Ratios

By studying this lesson you will be able to

- divide a quantity in a given ratio,
- find the total value or the values of the other terms when the value of a term of a ratio is given, and
- apply knowledge on ratios in practical situations.

21.1 Ratios and Equivalent Ratios

You have learnt in grade six that a ratio is a numerical relationship between two or more quantities expressed in similar units.

Let us focus on a few instances in daily life where ratios are applied.

A label pasted on a bottle of fruit juice recommends that, two parts juice be mixed with three parts water.

1 l 1 l 1 l 1 l 1 l

Fruit juice   Water

Therefore, to make a consumable drink from the bottled juice, one can mix 2 litres of juice with 3 litres of water.

We say that the fruit drink is made using juice and water in the ratio 2:3.

The mixed quantities (in litres) of juice and water are expressed by the ratio 2:3. This is read as 'two-to-three' or 'two-is-to-three'. The numbers 2 and 3 are called the **terms of the ratio**.

When we write a ratio, it is essential to write the terms in the correct order – that is, the order in which we mention the quantities. In the above example, we wrote juice first and water second. The same order is followed for the terms – we write 2 as the first term of the ratio and 3 as the second term.

When the terms of a given ratio are multiplied by the same positive whole number, we get an **equivalent ratio**.

That is, $1 : 3 = 2 : 6 = 3 : 9 = 4 : 12 = 5 : 15$.

Now let us consider an example where three items are mixed.

A concrete mixture is made by mixing together cement, sand and granite.

Cement

Sand

Granite

The ratio in which cement, sand and granite are mixed to prepare this concrete mixture is written as $1 : 3 : 4$. It is read '1 to 3 to 4' or '1 is to 3 is to 4'. Here, 1, 3 and 4 are the terms of the ratio.

Let us multiply each term of the ratio $1 : 3 : 4$ by 2.
Then we get the ratio $2 : 6 : 8$. The ratio $2 : 6 : 8$ is equivalent to the ratio $1 : 3 : 4$.

A ratio should be written such that its terms are whole numbers that cannot be simplified further.

If the terms of a given ratio are whole numbers, and if the highest common factor of the terms is 1, then the ratio is said to be in the simplest form.

When the terms of a ratio are whole numbers, to write the ratio in its simplest form,

☛ check whether the terms have common factors.
☛ if the terms have common factors, then divide each term of the ratio by the highest common factor of the terms.

Example 1

Write a ratio equivalent to 4 : 1 : 6.
Multiplying the terms in the ratio by 3 we obtain,

4 : 1 : 6 = 4 × 3 : 1 × 3 : 6 × 3
$\qquad$ = 12 : 3 : 18

Example 2

Express the ratio 8 : 4 : 12 in its simplest form.
The HCF of the terms 8, 4 and 12 is 4. Dividing the terms of the ratio by 4 we obtain,

8 : 4 : 12 = 8 ÷ 4 : 4 ÷ 4 : 12 ÷ 4
$\qquad$ = 2 : 1 : 3

Example 3

The sides of a triangle are 8 cm, 6 cm 5 mm and 50 mm. Find the ratio of the lengths of the sides of the triangle and express it in the simplest form.
Let us express the lengths in similar units.
8 cm = 80 mm, 6 cm 5 mm = 65 mm, 50 mm
The ratio of the lengths of the sides $\qquad$ = 80 : 65 : 50
The ratio of the lengths of the sides
$\qquad$ in the simplest form $\Big\}$ = 16 : 13 : 10

Exercise 21.1

(1) Write down the ratio for each of the following examples, and express it in the simplest form.
  (i) The number of boys in a class is 20 and the number of girls is 25.
  (ii) The price of a pen is Rs. 15, the price of a pencil is Rs. 10 and the price of an eraser is Rs. 5.
  (iii) The ingredients for a cake are 1 kg flour, 500 g sugar and 500 g margarine.
  (iv) The price of a mandarin is Rs. p, the price of an orange is Rs. q and the price of an apple is Rs r.

(2) For each of the following ratios, write down two equivalent ratios.
  (i) 2 : 3 $\qquad$ (ii) 6 : 5 : 7 $\qquad$ (iii) 1 : 4 : 5

(3) Express each of the following ratios in its simplest form.
  (i) 12 : 18 $\qquad$ (ii) 28 : 70 : 42 $\qquad$ (iii) 25 : 100 : 125

(4) The sides of a triangle are 7 cm, 50 mm and 6 cm 5 mm. Find the ratio of the lengths of the sides and express it in the simplest form.

21.2 Dividing in a ratio

● **Dividing a given quantity in a ratio**

There are instances in day-to-day life when people need to divide items among themselves. On some occasions, this is done in equal amounts, while on other occasions it is done in unequal amounts.

At the beginning of this lesson, we discussed about mixing juice and water in the ratio 2:3.

Five units of fruit drink are made by mixing 2 parts juice with 3 parts water.

The ingredients of this drink are juice and water.

Since the **number of parts of juice is 2** and the **number of parts of water is 3**, the number of parts of drink is 5.

Let us find the amount of each ingredient in the drink, if 10 litres of the drink were made.

$$\text{Ratio of juice to water} = 2 : 3$$
$$\text{Total number of parts} = 2 + 3$$
$$= 5$$
$$\text{Volume of five parts} = 10 \, l$$
$$\text{Volume of one part} = \frac{10 \, l}{5}$$
$$= 2 \, l$$

$$\text{Parts of juice} = 2$$
$$\text{Volume of juice} = 2 \, l \times 2$$
$$= 4 \, l$$

$$\text{Parts of water} = 3$$
$$\text{Volume of water} = 2 \, l \times 3$$
$$= 6 \, l$$

Juice		Water
2	:	3

Number of parts	Volume
5	10
2	?
3	?

Note

When using this method, problem solving is facilitated by writing the given ratio in its simplest form and then finding the total number of parts relevant to it.

Example 1

Cement, sand and granite in a concrete mixture are in the ratio $1 : 3 : 4$. Find the quantities of cement, sand and granite in 16 cubic metres of concrete.

$$\text{Ratio of cement to sand to granite} = 1 : 3 : 4$$
$$\text{Total number of parts} = 1 + 3 + 4 = 8$$
$$\text{Size of 8 parts} = 16 \text{ m}^3$$
$$\text{Size of a single part} = \frac{16}{8} \text{ m}^3 = 2 \text{ m}^3$$
$$\text{Number of parts of cement} = 1$$
$$\text{Quantity of cement} = 1 \times 2 \text{ m}^3 = 2 \text{ m}^3$$
$$\text{Number of parts of sand} = 3$$
$$\text{Quantity of sand} = 3 \times 2 \text{ m}^3 = 6 \text{ m}^3$$
$$\text{Number of parts of granite} = 4$$
$$\text{Quantity of granite} = 4 \times 2 \text{ m}^3 = 8 \text{ m}^3$$

Example 2

The ingredients to make 3 kg of cake are butter, sugar and flour, mixed in the ratio 1 : 2 : 3. Find the mass of each ingredient in the cake.

$$\text{Ratio of butter to sugar to flour} = 1 : 2 : 3$$
$$\text{Total number of parts} = 1 + 2 + 3 = 6$$
$$\text{Total mass of the 6 parts of cake mixture} = 3 \text{ kg}$$
$$\text{Mass of a single part} = \frac{3}{6} \text{kg}$$
$$= \frac{3000}{6} \text{g} = 500 \text{ g}$$
$$\text{Parts of butter} = 1$$
$$\text{Mass of butter} = 1 \times 500 \text{ g} = 500 \text{ g}$$
$$\text{Parts of flour} = 3$$
$$\text{Mass of flour} = 3 \times 500 \text{ g} = 1500 \text{ g}$$
$$= 1 \text{ kg } 500 \text{ g}$$
$$\text{Parts of sugar} = 2$$
$$\text{Mass of sugar} = 2 \times 500 \text{ g}$$
$$= 1000 \text{ g} = 1 \text{ kg}$$

Example 3

Nadaraja and Mohommad made a profit of Rs. 7000 from their small business. They decide to divide the profit in the ratio 3 : 4, which is the ratio in which they invested in the business. Find how much of the profit each person receives.

$$\left.\begin{array}{r}\text{The ratio in which the profit is}\\ \text{divided between Nadaraja and Mohommad}\end{array}\right\} = 3 : 4$$

Total number of parts $= 3 + 4 = 7$

Total profit $=$ Rs. 7000

Value of a single part $=$ Rs. $\dfrac{7000}{7}$

$=$ Rs. 1000

Number of parts Nadaraja receives $= 3$

Value of profit Nadaraja receives $=$ Rs. 1000×3

$=$ Rs. 3000

Number of parts Mohommad receives $= 4$

Value of profit Mohommad receives $=$ Rs. 1000×4

$=$ Rs. 4000

Exercise 21.2

(1) Rs. 1500 was divided between Sumudu and Kumudu in the ratio 2 : 3. Find the amount each received.

(2) Copper is added to gold in gold jewellery, such that the ratio of copper to gold is 1 : 11. Find the mass of gold and copper needed to make a necklace of mass 60 grammes.

(3) The ratio of boys to girls in a school is 5 : 4. If the total number of students in the school is 1800, find the number of boys and the number of girls there are in the school.

(4) A land owner divides his land of 1800 m² between his son and his daughter in the ratio 5:3. How much of the land does the son receive?

(5) Rice flour, sugar and coconut are mixed in the ratio 4 : 3: 1 to prepare a certain sweetmeat mixture. Find the mass of each ingredient in 2 kg of the sweetmeat.

(6) A high nutrient food item is made of green gram, soya and rice mixed in the ratio 2 : 1 : 3. Compute the amount of rice in a 840 g packet of this food item.

(7) The ratio of the high-school students enrolled in the science, technology and arts streams in a school is 3 : 5 : 7. If the total number of high-school students in the school is 600, how many students are enrolled in the arts stream?

(8) The ratio of length to breadth of a rectangular playground is 3 : 2. If its perimeter is 600 m, find its length and its breadth.

● **Calculating the total amount, when the amount of one item in a ratio is given**

The ratio of girls to boys in a class is 3 : 2. If the number of girls in the class is 24, let us find how many students there are in total in the class.

$$\text{Ratio of girls to boys} = 3 : 2$$
$$\text{Number of parts of girls in the class} = 3$$
$$\text{Number of girls} = 24$$

Since the number of students in 3 parts is 24,

$$\text{the number of students in one part} = \frac{24}{3} = 8$$
$$\text{Total number of parts} = 3 + 2 = 5$$
$$\text{Total number of students} = 8 \times 5$$
$$= 40$$

Girls Boys
3 : 2
Total Students ?

Example 1

A sum of money was divided between Ganesh and Suresh in the ratio 3 : 5. Suresh received Rs. 400. What was the total amount that was divided between the two of them?

The ratio in which the money was divided between Ganesh and Suresh $\left.\right\} = 3 : 5$

Ganesh		Suresh
3	:	5
?		400

Parts for Suresh = 5

Amount received by Suresh = Rs. 400

Therefore, the value of 5 parts = Rs. 400

Value of one part = Rs. 400 ÷ 5

= Rs. 80

Number of parts = 3 + 5 = 8

Total amount that was divided = Rs. 80 × 8

= Rs. 640

Exercise 21.3

(1) Sugar and flour were mixed in the ratio 3 : 5 to prepare a mixture for a sweetmeat. If the mass of sugar used was 750 g, find the total mass of the sweetmeat mixture.

(2) Sirimal rides a bike from his house to the bus halt and then takes a bus when he travels to school. The ratio of the distance he travels by bike to the distance he travels by bus is 2 : 7. If the distance he travels by bus is 14 km, what is the distance to the school from his house?

(3) A fruit drink is made using water and orange juice in the ratio 5 : 7. If the quantity of orange juice used is 350 ml, what is the total volume of the fruit drink that is made?

(4) The ratio of Nitrogen to Phosphorus to Potassium in a fertiliser is 5 : 2 : 1. If the mass of Phosphorus in a bag of this fertiliser is 250 g, what is the total mass of the bag?

Fertiliser

(5) When making a mixture of plaster, the ratio in which cement, lime and sand are mixed is 2:3:5. If the quantity of lime in a mixture of plaster is 6 pans, what is the total quantity of the mixture, measured in pans?

- **When the amount related to one term in a ratio is given, determining the other amounts**

The ratio in which a sum of money was divided between Siyam and Kandan is 2 : 3. If Siyam received Rs. 300, let us find how much money Kandan received.

$$\left.\begin{array}{c}\text{Ratio in which the money was divided}\\ \text{between Siyam and Kandan}\end{array}\right\} = 2:3$$
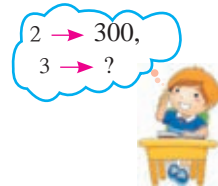
Parts Siyam received $= 2$

Money Siyam received $= Rs.300$

Since two parts is worth Rs. 300,

the value of one part $= Rs. \ 300 \div 2$

$= Rs. \ 150$

Parts Kandan received $= 3$

Money Kandan received $= Rs. \ 150 \times 3$

$= Rs. \ 450$

Example 1

The ratio in which cement, sand and granite are mixed in order to prepare a concrete mixture is 2 : 3 : 4. Let us find the quantities of cement and granite that should be mixed with 9 pans of sand and the total amount of concrete mixture that is made.

Parts of sand $= 3$

Pans of sand $= 9$

Quantity of sand in one part $= \dfrac{9}{3}$ pans $= 3$ pans

Parts of cement $= 2$

$\therefore$ quantity of cement $= 3 \times 2$ pans $= 6$ pans

Parts of granite $= 4$

$\therefore$ quantity of granite $= 3 \times 4$ pans $= 12$ pans

Total number of parts $= 2 + 3 + 4 = 9$

Quantity of concrete mixture $= 3 \times 9$ pans $= 27$ pans

Exercise 21.4

(1) Sesame balls are made by mixing sesame and jaggery in the ratio 5 : 4. How much jaggery is required to make sesame balls, if 500 g of sesame is used?

(2) The ratio of female workers to male workers in an office is 3 : 2. If there are 18 female workers, find the number of male workers.

(3) Tea and milk are mixed in the ratio 2 : 5 when making milk tea. How many millilitres of tea should be used to make milk tea, if 100 ml of milk is used?

(4) Mr. Perera's savings to expenditure ratio is 3:7. If his savings during a certain month was Rs. 6000, how much money did he spend that month?

(5) An alloy is made by mixing masses of zinc and copper in the ratio 2:5.

   (i) If the mass of zinc in a sample of this alloy is 6 kg, what is the mass of copper?

  (ii) If the mass of copper in a sample of this alloy is 10 kg, what is the mass of zinc?

 (iii) Find the mass of copper in 28 kg of the alloy.

 (iv) If the mass of zinc in a sample of this alloy is 2 kg, what is the mass of the sample?

Miscellaneous Exercise

(1) Silver and copper were mixed in the ratio 2 : 3 to make a statue. If the mass of the statue is 1425 g, find the mass of silver in the statue.

(2) Kamalini, Nimal and Tharaka divided several veralu (olives) among themselves in the ratio 1 : 3 : 5. If Tharaka received 15 veralu, how many veralu did Kamalini receive? Find also the number of veralu that Nimal received.

(3) The ratio of Sinhalese to Tamils to Muslims in a certain city is 5 : 4 : 3. If the total population in the city is 7200, find how many Sinhalese there are in the city.

Summary

- Multiplying (or dividing) the terms of a given ratio by a fixed number gives an equivalent ratio.

- If all the terms of a ratio are whole numbers and their HCF is one, the ratio is said to be in its simplest form.

- The total number of parts of a ratio is the sum of the terms of the ratio. The number of parts of each item is the term in the ratio associated with that item.

  For example, a concrete mixture in which cement, sand and granite are mixed in the ratio 3 : 6 : 8 has 3 parts of cement, 6 parts of sand and 8 parts of granite. Thus, the total number of parts is 17.

- When ingredients are mixed in a given ratio, if the amount of one ingredient or the total amount is known, it is possible to find the amount of a single part, by dividing the known amount by the relevant number of parts. Thereby the individual amounts of the other ingredients and the total amount can be determined.