Conducted by Field Work Centre, Thondaimanaru

In Collaboration with Provincial Department of Education

Northern Province

Term Examination, March - 2020

FWC

Grade – 13 (2020)	Combined Maths – I A	Time : 3 Hours 10 Minutes

Index No:						

Instructions

- This question paper consists of two parts; Part A (questions 1 - 10) and part B (questions 11 - 17).

Part - A

- Answer all questions. Answers should be written in the space provided on the questions paper. If additional space needed, you may use additional answer sheets.

Part - B

- Answer only 5 questions.
- After the allocated time hand over the paper to the supervisor with both parts attached together.
- Only part B of the paper is allowed to be taken out of the examination hall.

Combined mathematicsI		
Part	**Question**	**Marks**
A	1	
	2	
	3	
	4	
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	8	
	9	
	10	
B	11	
	12	
	13	
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	17	
	Total	

Combined Maths I	
Combined MathsII	
Total	
Final Marks	

Part – I A

1. Using the principle of Mathematical Induction, prove that $\sum_1^n \frac{1}{r(r+1)} = \frac{n}{n+1}$ for all $n \epsilon Z^+$

2. Draw the graphs of $y = |x - 2|$, $y = 2 - |x|$ in the same diagram. Hence find all real values of x . Satisfying the equation $|x - 2| + |x| = 2$.

3. On an Argand diagram, draw the locus of the complex number Z which satisfies the equation $Arg(Z + 1 + i) = \frac{\pi}{4}$. Hence or otherwise find the values of Z which satisfies the equations $Arg\ (Z + 1 + i) = \frac{\pi}{4}$ and $|Z - i| = 1$

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4. Find the value $\lim_{x \to 2} \frac{\cos\frac{\pi}{4}x}{\sqrt{x+3} - \sqrt{5}}$

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5. Let C be the curve given by $x = 2(1 - \cos\theta)$, $y = 2(\theta + \sin\theta)$ for $0 < \theta \leq \pi$. Show that $\frac{dy}{dx} = \cot\frac{\theta}{2}$, Find the equation of the tangent drawn to the curve C at $(4, 2\pi)$.

6. Show that $\frac{d}{dx}\left\{x\sqrt{1 - x^2}\right\} = \frac{1-2x^2}{\sqrt{1-x^2}}$. Hence find the value of $\int \frac{1+x^2}{\sqrt{1-x^2}}dx$

7. Show that the volume of solid made by rotating the area enclosed by the curve $y = e^x$ and the straight lines $y = o$, $x = o$, $x = 1$ by 2π radians around x axis is $\frac{\pi}{2}(e^2 - 1)$

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8. Find the equations of the straight lines, which intersect the coordinate axis at equal distance from origin and passes through the intersection point of the straight lines
$3x - 4y + 1 = 0$ and $5x + y - 1 = 0$

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9. Find the equation of the circle which passes through the points $(0,0), (0,2)$ and bisect the circumference of the circle $x^2 + y^2 - 2x + 4y - 6 = 0$

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10. Solve the equation $\tan^{-1}\left(\frac{x}{x-1}\right) + \tan^{-1}\left(\frac{x+2}{x+3}\right) = \tan^{-1}\left(\frac{2}{3}\right)$

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Grade – 13 (2020)	Combined Maths – I B	

Index number

11. a) Let $P\epsilon R$ and $0 < P \leq 1$ Show that 1 is not a root of the equation $(P+1)^2x^2 + 8x + 2(P+1) = 0$. Let α , β are the roots of the above equation. Show that α , β are real. Write $\alpha + \beta$, $\alpha\beta$ interms of P and show that $\frac{1}{(\alpha-1)\,(\beta-1)} = \frac{(P+1)^2}{P^2+4P+11}$.

Show that the equation having $\frac{\alpha}{\alpha-1}$, $\frac{\beta}{\beta-1}$ as roots is

$(P^2+4P+11)x^2 - 4(P+3)x + 2(P+1) = 0$ and also these roots are real

Hence deduce the equation having the roots $\left(1-\frac{1}{\propto}\right), \left(1-\frac{1}{\beta}\right)$

b) $n(> 2)$ is an odd positive whole number.

When $ax^n + b$ is divided by $x^2 - 1$ the remainder is $x + 2$. Find the values of a, b. Hence find the remainder when $x^7 + x^5 + x^3 + 6$ is divided by $x^2 - 1$.

12. a) 3 universities participate in a scientific seminar. Each universities sent 6 scientists such that 2 Scientists including a male and a female from each of the sectors Mathematics, physics and chemistry. Among 18 scientists a group of 3 scientists should be selected.

Calculate the number of groups that can be formed in the following instances.

i) All the scientists in the group are Male.

ii) No. of groups, having atleast one female.

iii) No. of groups having a scientist from each university

iv) No. of groups having 2 male and one female, one person from each university and one person from each sector.

b) let $U_r = \frac{r+3}{r(r+1)(r+2)}$ for $r \in Z^+$

$V_r = \frac{2r+3}{r(r+1)}$

Show that $V_r - V_{r+1} = 2U_r \quad r \in Z^+$

Hence show that $\sum_{r=1}^{n} U_r = \frac{5}{4} - \frac{2n+5}{2(n+1)(n+2)}$

Show that the series $\sum_{r=1}^{\propto} U_r$ is convergent and find the summation.

Also find the value of $\sum_{r=3}^{\propto} 3U_r$

13. a) Let $Z = x + iy$ for $Z \in \mathbb{C}$; here $x, y \in R$. Define $|Z|$ and $\overline{Z}$ of complex number Z.

Show that,

i) $Z.\overline{Z} = |Z|^2$

ii) $Z + \overline{Z} = 2ReZ$ and $Z - \overline{Z} = 2i\ Im\ Z$.

b) Express $Z = x + iy$ in polar co – ordinates form and write modulus of Z and arg (Z).

If $z_1, z_2\ \varepsilon\ \mathbb{C}$, Show that

i) $|Z_1\ Z_2| = |Z_1||\ Z_2|$

ii) $arg(Z_1 Z_2) = arg\ (Z_1) + arg\ (Z_2)$

c) Let $Z_1 = \frac{4}{1-i\sqrt{3}}$, $Z_2 = \frac{2}{1+i}$ be two complex numbers.

i) Express Z_1, Z_2 in polar co – ordinates form and find their modulus and arguments

ii) Find $|Z_1\ Z_2|$ and $Arg(Z_1\ Z_2)$ and then express $Z_1.Z_2$ in the polar coordinates form.

iii) P_1, P_2, P_3 denote the complex numbers $Z_1, Z_2, (Z_1 . Z_2)$ in the argand plane respectively. Mark these points on an Argand diagram.

iv) Using De Moivres Theorem, show that $Z_1^3 + 2\ Z\ _2^4 = -16$.

14 a) Let $f(x) = \frac{x+1}{(x+3)^2}$ for $x \neq -3$

Show that $f'(x) = -\frac{(x-1)}{(x+3)^3}$. for $x \neq -3$ and also $f''(x) = \frac{2(x-3)}{(x+3)^4}$

Here $f'(x), f''(x)$ represents the first and second derivatives of $f(x)$ by respectively.

Draw the graph $y = f(x)$ by showing their Asymtotes, turning points and inflection points.

b) A solid rectangular cone of radius 'r' and height 'h' is given . It's slant height is 3m. Show that the Volume (V) of the cone is given by $V = \frac{1}{3}\pi h\ (9 - h^2)$. Show that the volume of the cone is maximum when the semi vertex angle of the cone is $\tan^{-1}(\sqrt{2})$

15 a) i) Find $\frac{d}{dx}[ln(x^2 + 1)]$ deduce that $\int \frac{x}{x^2+1} dx$

ii) Express $\frac{x^3+4x^2-4x+4}{(x^2+1)(x^2-4)}$ as partial fractions.

Hence find $\int \frac{x^3+4x^2-4x+4}{(x^2+1)(x^2-4)}\ dx$

b) Using the integration by parts find the value of $\int x \sin^2 x\ dx$.

From this find the value of $\int x \cos^2 x\ dx$

c) For $0 \leq \theta \leq \frac{\pi}{2}$, using the substitution $x = 1 + 3\ \sin^2\theta$ find the value of $\int_1^4 \frac{dx}{\sqrt{(x-1)\ (4-x)}}$

16 If P is the point which divides the straight line connecting the points $A \equiv (5,0), B \equiv (10\ \cos\theta\ , 10\ \sin\theta)$ internally in the ratio 2 : 3, show that $P \equiv (4\ \cos\theta + 3, 4\ \sin\theta)$ the locus of is a circle of equation $S \equiv x^2 + y^2 - 6x - 7 = 0$ when θ changes. And find the center and radius also. Show that the point $Q \equiv (2, 5)$ lies outside the circle $S = O$. Find the equation of CD chord of contact of tangents drawn from Q to the circle $S = O$. Show that the equation of circle passes through the intersection points of $S = O$ and CD and passes also through $(1, 2)$ is $S^1 \equiv x^2 + y^2 - 4x - 10y + 19 = \odot$. Show that the locus of the center of the circle $S'' = O$ is a straight line of equation $4x - 2y + 17 = 0$. Here $S'' = O$ is the circle which intersects $S' = O$ orthogonally and which passes through the point $(0,\ 6)$

17 a) Write the expansion of $\sin(A + B)$ and obtain a similar expression for $\sin(A - B)$. By using the above results show that

$2 \sin A \cos B = \sin(A + B) + \sin(A - B)$. Hence deduce the relationship for $\sin 2A$

Show that $\sin\theta\,\{8 \cos\theta \cos 2\theta \cos 3\theta - 1\} = \sin 7\theta$. Here θ is not a multiple of 0 or π.

Hence solve $\cos\theta \cos 2\theta \cos 3\theta = \frac{1}{4}$ for $O < \theta < \frac{\pi}{2}$

b) Write down the sine rule for the triangle with usual notations. In a triangle ABC, $\hat{C} > \hat{B}$. The midpoint of the side BC is D.

$B\hat{A}D = \alpha\ , C\hat{A}D = \beta\ , A\hat{D}C = \theta\ \left(O < \theta < \frac{\pi}{2}\right)$

By using the sine rule for the triangle ABD Show that $\frac{a}{2\ \sin\ \alpha} = \frac{AD}{\sin\,(\theta - \alpha)}$

By obtaining another relationship for $\Delta\, ADC$. Show that $2\ \cot\theta = \cot\alpha - \cot\beta$

c) Show that $2\ \tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{6}{5}\right) = \frac{\pi}{2}$. Hence, show that $\sin\left\{\frac{\pi}{4} - \frac{1}{2}\tan^{-1}\left(\frac{6}{5}\right)\right\} = \frac{1}{\sqrt{26}}$

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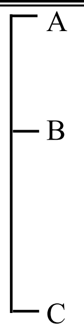
1) A particle falls from a certain height from a point A above the land. Particle takes 't' s to reach B. So that BC = h and also it takes $\frac{2t}{7}s$ to reach the point C from B. Draw the V – t graph for the motion of the particle from A to C and find t interms of h, g and find AC interms of h.



2) A particle is projected from a point O in a vertical plane at an angle of $\tan^{-1}(2)$ with thehorizontal with the speed of $35\,ms^{-1}$. The positional vector of the particle respect to O at time t is $x\,\underline{i} + 50\underline{j}$. [$\underline{i}$, $\underline{j}$ are unit vectors of the horizontal and vertical axis through 0]

i) Find t, x.

ii) Particle w is projected with the same speed at a different angle α it passes through the same point. Find .

3) Two smooth spheres P, Q with same radius have a mass of 2m, Km respectively are kept in a smooth horizontal table at a certain distance from each other. They are thrown along the table with velocities u, 3u respectively towards each other such that they collides head to head. After the collision, direction of motion of 2 spheres are interchanged and speed of ‘P’ becomes half the initial.

i) Calculate the speed of ‘Q’ after collision and rangeof values of K.

ii) If $k = \frac{1}{2}$, calculate the co – efficient of resitution of between P and Q.

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4) 1. A van weights of 3200 kg travels along the horizontal road with constant power of 36 kw against constant resistive force of ‘R’. Calculate the value of ‘R’ when the speed and acceleration of van are 20 ms^{-1} and 0.2 ms^{-2} respectively.

2. Engine of van increases its power when it travels upwardly with same resistive force. along the path which makes 30° with horizontal. Calculate the power of engine when van maintains a speed of 30 ms^{-1}? (g = 10 ms^{-2})

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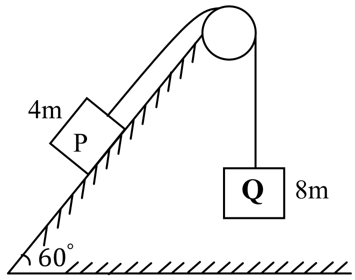
5) One end of an inelastic string is connected to a particle 'P' of mass 4m which is kept above the rough plane which makes 60° with horizontal. The other end is connected to a particle Q of mass 8m Such that string passes through smooth pulley fixed on top o plane. System releases from rest such that all parts of string is in stretch position. Calculate the tension of string and accleration of particles? (Co – efficient of friction between 'P' and plane is $\frac{1}{4}$)



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6) $\underline{R}$ is the resultant of $\underline{P}$ and $\underline{Q}$ forces. OA, OB and OC are line of action of forces. $\underline{P}$, $\underline{Q}$ and $\underline{R}$ respectively. A straight line cuts OA, OB, OC at points L, M, N respectively. Show that $\frac{P}{OL} + \frac{Q}{OM} = \frac{R}{ON}$

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7) The uniform rod AB has a weight of ‘W’ and length ‘8 a’. The end ‘A’ of the rod holds on smooth vertical wall. The one end of a string is attached to a point ‘C’ on a rod. The other end of string is attached to point D Such that, D is at a height of 6a from A. The rod makes 60° with downward vertical and it is in equillibrium. Find the lenght of BC?

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8) $\underline{a}\,,\underline{b}$ are the positional vectors of points A, B respectively with respect to point ‘O’. The point ‘C’ is in straight line AB such that $AC:CB = \mu:\lambda$. Show that positional vector of ‘C’ is given by $\underline{C} = \frac{\lambda\underline{a}+\mu\underline{b}}{\lambda+\mu}$

Hence, If $\propto \underline{a} + \beta\underline{b} + \gamma\underline{c} = \underline{o}$ and $\propto + \beta + \gamma = o$ show that A, B and C are in straight line

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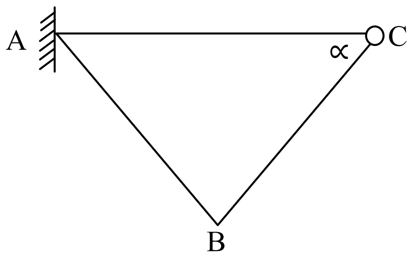
9) The uniform rod AB has a weight of 4W and length of 2a. Rod is fixed smoothy on A. Ring 'C' of weight 'W' is fixed in horizontally adjusted string through 'A' such that ring moves freely B and C are connected by a string of length 2a. Co – efficient of friction between string and ring is μ. Show that˙ $\tan \propto = \frac{1}{2\mu}$ when it is in limitting equilibrium.



10) A and B are two events. It $P(A \cap B^1) = \frac{8}{25}$ $P(A^1 \cap B) = \frac{11}{100}$, $P(A \cup B) = \frac{13}{20}$

Calculate (i) $P(A \cap B)$ (ii) $P(A)$ (iii) $P(B)$ (iv) $P(A/B)$

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Grade – 13 (2020)	Combined Maths – II B	

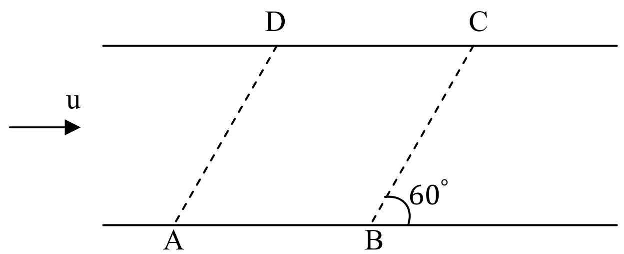
11) a) A motor car which travels with a uniform acceleration on a straight line passes the point O at $t = o$ with the velocity u. Motor car moves a distance p in the first t seconds and moves a distance q in the next t seconds and moves a distance r in the next t seconds.

i) Draw the velocity – time graph for the motion of the motor car for the first $4t$ seconds.

ii) Using velocity – time graph, show that

1. $2q = p + r$
2. the distance travelled during time $3t$ to $4t$ seconds is $2r - q$

b)



A river having parallel straight shores and width 3a flows with uniform velocity u. A, B, C, D are the vertices of the rhombus on shores as shown in the figure. Two persons P, Q who can swim with a speed $v(v \geq u)$ with relative to water, start to swim from A at same time. P wants to swim from A to C and Q wants to swim from A to D. Using relative velocity theory, Draw the velocity triangles for the motion of P, Q on the same diagram .

i) Show that the speed of P for the motion from A to C is $\frac{\sqrt{3}\,u+\sqrt{4V^2-u^2}}{2}$.

ii) Show that the speed of Q for the motion from A to D is $\frac{u+\sqrt{4V^2-3u^2}}{2}$

iii) Show that $v = u$, if P, Q reaches C, D on same time.

12)a)

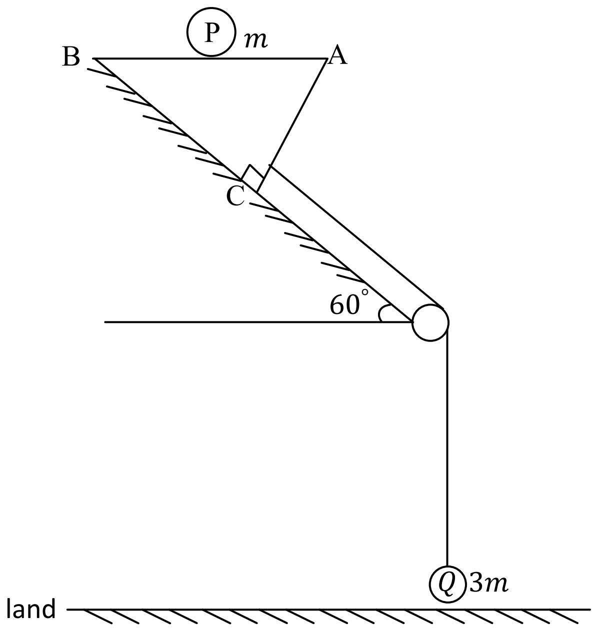


Figure shows a cross section of a wedge ABC of mass 4m such that $A\hat{C}B = \frac{\pi}{2}$. The face BC of the wedge is placed on an inclined plane incline at an angle of 60° with the horizontal as shown in the figure. The face AB is horizontal a particle 'P' of mass 'm' is placed on the face AB. one end of a light inextensible string is attached to the wedge. and passes through a light inclined plane. The other end of the string is attached with the particle Q of mass 3m. The string is kept tight and the system is allowed to move slowly.

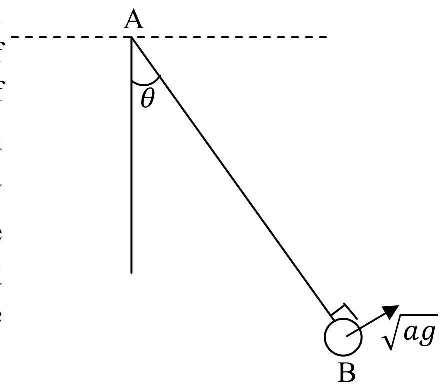
Assume that the particle Q does not touch the floor in it's continuous motion.

(i) Mark clearly the forces, accelerations act on the system.

(ii) Obtain the equations sufficient to find the acceleration of particles P, Q and wedge and also the tension in the string .

(iii) At $t = o$, the particle P is released from A. Find the time taken to the particle to reach B such that $BC = 2\ell$.

b) One end of a light inextensible string of length 3a is connected to a fixed point A which is at a certain height. The other end 'B' of the string is connected to a particle of mass 'm'. The string is kept tight at an angle of θ $\left(\cos\theta = \frac{2}{3}\right)$ with the downward vertical and projected in the direction. perpendicular to the string AB with a velocity of $\sqrt{ag}$. In the continuous motion of the particle, when the string is tied and it makes an angle β with downward vertical, the speed of the particle is B. The tension in the string at that moment is T.



i) Show that $V^2 = 3ag(2\ \sin\beta - 1)$

ii) Find the minimum and maximum values of 'T'

13) a) One end of a light elastic string of natural length $3a$ is connected to a fixed point O which is at a height h $(h > 8a)$ from the ground and a particle of mass 4m is connected to the order end string and the mass is at equilibrium at a distance $5a$ below O. When a velocity is given downwards to the particle at equilibrium, it comes to an instantaneous rest at a depth $8a$ from O.

i) Show that the elastic modulus of string is 6 mg.

ii) Show that the particle satisfies the equation of motion $\ddot{x} + \frac{g}{2a}x = 0$
When the particle is at a distance $5a + x$ from O.

iii) If $\ddot{x} + \frac{g}{2a}x = 0$ satisfies the equation $x = A\ \cos w\,t + B\ \sin w\,t$, find A, B, w in terms of u, g, a

iv) Using part (iii), by considering the instantaneous rest position of the particle, find u and then find the amplitude of simple harmonic motion.

v) Find the time taken from the start of the motion of the particle until the string slag.

14) a) The positional Vectors of the points A, B are $\underline{a}\,, \underline{b}$ respectively. The positional vector of point C on AB is $\underline{c}$. Show that $\underline{c}$ can be given by $\underline{c} = \propto \underline{a} + (1-\propto)\underline{b}$. Here $\propto -$ parameter.
The positional vectors of points A, B, C, D are $\underline{a}\,, \underline{b}\,, 3\,\underline{a}\,,\ 5\,\underline{b}$ respectively.

i) Give the positional vector of any point in AD interms of $\underline{a}\,, \underline{b}$ and parameter $'\lambda'$

ii) Give the positional vector of any point in BC interms of $\underline{a}\,, \underline{b}$ and parameter 'μ'
By using the above result find the positional vector of the intersecting point of AD, BC.
Find the ration at which the intersection point cuts the line AD.

b) Forces $2\underline{i} + \underline{j}\,, 5\underline{i} - 4\underline{j}\,, a\underline{j}$ are acting on the points $3\underline{i} + 4\underline{j}\,, -2\underline{i} - 3\underline{j}\,, 2\underline{i}$ respectively, with respect to O. This system of forces is equivalent to a force $\underline{F}$ acts through O and a couple of 24 Nm.

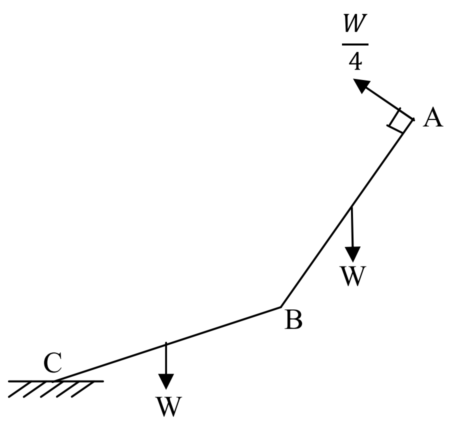
i) Mark clearly the forces acting, on a cartesion plane.

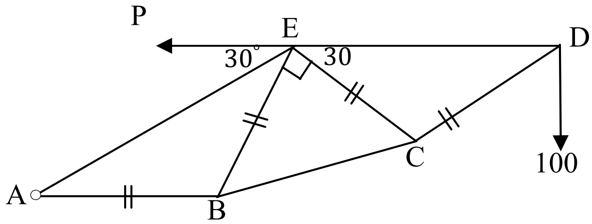
ii) Find the values of a.

iii) Find $\underline{F}$ for each values of a.

iv) Find the equation of the line of action of the force equivalent to the system for the positive value of a.

15) a) The roads AB, BC each having weight W and length 2a are joined freely at B and it is at equilibrium by having C on ground and by applying a force $\frac{W}{4}$ at A perpendicular to AB .
Find the angles made by AB, BC with horizontal and find the vertical, horizontal components of reaction at B.



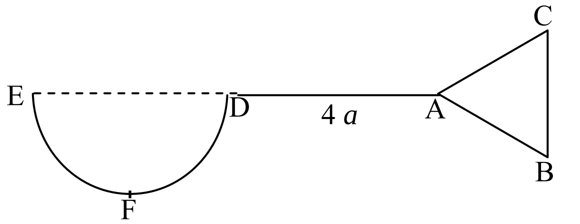
b) 

The frame work made of seven light rods is at equilibrium. Such that it is hinged freely at A and 100 N load is tied to D, and a horizontal force P is applied on E.
AB, ED are horizontal

i) Give the direction of reaction at A without any calculations.
ii) Draw the stress diagram using Bow's Notation.
iii) Hence, distinguish the stresses as tension and trusts and find the values. Find also the value of P.

16) a) AB, BC, CA are uniform heavy rods made of same material and having length $5a, 6a, 5a$ respectively. A triangular shape ABC is made by these rods jointed rigidly on their ends. Show that the centre of mass of this shape lies in the median through A at a distance $\frac{11a}{4}$ from A.

b) By using integration, find the centre of mass of the uniform hollow hemisphere of radius a.

c) 

A spoon is made by joining a hemispherical bowl DEF of radius a and a rod DA of length 4a as shown in the figure and triangular shape mentioned in part (a) is connected to end A. The shape is made of same material
Find the centre of mass of this combined object from BC, AD.

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17) a) A, B are two events in a sample space define $P(A/B)$, the probability for event A to happen if it is given that event B happened.

b) State the total probability theorem

c) A lecturer can inform a message to his student through email or letter or by telephone. The probability for sending the messages through them are $\frac{2}{5}, \frac{1}{10}, \frac{1}{2}$ respectively.
He will send messages through one method only.
The probability of obtaining these messages by a student when the lecturer sends messages is $\frac{3}{5}, \frac{4}{5}, 1$ respectively.

i) What is the probability for obtaining messages by student?
ii) What is the probability for student to obtain message through email given that student obtains message?