Solids

25

By studying this lesson you will be able to

- prepare models of a square pyramid and a triangular prism,
- draw the net of a square pyramid and a triangular prism on a square ruled paper, and
- know Euler's relationship for the above solids by considering the number of edges, vertices and faces of these solids.
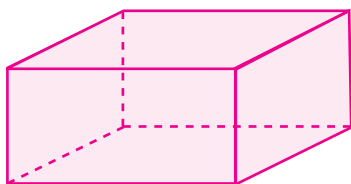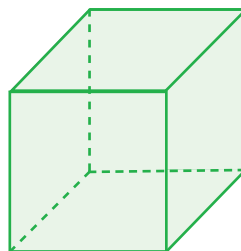
25.1 Introduction of Solids
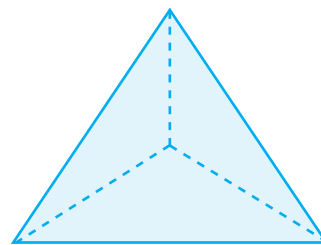
A die

A brick

A putt

A concrete post

You have learnt that an object such as a die, an iron ball or a concrete pillar, which has a specific shape and which occupies a certain amount of space is called a solid object. You have also learnt in grade 6 that the surfaces of solid objects can be plane surfaces or curved surfaces.

A cuboid

A cube

A regular tetrahedron

Do the review exercise to recall the facts you have learnt about solids.

(1) (i) Write down the number of faces, edges and vertices of a cuboid.

   (ii) Draw a net that can be used to construct a cuboid.

(2) (i) What is the shape of a face of a cube?

   (ii) Draw a net that can be used to construct a cube.

(3) Write down the number of faces, edges and vertices of a regular tetrahedron.

(4) (i) Draw the shape of a face of a regular tetrahedron.

   (ii) Draw a net that can be used to make a regular tetrahedron.

(5) Below is the figure of a solid object constructed by pasting two faces of two identical regular tetrahedrons, one on the other.

   (i) How many faces are there in this solid?
   (ii) How many edges are there in this solid?
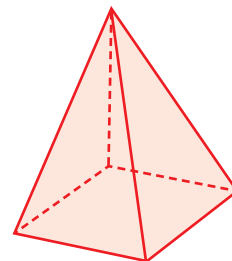   (iii) How many vertices are there in this solid?
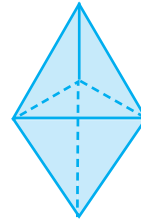
25.2 Square pyramid

Tombs of the Pharaohs who ruled in Egypt were built in this shape. They are called pyramids.

A solid object with a square base and four equal triangular faces is called a **square pyramid**. The figure illustrates a square pyramid.
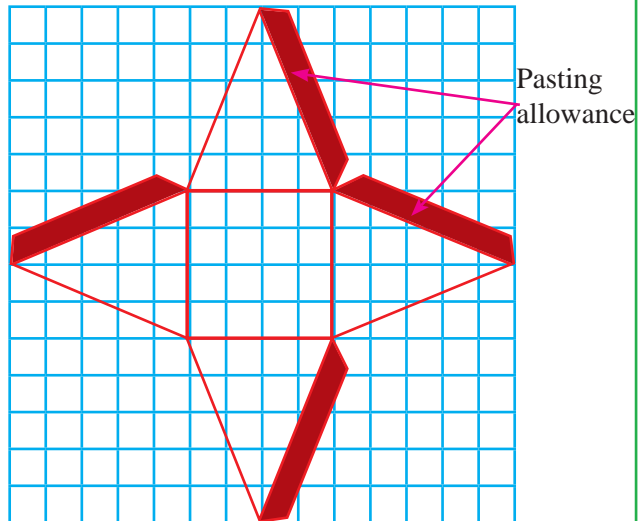
Let us identify the characteristics of a square pyramid by engaging in the following activity.

Activity 1

Step 1- Draw the given figure on a square ruled paper. Cut out the figure that you drew and either copy it or paste it on a thick piece of paper such as a Bristol board.

Pasting allowance

Step 2 - Cut out the figure drawn or pasted on the Bristol board and prepare a model of a square pyramid by folding along the edges and pasting along the pasting allowances.

Step 3 - Based on the model you prepared, find the number of faces, edges and vertices of a square pyramid. Examine the specific features of the model.

Step 4 - Write down the specific features you identified in your exercise book.

Step 5 - Measure and write down the lengths of the edges of the model.

The figure you obtain by removing the pasting allowances of the above figure which was used to prepare a model of a square pyramid, is called the "**net of the square pyramid**".
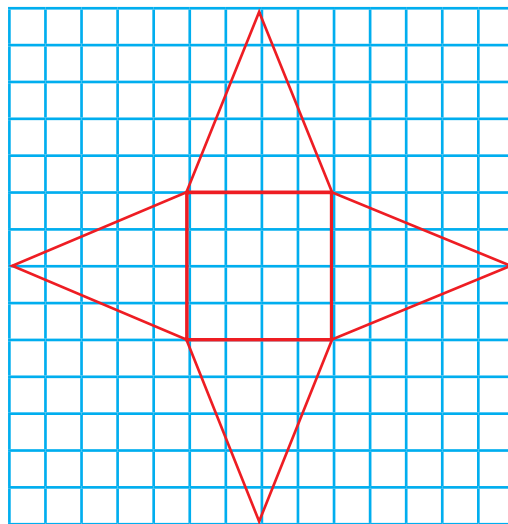
The object you prepared during the above activity is a model of a square pyramid.

Features you can identify in a square pyramid
- There are 5 faces in a square pyramid.
- One face has a square shape.
- The other 4 faces take the shape of equal triangles.
- There are 5 vertices in a square pyramid.
- There are 8 edges in a square pyramid. All are straight edges.

Activity 2

(1) Draw each shape given in the figure on a square ruled paper.

(2) Cut out each shape, fold along the edges and paste them using sellotape.

(3) What is the name of each of the solids you get?
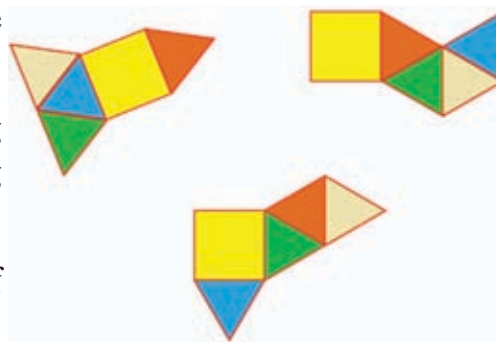
25.3 Triangular Prism

A figure of a kaleidoscope which is an object through which a pattern of multiple images can be observed is given here. It is made out of 3 rectangular plane mirrors.

Pattern seen through this kaleidoscope
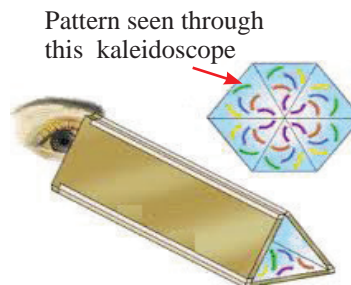
A solid object which has 3 rectangular plane faces and two triangular faces is called a "**triangular prism**".

Let us identify the characteristics of a triangular prism by engaging in the following activity.

Step 1 - Draw the given figure on a square ruled paper. Cut out the figure that you drew and either copy it or paste it on a thick piece of paper such as a Bristol board.
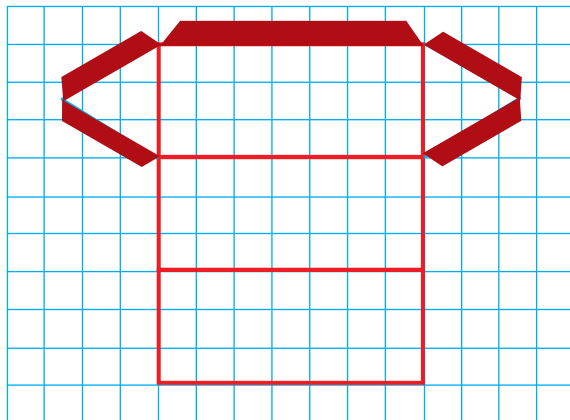
Step 2 - Cut out the figure drawn or pasted on the Bristol board and prepare a model of a triangular prism by folding along the edges and pasting along the pasting allowances.

Step 3 - Based on the model you prepared, find the number of faces, edges and vertices of a triangular prism. Examine other specific features of the model.

Step 4 - Write down the specific features you identified in your exercise book.

The figure you obtain by removing the pasting allowances of the above figure which was used to prepare a model of a triangular prism, is called the "**net of the triangular prism**".
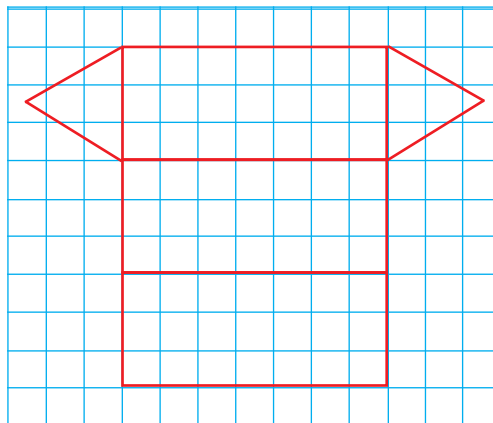
Features you can identify in a triangular prism

- There are 5 faces in a triangular prism.
- There are 2 triangular shaped faces in a triangular prism. They are equal in size and shape.
- The other 3 faces of a triangular prism are of rectangular shape.
- There are 9 edges in a triangular prism. All are straight edges.
- There are 6 vertices in a triangular prism.

Exercise 25.1

(1) Write down the number of faces, edges and vertices of a square pyramid.

(2) Construct two square pyramids of equal measurements using Bristol board.
   (i) Paste together the square faces of the two pyramids you constructed.
   (ii) Write down the number of faces, edges and vertices of the solid object you obtained in the above step.

(3) Draw the figure of another net which can be used to prepare a square pyramid.

(4) Write down the number of faces, edges and vertices of a triangular prism.

(5) Write down the number of faces, edges and vertices of the solid you obtain by overlapping and pasting together two equal rectangular faces of two identical triangular prisms.

(6) Draw different nets that can be used to construct a triangular prism.

25.4 Euler's Relationship

Fill in the blanks in the table given below based on the solids you studied in grade 6 and by observing the solids you constructed in activities 1 and 3.

Solid	Number of vertices (V)	Number of faces (F)	Sum of the number of vertices and the number of faces (V + F)	Number of edges (E)
Cube	8	6	8 + 6 = 14	12
Cuboid	....	....	....	....
Regular tetrahedron	....	....	....	....
Square pyramid	....	....	....	....
Triangular prism	....	....	....	....

After completing the table, turn your attention to the $(V + F)$ column and the "E" column. With regard to the above solids, notice that the values in the $(V + F)$ column are always greater by 2 than the values in the "E" column.

Accordingly, for the above solids, the sum of the number of faces and the number of vertices is equal to the value obtained by adding 2 to the number of edges.

Number of Vertices	+	Number of Faces	=	Number of Edges + 2
V	+	F	=	E + 2

The above relationship which is true for solids with plane faces only, was first presented in the 18th century by a Swiss mathematician called "Leonhard Euler" who lived in Switzerland. Therefore this relationship was later called Euler's formula.

Exercise 25.2

(1) A certain solid has 6 faces and 8 vertices. Find the number of edges of the solid using Euler's relationship.

(2) If a certain solid has 8 edges and 5 faces, find the number of vertices of the solid.

(3) Verify Euler's relationship for a triangular prism, by considering the number of faces, vertices and edges it has.

(4) A solid constructed by coinciding and pasting the square faces of two identical square pyramids is shown in the figure.

   (i) Find the number of edges, faces and vertices of this solid.

   (ii) Show that the above values satisfy Euler's relationship.
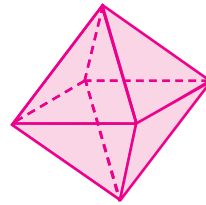
(5) A solid constructed by combining a cube and a square pyramid is shown in the figure. Find the number of edges, faces and vertices of the solid and check whether they satisfy Euler's relationship.
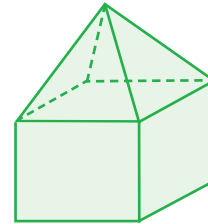
(6) The solid shown in the figure has been constructed using a cuboid and a triangular prism. Validate Euler's relationship for this solid.
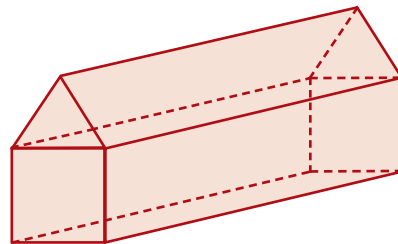
(7) Construct a cube and 6 square pyramids with bases that are equal to a face of the cube. Construct a composite solid by pasting the square faces of the 6 pyramids on the six faces of the cube.

   (i) How many edges, faces and vertices are there in the composite solid?

   (ii) Do these values agree with Euler's relationship?

Summary

- A solid that consists of a square base and 4 identical triangular faces having a common vertex is called a square pyramid.
- A square pyramid consists of 8 edges, 5 faces and 5 vertices.
- A solid with 3 rectangular faces and 2 parallel triangular faces is called a triangular prism.
- A triangular prism consists of 9 edges, 5 faces and 6 vertices.
- If a solid has E number of edges, F number of faces and V number of vertices, $V + F = E + 2$ denotes Euler's relationship.