \theta - \frac{1}{2}gt^2 \quad (u \sin^2 \theta) \quad (3) \\
 50 &= x \cdot 2 - \frac{9.8x^2}{2 \cdot 35^2} \quad (5) \\
 x^2 - 100x + 2500 &= 0 \quad (5) \\
 (x-50)^2 &= 0 \\
 x &= 50 \quad (5) \\
 \Rightarrow t &= \frac{10\sqrt{5}}{7} \quad (5)
 \end{aligned}$$

④

$$\begin{aligned}
 R \leftarrow \square \rightarrow F \\
 36 \times 1600 &= F \times 20 \quad (5) \\
 F &= 1800 \text{ N} \\
 \rightarrow F - R &= 3200 \text{ a} \quad (5) \\
 1800 - R &= 3200 \times 0.2 \\
 R &= 1160 \text{ N} \quad (5) \\
 \rightarrow F' - R - 3200g \sin 30^\circ &= 0 \quad (5) \\
 F' &= 17160 \text{ N} \\
 P &= 17160 \times 30 \\
 &= 514800 \text{ W} \quad (5) \\
 &= 514.8 \text{ kW}
 \end{aligned}$$

25

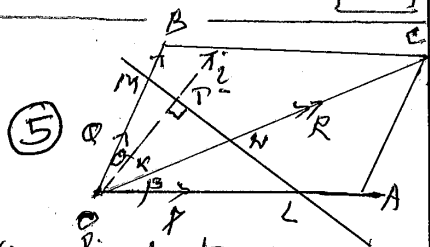
5



$R = 4mg \cos 60 = 2mg$  (5)  
 $\rightarrow T = \mu R - 4mg \sin 60 = 4ma$  (5)  
 $\downarrow 8mg - T = 8ma$  (5)  
 $a = \frac{g}{24} (15 - 4\sqrt{3})$  (5)  
 $T = \frac{g}{3} (9 + 4\sqrt{3})$  (5)

25

6

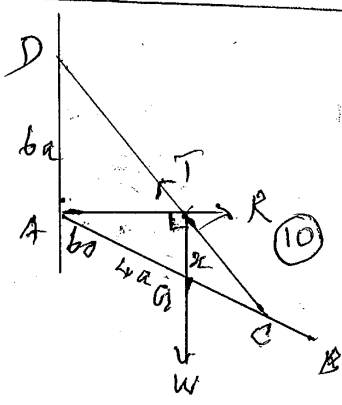


$OT$  is a perpendicular to  $LMN$   
 let  $\underline{i}$  is a unit vector in the direction of  $\vec{OT}$   
 $\underline{p} + \underline{q} = \underline{r}$   
 $\underline{p} \cdot \underline{i} + \underline{q} \cdot \underline{i} = \underline{r} \cdot \underline{i}$   
 $|\underline{p}| \cdot 1 \cdot \cos(\alpha + \beta) + |\underline{q}| \cdot 1 \cdot \cos \alpha = |\underline{r}| \cdot 1 \cdot \cos \alpha$  (10)  
 $\underline{p} \cdot \frac{\vec{OT}}{OL} + \underline{q} \cdot \frac{\vec{OT}}{OM} = \underline{r} \cdot \frac{\vec{OT}}{ON}$  (5)  
 $\Rightarrow \frac{\underline{p}}{OL} + \frac{\underline{q}}{OM} = \frac{\underline{r}}{ON}$  (5)

25

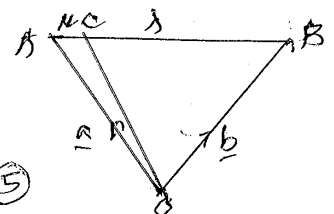
7

$x = 2a$   
 $\frac{AC}{CG} = \frac{AD}{x} = \frac{6a}{2a}$  (10)  
 $\therefore CG = 2a$   
 $\therefore BC = 2a$  (5)



25

8

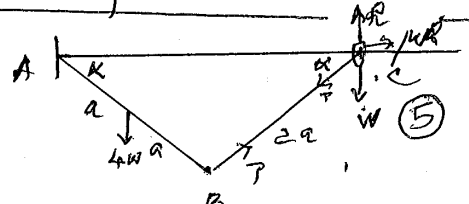


$\frac{AC}{CB} = \frac{\mu}{\lambda}$   
 $\lambda \vec{AC} = \mu \vec{CB}$  (5)  
 $\lambda(\underline{c} - \underline{a}) = \mu(\underline{b} - \underline{c})$  (5)  
 $(\lambda + \mu)\underline{c} = \lambda \underline{a} + \mu \underline{b}$   
 $\underline{c} = \frac{\lambda \underline{a} + \mu \underline{b}}{\lambda + \mu}$  (5)  
 $\alpha \underline{a} + \beta \underline{b} + \gamma \underline{c} = \underline{0}$  and  $\alpha + \beta + \gamma = 0$   
 $\alpha \underline{a} + \beta \underline{b} = -\gamma \underline{c}$  and  $\alpha + \beta = -\gamma$   
 $\therefore \frac{\alpha \underline{a} + \beta \underline{b}}{\alpha + \beta} = \frac{-\gamma \underline{c}}{-\gamma}$  (5)  
 $\Rightarrow \frac{\alpha \underline{a} + \beta \underline{b}}{\alpha + \beta} = \underline{c}$  (5)

25

from the 1st part A, B, C are collinear.

9



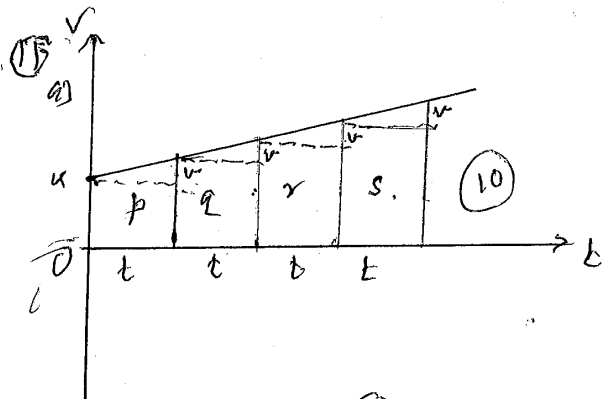
for Ring  $\rightarrow T \cos \alpha - \mu R \cos \alpha = 0$  (5)  
 $\mu R = T$  (5)  
 for Rod & Ring A  
 $4a \cos \alpha (R - W) - 4wa \cos \alpha = 0$  (5)  
 $R = 2W$   
 $\frac{T}{\mu} = \tan \alpha = \frac{R - W}{\mu R} = \frac{1}{2\mu}$  (5)  
 $\tan \alpha = \frac{1}{2\mu}$  (5)

25

$P(A \cap B') = \frac{8}{25}, P(A' \cap B) = \frac{11}{100}$

$P(A \cup B) = \frac{13}{20}$   
 $P(A \cup B) = P(A \cup B') + P(A' \cap B) + P(A \cap B)$  (5)  
 $\frac{13}{20} = \frac{8}{25} + \frac{11}{100} + P(A \cap B)$   
 $\therefore P(A \cap B) = \frac{22}{100}$  (5)  
 $P(A) = P(A \cap B) + P(A \cap B')$   
 $= \frac{22}{100} + \frac{8}{25} = \frac{54}{100}$  (5)  
 $P(B) = P(A \cap B) + P(A' \cap B) = \frac{33}{100}$  (5)  
 $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{2}{3}$  (5)

25



$$p = \frac{1}{2}(2u+v)t \quad (10)$$

$$q = \frac{1}{2}(2u+3v)t \quad (10)$$

$$r = \frac{1}{2}(2u+5v)t \quad (10)$$

$$\Rightarrow p+r=2q$$

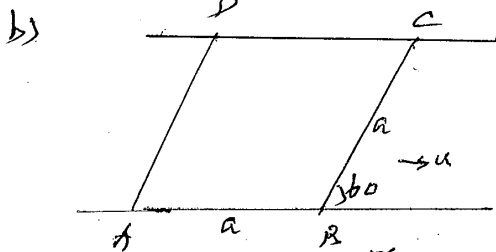
$$s = \frac{t}{2}(2u+7v) \quad (5)$$

$$2r-q = \frac{t}{2}[2(2u+5v)-(2u+3v)] \quad (10)$$

$$= \frac{t}{2}[2u+7v]$$

$$\Rightarrow s = 2r - q \quad (5)$$

60

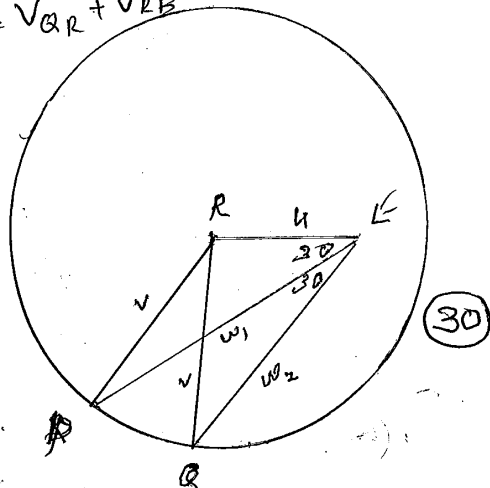


$$V_{RE} \rightarrow u \quad V_{PE} \rightarrow 30^\circ \quad V_{PR} \quad (V)$$

$$V_{RE} \rightarrow 60^\circ \quad (5) \quad V_{QR} \quad (V)$$

$$V_{PE} = V_{PR} + V_{RE} \quad (5)$$

$$V_{QE} = V_{QR} + V_{RE}$$



30

$$V_{PE} = w_1 = u \cos 30 + \sqrt{v^2 - u^2 \sin^2 30} \quad (10)$$

$$= \frac{u\sqrt{3}}{2} + \frac{\sqrt{4v^2 - u^2}}{2}$$

$$= \frac{1}{2} [\sqrt{3}u + \sqrt{4v^2 - u^2}] \quad (5)$$

$$V_{QE} = w_2 = u \cos 60 + \sqrt{v^2 - u^2 \sin^2 60} \quad (10)$$

$$= \frac{u}{2} + \frac{\sqrt{4v^2 - 3u^2}}{2}$$

$$= \frac{1}{2} [u + \sqrt{4v^2 - 3u^2}] \quad (5)$$

$$t_{AC} = t_{AD}$$

$$\frac{u \cos 30}{w_1} = \frac{2a}{w_2} \quad (10)$$

$$\sqrt{3}w_2 = w_1$$

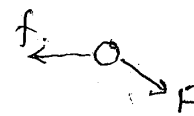
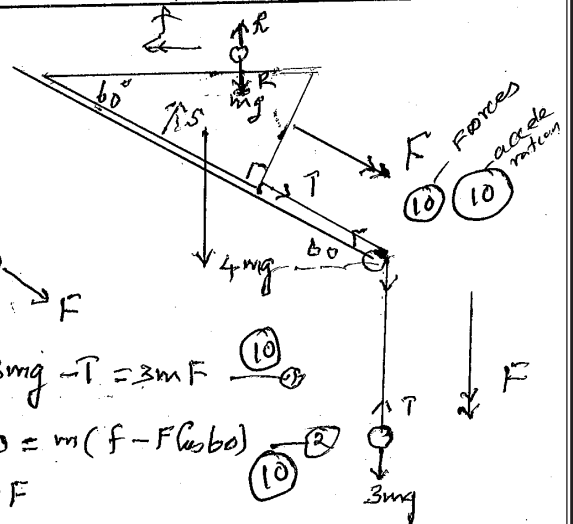
$$\frac{\sqrt{3}}{2} (u + \sqrt{4v^2 - 3u^2}) = (\sqrt{3}u + \sqrt{4v^2 - u^2}) \frac{1}{2} \quad (5)$$

$$\sqrt{3} \sqrt{4v^2 - 3u^2} = \sqrt{3}u + \sqrt{4v^2 - u^2}$$

90

$$v = u$$

12



$$\downarrow 3mg - T = 3mF \quad (10)$$

$$p \leftarrow 0 = m(f - F \cos 60) \quad (10)$$

$$2f = F$$

for Wedge at P

$$\rightarrow T + 5mg \sin 60 = 4mF + m(F - f \cos 60) \quad (15)$$

$$\Rightarrow (3 + 5\sqrt{3})g = 8F - \frac{f}{2}$$

$$\frac{b + 5\sqrt{3}}{2}g = \frac{31}{2}f$$

$$f = \frac{b + 5\sqrt{3}}{31}g \quad (5)$$

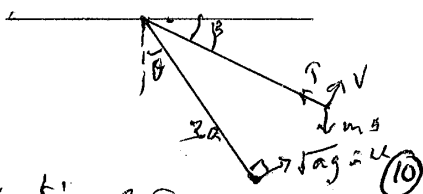
$$\text{for } P \leftarrow s = ut + \frac{1}{2}at^2 \quad (10)$$

$$2l = 0 + \frac{1}{2} \left( \frac{b + 5\sqrt{3}}{31}g \right) t^2 \quad (10)$$

$$t = \sqrt{\frac{124l}{b + 5\sqrt{3}}} \quad (5)$$

75

12b)



Using conservation of Energy.

$$\frac{1}{2} m u^2 - mg \cdot 2a \cos \theta = \frac{1}{2} m v^2 - mg \cdot 2a \sin \theta \quad (15)$$

$$v^2 = 2ag(2 \sin \theta - \cos \theta) \quad (5)$$

$$T - mg \sin \theta = m \frac{v^2}{2a} \quad (10)$$

$$T = mg(3 \sin \theta - 1) \quad (10)$$

$$v = 0 \Rightarrow \theta = \pi/6 \quad (5)$$

$$\Rightarrow T > 0$$

$$\therefore T_{\min} = \frac{mg}{2} \quad (10) \quad \theta = \frac{\pi}{6}$$

$$T_{\max} = 2mg \quad (10) \quad \theta = \frac{\pi}{2}$$

75

$$(13)(i) T_0 = 4mg \quad (5)$$

$$\frac{\lambda \times 2a}{3a} = 4mg \quad (5)$$

$$\lambda = 6mg \quad (5)$$

15

(ii)

at P

$$4mg - T = 4m \ddot{x} \quad (10)$$

$$4mg - \frac{6mg(2a+x)}{3a} = 4m \ddot{x} \quad (10)$$

$$\ddot{x} = -\frac{g}{3} x \quad (5)$$

35

$$(iv) x=0 \text{ is centre } \left\{ \begin{array}{l} 2a \\ 2a \end{array} \right.$$

$$x = A \cos \omega t + B \sin \omega t$$

$$(5) \dot{x} = -A\omega \sin \omega t + B\omega \cos \omega t$$

$$(5) \ddot{x} = -\omega^2(A \cos \omega t + B \sin \omega t)$$

$$c = -\omega^2 x$$

$$\therefore \omega = \sqrt{\frac{g}{2a}} \quad (5)$$

$$t=0, x=0, \dot{x}=u \quad (5)$$

$$\Rightarrow A=0 \quad (5)$$

$$u = B\omega \Rightarrow B = \frac{u}{\omega} = u \sqrt{\frac{2a}{g}} \quad (5)$$

35

$$(5) \therefore x = \frac{u}{\omega} \sin \omega t \Rightarrow \dot{x} = u \cos \omega t \quad (5)$$

$$x = 3a, \dot{x} = 0, \text{ at } P \Rightarrow 0 = u \cos \omega t \quad (5)$$

$$\therefore 3a = \frac{u}{\omega} \sin \frac{\pi}{2} \quad (5)$$

$$u = 3a\omega = 3\sqrt{\frac{2ag}{2}} \quad (5)$$

$$\text{amplitude is } 3a \quad (5)$$

40

$$t_{BA} = \frac{\pi - \theta}{\omega} \quad (5)$$

$$\cos \theta = \frac{2}{3}$$

$$t_{CB} = \frac{\pi}{2\omega} \quad (5)$$

$$\text{Total time} = \frac{\pi}{2\omega} + \frac{\pi - \theta}{\omega} \quad (5)$$

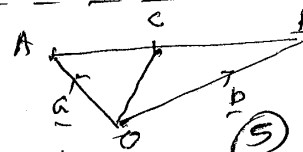
$$= \frac{1}{\omega} \left( \frac{\pi}{2} + \pi - \theta \right)$$

$$= \frac{1}{\omega} \left( \frac{3\pi}{2} - \theta \right)$$

$$= \sqrt{\frac{2a}{g}} \left[ \frac{3\pi}{2} - \cos^{-1} \frac{2}{3} \right] \quad (10)$$

25

(14) (a)



$$\vec{OC} = \vec{OA} + \vec{AC}$$

$$c = a + \lambda(A\vec{B}) \quad (10)$$

$$= a + \lambda(b-a) \quad (5)$$

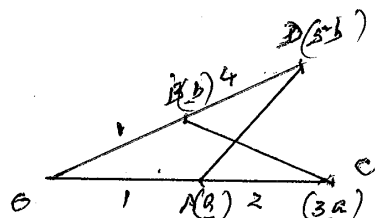
$$= (1-\lambda)a + \lambda b$$

$$\text{let } 1-\lambda = \alpha$$

$$\therefore \lambda = 1-\alpha$$

$$c = \alpha a + (1-\alpha)b \quad (5)$$

25



Let M is on OA and N is on BC from the first part  $m = \lambda a + (1-\lambda)b \quad (10)$

$$n = \mu 3a + (1-\mu)b \quad (10)$$

where  $\lambda, \mu$  are parameters to find the intersecting point

$$m = n \quad (5)$$

$$\Rightarrow \lambda a + (1-\lambda)b = \mu 3a + (1-\mu)b \quad (5)$$

$$(\lambda - 3\mu)a + [5(1-\lambda) - (1-\mu)]b = 0$$

$$\Rightarrow \lambda - 3\mu = 0 \quad \& \quad 5\lambda - \mu = 4 \quad (5)$$

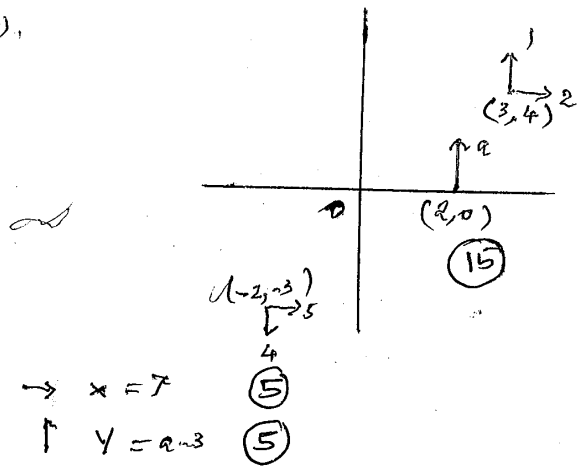
$$\mu = \frac{2}{7}, \lambda = \frac{6}{7} \quad (5)$$

$$m = \frac{6}{7}a + \frac{5}{7}b \quad (5)$$

50

M divides AD in 1:6

14b)



$a \cdot 2 + 1 \cdot 3 - 2 \cdot 4 + 5 \cdot 3 + 4 \cdot 2 = \pm 24$  (10)

$2a + 18 = \pm 24$

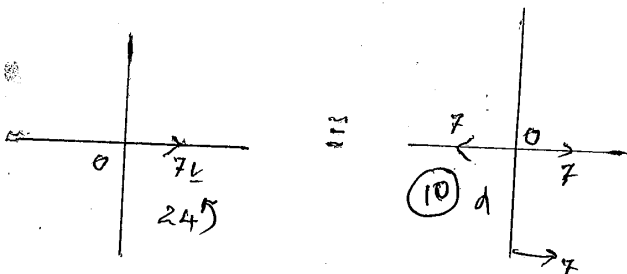
$2a = 6, -42$

$a = 3, -21$  (5)

$a = 3 \Rightarrow F = 7i$  (5)

$a = -21 \Rightarrow F = 7i - 24j$  (5)

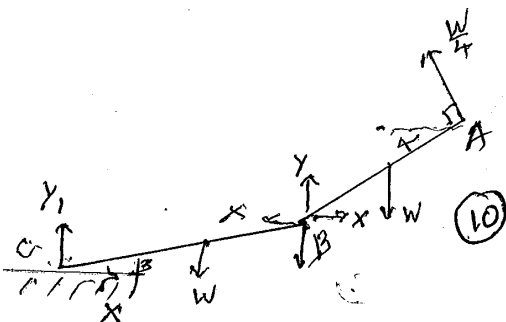
$a = 3 \Rightarrow G = 24j$  (5)



$d = \frac{G}{F} = \frac{24}{7}$

Line Eq<sup>n</sup> of the line of action of the equivalent force is  $y = -\frac{24}{7}x$  (5)

(15) (10)



for AB  $\frac{W}{4} \cdot 2a - W \cdot a \sin \alpha = 0$  (10)

$\cos \alpha = \frac{1}{2}$   
 $\alpha = \frac{\pi}{3}$  (5)

$\rightarrow X = \frac{W}{4} \cos 30 = \frac{\sqrt{3}W}{8}$  (5)

$\uparrow Y = W - T \sin 30$  (5)  
 $= W - \frac{W}{4} \cdot \frac{1}{2}$  (5)  
 $= \frac{7W}{8}$  (5)

for BC

$X \cdot 2a \sin \beta + Y \cdot 2a \cos \beta - W a \sin \beta = 0$  (10)

$\frac{\sqrt{3}W}{8} \cdot 2a \sin \beta - \frac{7W}{8} \cdot 2a \cos \beta - W a \sin \beta = 0$  (5)

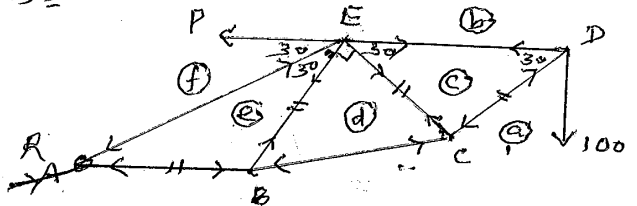
$\sqrt{3} \tan \beta = 11$

$\tan \beta = \frac{11}{\sqrt{3}}$

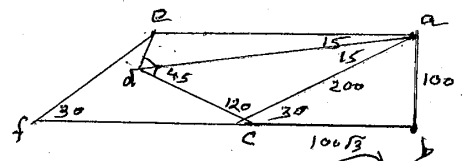
$\beta = \tan^{-1} \left( \frac{11}{\sqrt{3}} \right)$  (5)

60

15b



Reaction on A is in the direction  $\vec{AD}$  for the equilibrium of system, these forces 100, P, R & their line of action of these forces meet at D. (10)



faae is a Rhombus  $\Rightarrow ef = 200$  (30)

$\frac{de}{\sin 15} = \frac{ad}{\sin 120} = \frac{200}{\sin 45}$  (10)

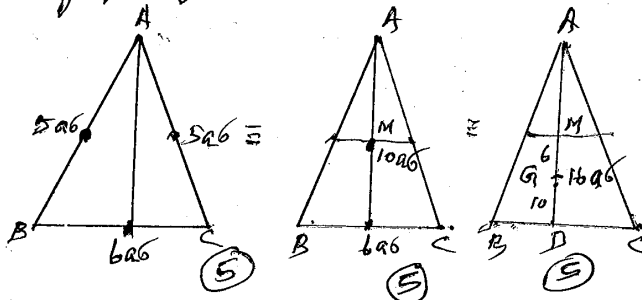
$ad = 100\sqrt{6}, de = 100(\sqrt{2}-1) = de$

$ae = 200 = cf$

| • Rod | Notation | Thrust | Tension   |
|-------|----------|--------|-----------|
| AB    | ae       | 200    | -         |
| BC    | da       | 100√3  | -         |
| CD    | ca       | 200    | -         |
| DE    | be       | -      | 100√3     |
| EC    | ed       | -      | 100(√3-1) |
| BE    | ea       | -      | 100(√3-1) |
| AE    | fe       | 200    | -         |

$P = 200 + 100\sqrt{3} = 100(2 + \sqrt{3})$  (5) 90

(1b) a) by symmetry C.M. lies on median through A

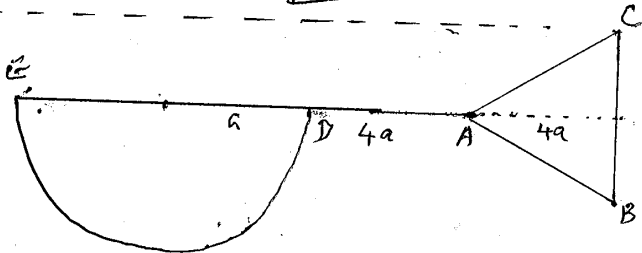


$\frac{MG}{GD} = \frac{6}{10} = \frac{3}{5}$  (5)

$MG = \frac{3}{8} \cdot 2a = \frac{3}{4}a$  (5)

$AG = 2a + \frac{3}{4}a = \frac{11a}{4}$  (5) 30

b) Theory 40



|             | Mass                             | C.M. from BC       | C.M. from AD      |
|-------------|----------------------------------|--------------------|-------------------|
| Hemi Sphere | $2\pi a^2 \rho$ (5)              | $9a$ (5)           | $\frac{R}{2}$ (5) |
| AD          | $4a^2 \rho$ (5)                  | $6a$ (5)           | $0$ (5)           |
| ABC         | $16a^2 \rho$ (5)                 | $\frac{5a}{4}$ (5) | $0$ (5)           |
| System      | $2\pi a^2 \rho + 20a^2 \rho$ (5) | $\bar{x}$          | $\bar{y}$         |

$(2\pi a^2 + 20a^2)\bar{x} = 2\pi a^2 \cdot 9a + 4a^2 \rho \cdot 6a + 16a^2 \rho \cdot \frac{5a}{4}$  (10)

$\bar{x} = \frac{9\pi a^2 + 22a}{\pi a + 10}$  (5)

$(2\pi a^2 + 20a^2)\bar{y} = 2\pi a^2 \rho \cdot \frac{R}{2}$  (10)

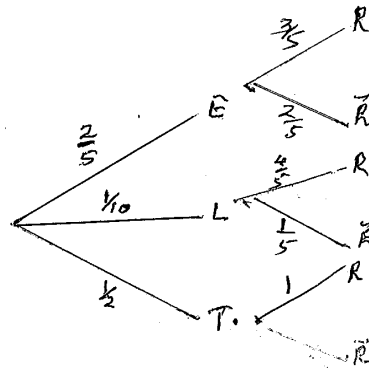
$\bar{y} = \frac{\pi a^2 \rho}{2(\pi a + 10)}$  (5) 80

17) a)  $P(A/B) = \frac{P(A \cap B)}{P(B)}$  (10)  $P(B) > 0$

b)  $A_i$  are partitions in sample space  $S$ . &  $B$  is a event.

$P(B) = \sum_i P(A_i) \cdot P(B/A_i)$  (20)

c)  $R = \{\text{Student receiving the message}\}$



$P(R) = \frac{2}{5} \cdot \frac{3}{5} + \frac{1}{5} \cdot \frac{4}{5} + \frac{1}{2} \cdot 1$  (20)  
 $= \frac{41}{50}$  (10)

$P(E/R) = \frac{P(E) \cdot P(R/E)}{P(R)}$  (30)

$= \frac{\frac{2}{5} \cdot \frac{3}{5}}{\frac{41}{50}}$  (10)

$= \frac{12}{41}$  (10) 150