

Department of Examinations - Sri Lanka

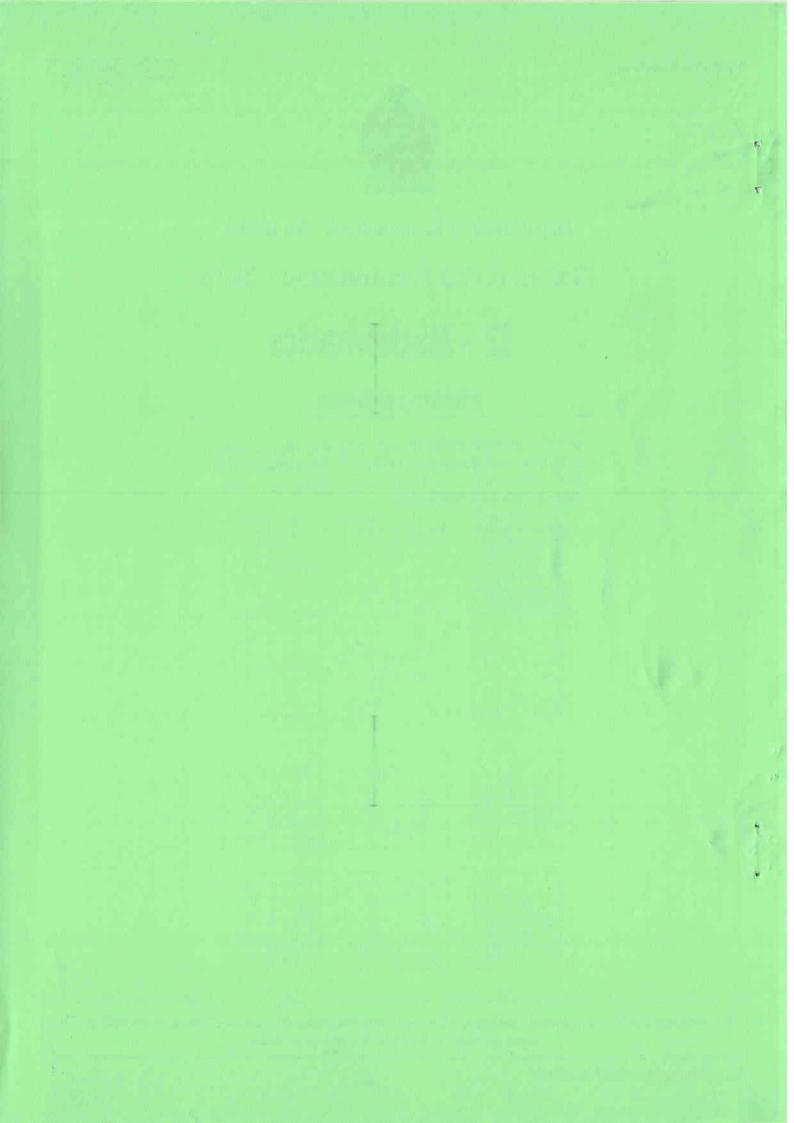
G.C.E. (O/L) Examination - 2018

## 32 - Mathematics

### **Marking Scheme**

In this "supermagic square", not only do the rows, columns and diagonals add up to 34, but so do all the combinations of 4 numbers marked by linked dots in the squares below:

This document has been prepared for the use of Marking Examiners. Some changes would be made according to the views presented at the Chief Examiners' meeting.



#### Part A

Answer all questions on this question paper itself.

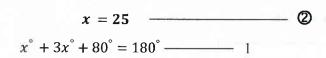
- Area of the curved surface of a right circular cylinder of radius r and height h is  $2\pi rh$ .
- Wherever necessary, use  $\frac{22}{7}$  for the value of  $\pi$ .
- 1. It has been estimated that it will take 10 men 6 days to complete a certain task. Find the number of days it will take 8 men to complete a job which is double that task.

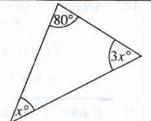
Amount of work = 
$$10 \times 6 \times 2$$
 man days \_\_\_\_\_\_ 1

2. Factorize:  $2x^2 + x - 6$ 

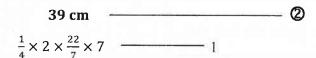
$$(x+2)(2x-3)$$
 2 $x^2 + 4x - 3x - 6$  1

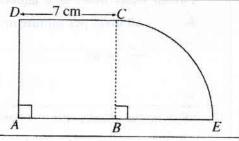
3. Find the value of x based on the information given in the figure.





4. In the figure, ABCD is a square; BCE is a sector. Find the perimeter of the composite figure.





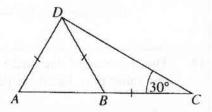
 $5. \quad \text{Simplify:} \quad \frac{4}{x} - \frac{1}{2x}$ 

$$\frac{7}{2x}$$

6. In the figure, ABC is a straight line. Find the magnitude of  $D\hat{A}B$  based on the given information.

$$D\widehat{A}B = 60^{\circ}$$

$$B\widehat{D}C = 30^{\circ}$$



7.  $26.3 = 10^{1.42}$ .

What is the value of lg 26.3?

1.42

21. Write the 7th term of the geometric progression with first term 8 and common ratio 2, as a power of 2.

$$T_7 = 2^9$$
 — 2
 $T_7 = 8 \times 2^6$  — 1

22. Find the gradient of the straight line that passes through the points (0, 8) and (2, 4).

Gradient = -2 
$$4 = m \times 2 + 8 \text{ or } \frac{8-4}{0-2}$$
 1

23. The first quartile of an array of data that has been arranged in ascending order is in the 7th position. How many data are there in this array?

$$\frac{1}{4}(n+1) = 7 - 1$$

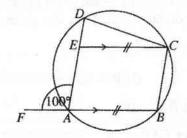
24. Simplify: 
$$\frac{3a}{10b} \div \frac{9}{5b}$$

$$\frac{a}{6} - 2$$

$$\frac{3a}{10b} \times \frac{5b}{9} - 1$$

25. In the given figure, ABCE is a parallelogram. The 4 points A, B, C and D lie on the circle. Find the magnitude of  $E\hat{C}D$  based on the given information.

$$E\hat{C}D = 20^{\circ}$$
 ②
 $B\hat{C}D = 100^{\circ} \text{ or } B\hat{C}E = 80^{\circ}$  ①



# Part B Answer all questions on this question paper itself.

- 1. A man intended to distribute a certain amount of money he had, by giving  $\frac{2}{5}$  to his wife and the remaining amount equally to his three sons. However, he had to give  $\frac{1}{6}$  of this amount to his brother before he distributed it as intended. He distributed the remaining amount as originally intended.
  - (i) What fraction of the initial amount that the man had, did the wife receive?

Fraction the wife received  $=\frac{2}{5}$  of  $\frac{5}{6}$  = 1+1  $=\frac{1}{3}$  = 1

(ii) What fraction of the initial amount did he have remaining after giving his brother and his wife?

wife?
Portion given to his brother and wife  $=\frac{1}{6} + \frac{1}{3}$  or  $\frac{5}{6} - \frac{1}{3}$  1  $= \frac{1+2}{6} \text{ or } \frac{5-2}{6}$  1
Remaining portion  $=\frac{1}{2}$  1

(iii) The amount a son received was 40 000 rupees less than the amount he was to receive originally. Find the amount the man had initially.

Portion received by a son now  $=\frac{1}{3}$  of  $\frac{1}{2}=\frac{1}{6}$  1

Portion a son was to receive  $=\frac{1}{3}$  of  $\frac{3}{5}=\frac{1}{5}$  1

Reduced portion  $= \frac{1}{5} - \frac{1}{6} = \frac{1}{30}$ 

Amount = Rs. 1200000

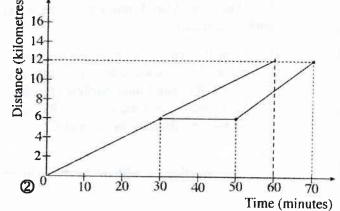
10

- 2. How a student travelled from his home to school is shown in the given distance-time graph.
  - (i) For how long did the student stop in between?

20 minutes ———— 1 ①

(ii) Find the speed at which he travelled during the initial 30 minutes in kilometres per hour.

Speed =  $\frac{6}{1/2}$  1 = 12 kilometres per hour



(iii) What multiple of the speed at which he travelled the initial 30 minutes is the speed at which he travelled the final 20 minutes?

Speed in the final 20 minutes =  $\frac{6}{1/3}$  = 18 kilometres per hour — 1+1  $\frac{18}{12}$  — 1

The final speed is  $1\frac{1}{2}$  times the intial speed [1] (iv) If he travelled the whole distance without stopping, in the same speed at which he travelled the initial 30 minutes, draw the relevant graph on this figure itself.

In this case, how many minutes earlier would the student be able to complete the journey?

10 minutes earlier ————

3

10

<b>3</b> .	(a)	Customs	duty	y of 3	0% is	charge	d when	ele	ctrica	litem	s ar	e importe	ed. If	9 00	)0 n	upees	has	s to
		be paid	as c	ustom	s duty	when	an iten	n of	this	type	is i	mported,	what	is t	he	value	of	the
		item wh	ich i	s bein	g impo	orted?				• •								

Value = Rs. 
$$9000 \times \frac{100}{30}$$
 \_\_\_\_\_\_ 2  
= Rs.  $30000$  \_\_\_\_\_\_ 1

(b) (i) The annual assessed value of a house is 30 000 rupees. If the municipal council charges annual rates of 8% on this property, find how much has to be paid as rates for a quarter.

Annual rates = Rs. 
$$30\ 000 \times \frac{8}{100} = 1$$

Rates for a quarter = Rs.  $\frac{2400}{4} = 1$ 

= Rs.  $600 = 1$ 

(ii) After several years, the assessed value of the house changed. The annual rates percentage that the municipal council charges also increased to 9%. If the amount to be paid as rates for a quarter increased by 30 rupees as a result, find the new annual assessed value of the house.

10

4. (a) A bag contains 3 vanilla flavoured milk packets and 2 chocolate flavoured milk packets of the same size. After Kamala takes out a milk packet randomly, Nimala also takes out a milk packet randomly.

(i) Using the symbol 'x', represent the sample space of the above experiment in the given grid. The vanilla flavoured milk packets are denoted by  $V_1$ ,  $V_2$  and  $V_3$  and the chocolate flavoured milk packets are denoted by  $C_1$  and  $C_2$ .

Marking 'x' without the diagonal =

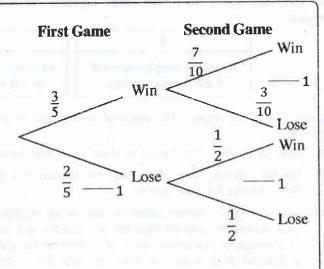
(ii) In the grid, encircle the event of both of them taking out vanilla flavoured milk packets and find its probability.

Encircling \_\_\_\_\_ 1

Identifying 20 elements in the sample space

Probability =  $\frac{6}{20}$  or  $\frac{3}{10}$  \_\_\_\_\_ 1

(b) The probability of a certain sports team winning the first game they participate in is  $\frac{3}{5}$ . If they win the first game, then the probability of them winning the second game is  $\frac{7}{10}$ . If they lose the first game, then the probability of them winning the second game is  $\frac{1}{2}$ . An incomplete tree diagram drawn to represent this information is shown in the figure.



- (i) Complete the tree diagram by indicating the relevant probabilities.
- (ii) Find the probability of the team winning at least one game.

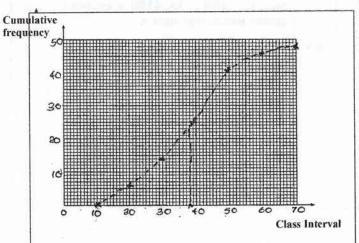
$$\left(\frac{3}{5} \times \frac{7}{10}\right) + \left(\frac{3}{5} \times \frac{3}{10}\right) + \left(\frac{2}{5} \times \frac{1}{2}\right) = 1$$

$$= \frac{40}{50} \quad \text{or} \quad \frac{4}{5} = 1$$

10

5. Given below is a grouped frequency distribution of 48 continuous data. All the data which are greater or equal to 10 but less than 20 belong to the class interval 10 - 20. Likewise, the other class intervals.

Class Interval	Frequency	Cumulative frequency		
10 - 20	6	6		
20 - 30	8	14		
30 - 40	12	26		
40 - 50	15	41		
50 - 60	5	46		
60 - 70	2.	48		



- (i) Fill in the blanks in the table.
- (ii) Draw the cumulative frequency curve on the given coordinate plane and thereby obtain the median of the frequency distribution.

of the frequency distribution.

Marking the axes

Joining to the point (10, 0)

Marking at least four points other than (10, 0) correctly

1

Drawing the curve

1

Median 38 or 39

(iii) By how much does the median that was obtained in part (ii) above deviate from the midpoint of the class interval it belongs to?

3 or 4 \_\_\_\_\_\_ 1

2

10

### Paper II (Part A)

1. The following notices have been issued regarding the interest paid by two banks A and B for deposits.

A	В
An annual simple interest of 5.2% for your deposit!	An annual compound interest of 5 % for your deposit!

Saman had  $80\,000$  rupees. He deposited exactly half of it in bank  ${\bf A}$  and the remaining half in bank  ${\bf B}$ .

- (i) Find the interest that Saman receives for a year from his deposit in bank A.
- (ii) For his deposits, from which bank will he receive a greater income at the end of two years? Give reasons for your answer.
- (iii) After two years Saman added the amount he initially deposited and an extra amount to the total income he received from the two deposits and invested this whole amount to buy shares of a company. The market price of a share of this company is 50 rupees. The company pays a dividend of 2 rupees per share annually. He received a dividend income of 3600 rupees at the end of a year. Find the extra amount he added when he bought the shares.

Question No.		Marking Scheme	ann	Mark	S	Other facts	
1	(i)	Interest received by Saman = Rs. $40000 \times \frac{5.2}{100}$ = Rs. 2080	1	2			
	(ii)	Income for two years from bank A = Rs. 4160 Income for the first year from bank B = Rs. $40000 \times \frac{5}{100}$ Income for the second year = Rs. $42000 \times \frac{5}{100}$ Total income from bank B = Rs. 4100	1 1 1		avya.		
		Since Rs. 4160 > Rs. 4100 he receives a greater income from bank A	1	(5)			
	(iii)	Number of shares = 1800 Amount invested = Rs. 1800 × 50 ∴ Extra amount added = Rs. 1740	1 1 1	3			
					10		

2. The sum of the lengths of two adjacent sides of a rectangle is 16 cm and the length of a diagonal is 14 cm. Show that, when the breadth of the rectangle is taken as x cm, it satisfies the quadratic equation  $x^2 - 16x + 30 = 0$ , and find separately the length and the breadth of the rectangle to the first decimal place.

(Use 5.83 for the value of  $\sqrt{34}$ .)

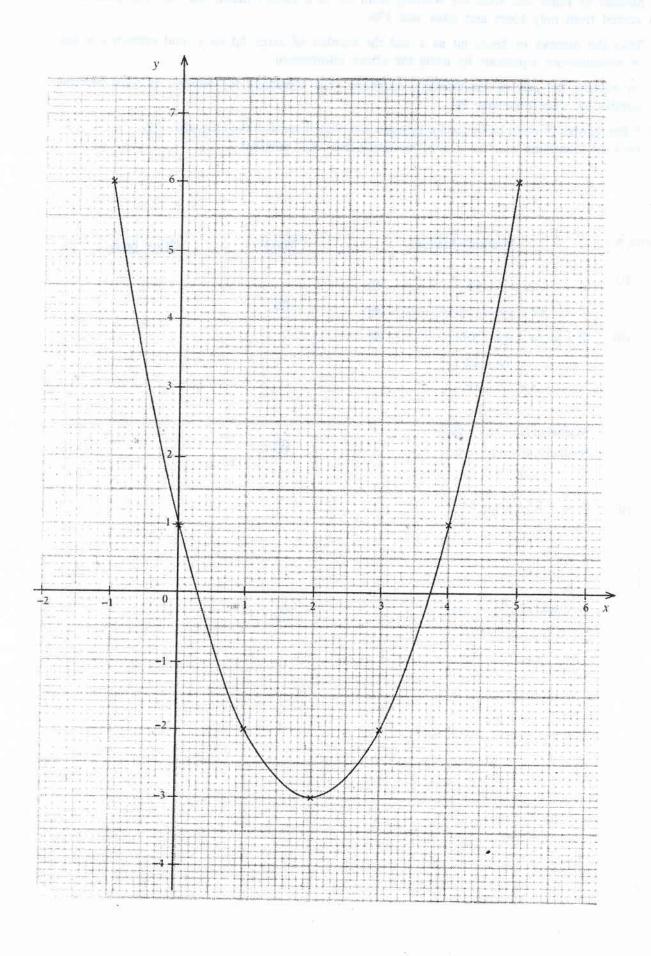
Ques	tion No.	Marking Scheme	Marks	Other facts		
Ques:	(i)	Marking Scheme  If the breadth of the rectangle is $x$ cm, the length = $(16 - x)$ cm  By Pythagoras' theorem $x^2 + (16 - x)^2 = 14^2$ $x^2 + 256 - 32x + x^2 = 196$ $2x^2 - 32x + 60 = 0$ $x^2 - 16x + 30 = 0$ $(x - 8)^2 = -30 + 64$ $x - 8 = \pm \sqrt{34}$ $x = 8 + 5.83 \text{ or } x = 8 - 5.83$ $x = 13.83 \text{ or } x = 2.17$ $\therefore$ Length = 13.8 cm  Breadth = 2.2 cm	Marks  1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	Other facts $x = \frac{16 \pm \sqrt{256 - 4 \times 1 \times 30}}{2}$ $x = 8 \pm \sqrt{34}$		
			10			

3. y is a quadratic function of x. An incomplete table containing the values of y corresponding to several values of x is given below.

ſ	х	-1	0	1	2	3	4	5
1	y	6	1	-2	-3	-2		6

- (i) By considering the symmetry of the quadratic function, obtain the value of y when x = 4.
- (ii) Using the standard system of axes and a suitable scale, draw the graph of the quadratic function on a graph paper based on the above table of values.
- (iii) Describe the behaviour of y as the value of x increases from 0 to 2.
- (iv) Express the quadratic function in the form  $y = (x a)^2 + b$ .
- (v) y = t is a straight line parallel to the x-axis. What is the interval in which t should lie for this straight line and the graph of the quadratic function to intersect at two points with positive x-coordinates?

ion No.	Marking Scheme		Mark	S	Other facts
(i)	y=1 when $x=4$	1	①		10 at 1
(ii)	Correct scale  Marking 5 points correctly  Smooth curve	1 1 1	3		
(iii)	Positive and decreasing from 1 to 0  Negative and decreasing from 0 to -3	1	2		The state of the s
(iv)	$y = (x-2)^2 - 3$	1+1	2		
(v)	-3 < t < 1	1+1	2	20	
		6		10	
	(iii)	<ul> <li>(ii) Correct scale Marking 5 points correctly Smooth curve</li> <li>(iii) Positive and decreasing from 1 to 0 Negative and decreasing from 0 to -3</li> <li>(iv) y = (x - 2)<sup>2</sup> - 3</li> <li>(v) -3 &lt; t &lt; 1</li> </ul>	(iii) Correct scale Marking 5 points correctly Smooth curve  (iii) Positive and decreasing from 1 to 0 Negative and decreasing from 0 to -3  (iv) $y = (x-2)^2 - 3$ (v) $-3 < t < 1$	(iii) Correct scale Marking 5 points correctly Smooth curve  (iii) Positive and decreasing from 1 to 0 Negative and decreasing from 0 to -3  (iv) $y = (x-2)^2 - 3$ (v) $-3 < t < 1$ 1 2  1 2  1+1 2  1+1 2	(iii) Correct scale Marking 5 points correctly Smooth curve  (iii) Positive and decreasing from 1 to 0 Negative and decreasing from 0 to -3  (iv) $y = (x-2)^2 - 3$ (v) $-3 < t < 1$ 1  2  1-1  2  1-1  2  1-1  2



- 4. The number of fours and sixes the winning team hit in a cricket match was 38. The number of runs scored from only fours and sixes was 176.
  - (i) Take the number of fours hit as x and the number of sixes hit as y, and construct a pair of simultaneous equations by using the above information.
  - (ii) By solving the pair of simultaneous equations, find separately the number of fours and the number of sixes that were hit.
  - (iii) If the number of sixes hit by the losing team is a, then it satisfies the inequality  $2(2a-5)+3a \le 54$ . Find the **maximum** number of sixes the losing team may have hit.

Quest	tion No.	Marking Scheme		Marks	Other facts		
4	(i) (ii)	Warking Scheme $x + y = 38 \qquad 0$ $4x + 6y = 176 \qquad 2$ $0 \times 4, \ 4x + 4y = 152 \qquad 3$ $y = 12$ $x + 12 = 38$ $x = 26$ Number of fours hit = 26 Number of sixes hit = 12	1 1 1 1 1 1	② ⑤	Other facts		
	(iii)	$2(2a-5) + 3a \le 54$ $7a \le 64$ $a \le \frac{64}{7}$	1				
		Maximum number of sixes = 9	1	10			

- 5. The base of a cuboid shaped glass container of height one metre is a square. The length of a side of the base is 25 cm. The container is filled with water to exactly half its height.
  - (i) Find the volume of water in the container in cubic centimetres.
  - (ii) Rani has several identical solid right circular metal cylinders of unknown base radius and height 10 cm. To find the base radius r of a cylinder, she puts them one by one into the above container half filled with water. When exactly 25 of them are put, the water reaches the level of the container being completely filled.

Show that  $r = 5\sqrt{\frac{5}{\pi}}$  cm.

(iii) Find the value of r in centimetres to the first decimal place, by using 3.14 for the value of  $\pi$ .

Quest	ion No.	Marking Scheme		Mar	ks	Other facts		
3	(i)	Volume of water $= 25 \times 25 \times 50$ $= 31250 \text{ cm}^3$	1	0				
	(ii)	Volume of the 25 cylinders $= \pi \times r^2 \times 10 \times 25$	741			$\pi \times r^2 \times 10 \times 25 = 31256$		
		$\pi \times r^2 \times 10 \times 25 = 25 \times 25 \times 50$	1					
		$r^2 = \frac{125}{\pi}$	1					
		$r^2 = \frac{25 \times 5}{\pi}$ $r = 5\sqrt{\frac{5}{\pi}}$	1	4				
		$r = 5 \times \sqrt{\frac{5}{3.14}}$	1			$5 \times \sqrt{\frac{5}{3.14}}$ $5\sqrt{1.592} \longrightarrow 1$		
		$\lg r = \lg 5 + \frac{1}{2} \{ \lg 5 - \lg 3.14 \}$				5 × (1.261) 2		
		$=0.6990+\frac{1}{2}\{0.6990-0.4969\}$	1+1			. 205		
		= 0.8001	1		4	6.305 ——1 6.3 cm—— 1		
		r = 6.3  cm	1	(5)	10			
					10			

6. Nimal is involved in a small industry which produces sports items. Information regarding the number of items he produced each day during a period of 50 days is shown in the following frequency distribution.

Number of Items	20 - 30	20 - 30   30 - 40   40 - 50   50 - 60		50 - 60	60 - 70	70 - 80	
Number of Days	5	8	10	12	9	6	

Nimal gains a profit of 60 rupees by selling one of these items. He expects to gain a profit of 370 000 rupees during the next 120 days by working and selling the items in the above manner. Find the mean number of sports items he produces in a day, and show with reasons whether his expectation is fulfilled.

Ques	tion No.		Mar	king Scl	heme			Mark	S	Other facts	
6	(i)	Number of items  20-30 30-40 40-50 50-60 60-70 70-80	Number of days (f)  5  8  10  12  9  6	Midvalue (x)  25  35  45  55  65  75	(fx)  125 280 450 660 585 450  \$\sum_{fx}\$ = 2550				- 10		
			mber of sp $= \frac{\Sigma f x}{\Sigma f}$ $= \frac{2550}{50}$ $= 51$	g 120 days			1 1 1			fd column ——— 2 ( Disregard one error in the fx or fd column ) Σfd ———— 1	
			Rs.367200 xpectation	= < Rs.370	- F	20	1+1		10		

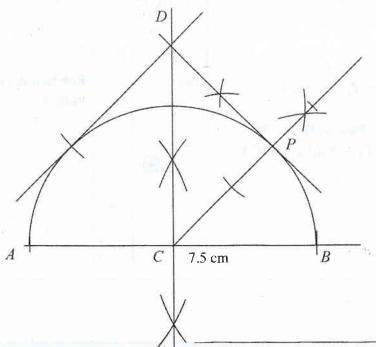
### Paper II (Part B)

- 7. A decoration consists of several circles containing small bulbs. There are 5 bulbs in the first circle, 9 bulbs in the second circle, 13 bulbs in the third circle, and so on. Starting from the first circle, when the number of bulbs in each of the circles is considered in order, they are in an arithmetic progression.
  - (i) How many bulbs are there in the 10th circle?
  - (ii) If the total number of bulbs in the first n circles is  $S_n$ , show that  $S_n = n(2n + 3)$ .
  - (iii) If the decoration consists of 40 circles, find the total number of bulbs in the decoration.
  - (iv) Among the circles, starting from the 10th circle, every circle which counts as a multiple of 5 consists of only yellow bulbs while all the other bulbs are red. Find the number of red bulbs in the decoration.

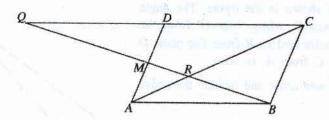
Questi	ion No.	Marking Scheme		Marks		Other facts
Ø	(i)	$\mathbf{T}_{n} = a + (n-1)d$	1	980		alemili ale
		$T_{10} = 5 + (10 - 1) \times 4$	1			ha as year at All
		= 41	1	3		
	(ii)	$S_n = \frac{n}{2} \{2a + (n-1)d\} \text{ or }$	1			man ages all
		$= \frac{n}{2} \{2 \times 5 + (n-1)4\}$				
	=	$= \frac{n}{2}(4n+6)$ $= n(2n+3)$	1	2		
	(iii)	$\mathbf{S_{40}} = 40 \left(2 \times 40 + 3\right)$			i.	
		= 3320	1	<b>①</b>		
	(iv)	a = 41,  n = 7,  d = 20	1+1			For two correct values
		Number of yellow bulbs = 707 ∴ Number of red bulbs = 2613	1 1	4		
					10	1
		ļ' <del>-</del>			10	

- 8. Use only a straight edge with a cm/mm scale and a pair of compasses for the following constructions. Show the construction lines clearly.
  - (i) Draw a straight line segment AB of length 7.5 cm and construct its perpendicular bisector.
  - (ii) Take the midpoint of AB as C and construct a semicircle with C as the centre and AB as the diameter.
  - (iii) Construct the locus of a point that moves at an equal distance from the perpendicular bisector of AB and the line CB and name the point at which it intersects the semicircle as P.
  - (iv) Construct the tangent to the semicircle at P and name the point at which it meets the perpendicular bisector of AB as D.
  - (v) Construct the other tangent that can be drawn to the semicircle from D and give reasons why this tangent is parallel to the line PC.

Question No.		Marking Scheme	Marks			Other facts
8	(i)	The straight line <i>AB</i> The perpendicular bisector	1 2	3		
	(ii)	Semicircle	1	0		
	(iii)	Angle bisector	1	0		
	(iv)	Tangent	2	2		
	(v)	The other tangent from $D$ Obtaining $E\widehat{D}C = 45^{\circ}$ Giving reasons for being parallel	1 1 1	3	10	
					10	5111 687 1111



9. In the parallelogram ABCD shown in the figure, M is the midpoint of the side AD. The point intersection of BM and AC is R. Moreover, the lines BM and CD produced meet at Q.

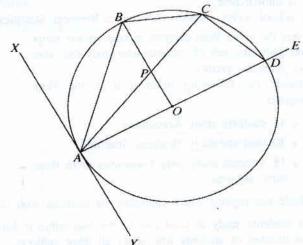


Copy this figure in your answer script.

- (i) Join AQ and BD, and show that ABDQ is a parallelogram.
- (ii) Show that  $\frac{MR}{RB} = \frac{1}{2}$  and that QR = 2RB.

Question No.		Marking Scheme	Marks			Other facts
9		C A B				
	(ii)	In the triangles $QDM$ and $AMB$ , $DM = MA$ (given) $Q\widehat{D}M = M\widehat{A}B \text{ (alternate angles, } QC \parallel AB)$ $D\widehat{Q}M = M\widehat{B}A \text{ (alternate angles, } QC \parallel AB)$ $\therefore \Delta QDM \equiv \Delta AMB \text{ (A.A.S.)}$ $\therefore QM = MB \text{ (corresponding sides of congruent } \Delta S \text{ )}$ $\therefore ABDQ \text{ is a parallelogram (diagonals bisect)}$ In the triangles $AMR$ and $BCR$ , $M\widehat{A}R = B\widehat{C}R$ (alternate angles, $AD \parallel BC$ ) $M\widehat{R}A = B\widehat{R}C$ (vertically opposite angles) $A\widehat{M}R = R\widehat{B}C \text{ (remaining angles)}$ $\therefore \Delta AMR \text{ and } \Delta BCR \text{ are equiangular}$ $\therefore \frac{MR}{RB} = \frac{AM}{BC}$ $2AM = BC \text{ (}M \text{ is the midpoint of } AB \text{ )}$	1 1 1 1	4		
		$ \frac{MR}{RB} = \frac{AM}{2AM} $ $ \frac{MR}{RB} = \frac{1}{2} $ $\frac{MR}{RB} = \frac{1}{2} $ $$	1 1	6	10	

- 12. In the given figure, the tangent drawn to the circle with centre O, at the point A, is XAY. The chord AB bisects  $X\widehat{A}O$ . The diameter AD has been produced to E and the point C lies on the circle between the points B and D. Moreover, the point of intersection of AC and OB is P. With reasons show that,
  - (i)  $A\hat{C}B = 45^{\circ}$
  - (ii)  $Y\hat{A}C = C\hat{D}E$
  - (iii)  $B\hat{P}C = O\hat{D}C$ .



Question No	Marking Scheme		Mark	s	Other facts		
2.	X O E						
(i)	$O\widehat{A}X = 90^{\circ}$ (angle between the tangent and radius) $B\widehat{A}X = B\widehat{A}O = 45^{\circ}$ ( $O\widehat{A}X$ is bisected by $AB$ ) $B\widehat{A}X = A\widehat{C}B$ (angle in the alternate segment) $\therefore A\widehat{C}B = 45^{\circ}$	1 1 1+1	4				
(ii)	quadrilateral is equal to its interior opposite angle) $\widehat{YAC} = \widehat{ABC}$ (angle in the alternate segment) $\widehat{YAC} = \widehat{CDE}$	1	2				
(iii	$B\widehat{O}A = 90^{\circ}$ (2 $B\widehat{C}A = B\widehat{O}A$ ) $A\widehat{C}D = 90^{\circ}$ (angle in a semicircle) $\therefore PODC$ is a cyclic quadrilateral (opposite angles are supplementary) $\therefore B\widehat{P}C = O\widehat{D}C$ (Exterior angle of a cyclic quadrilateral is equal to its interior opposite angle)	1 1 1	4	10	Securities in material  (a) apply - ac  (b) apply - ac  (c) apply - ac  (d) ap		