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இலங்கைப் பரீட்சைத் திணைக்களம்
Department of Examinations, Sri Lanka

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2018 අගෝස්තු
கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2018 ஓகஸ்ற்
General Certificate of Education (Adv. Level) Examination, August 2018

සංයුක්ත ගණිතය I
இணைந்த கணிதம் I
Combined Mathematics I

10 E I

06.08.2018 / 0830 – 1140

පැය තුනයි
மூன்று மணித்தியாலம்
Three hours

අමතර කියවීම් කාලය - මිනිත්තු 10 යි
மேலதிக வாசிப்பு நேரம் - 10 நிமிடங்கள்
Additional Reading Time - 10 minutes

Use **additional reading time** to go through the question paper, select the questions and decide on the questions that you give priority in answering.

Index Number | | | | | | | |

Instructions:

- *This question paper consists of two parts;*
  Part A (Questions 1 - 10) and **Part B** (Questions 11 - 17).
- **Part A:**
  *Answer **all** questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.*
- **Part B:**
  *Answer **five** questions only. Write your answers on the sheets provided.*
- *At the end of the time allotted, tie the answer scripts of the two parts together so that* **Part A** *is on top of* **Part B** *and hand them over to the supervisor.*
- *You are permitted to remove **only** **Part B** of the question paper from the Examination Hall.*

For Examiners' Use only

(10) Combined Mathematics I		
Part	**Question No.**	**Marks**
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
	Total	
	Percentage	

Paper I	
Paper II	
Total	
Final Marks	

Final Marks

In Numbers	
In Words	

Code Numbers

Marking Examiner		
Checked by:	1	
	2	
Supervised by:		

[see page two

Part A

1. Using the **Principle of Mathematical Induction**, prove that $\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$ for all $n \in \mathbb{Z}^+$.

2. Sketch the graphs of $y = 3 - |x|$ and $y = |x - 1|$ in the same diagram.
Hence or otherwise, find all real values of x satisfying the inequality $|x| + |x-1| \leq 3$.

[see page three

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3. Sketch, in an Argand diagram, the locus of the points that represent complex numbers z satisfying $\mathrm{Arg}(z-3i) = -\frac{\pi}{3}$.

Hence or otherwise, find the minimum value of $|z-1|$ such that $\mathrm{Arg}(\bar{z}+3i)=\frac{\pi}{3}$.

4. The coefficients of x and x^4 of the binomial expansion of $\left(x^2+\frac{3k}{x}\right)^8$ are equal. Find the value of the constant k.

[see page four

5. Show that $\lim_{x\to 0} \dfrac{1-\cos\left(\dfrac{\pi x}{4}\right)}{x^2(x+1)} = \dfrac{\pi^2}{32}$.

6. Show that the area of the region enclosed by the curves $y=e^{2x}$, $y=e^{3-x}$, $x=0$, $x=3$ and $y=0$ is $\frac{3}{2}\left(e^2-1\right)$ square units.

[see page five

7. A curve C is given by the parametric equations $x = \ln\left(\tan\frac{t}{2}\right)$ and $y = \sin t$ for $\frac{\pi}{2} < t < \pi$.

Show that $\frac{dy}{dx} = \cos t \sin t$.

Deduce that the gradient of the tangent line drawn to the curve C at the point corresponding to $t = \frac{2\pi}{3}$ is $-\frac{\sqrt{3}}{4}$.

8. Let l_1 be the straight line $x + y - 5 = 0$. Find the equation of the straight line l_2 passing through the point $P \equiv (3, 4)$ and perpendicular to l_1.

Let Q be the point of intersection of l_1 and l_2, and let R be the point on l_2 such that $PQ : QR = 1 : 2$. Find the coordinates of R.

[see page six

9. Let $P \equiv (1,2)$ and $Q \equiv (7,10)$. Write down the values of the constants a and b such that the equation of the circle with points P and Q as the ends of a diameter is $S \equiv (x-1)(x-a)+(y-2)(y-b)=0$.

Let $S' \equiv S + \lambda(4x-3y+2)=0$, where $\lambda \in \mathbb{R}$. Show that the points P and Q lie on the circle $S'=0$, and find the value of λ such that this circle passes through the point $R \equiv (1,4)$.

10. Show that $\sec^3 x + 2\sec^2 x \tan x + \sec x \tan^2 x = \dfrac{\cos x}{(1-\sin x)^2}$ for $x \neq (2n+1)\dfrac{\pi}{2}$, where $n \in \mathbb{Z}$.

Department of Examinations, Sri Lanka

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2018 අගෝස්තු
கல்விப் பொதுத் தராதரப் பத்திர (உயர் தரப் பரீட்சை, 2018 ஓகஸ்ற்
General Certificate of Education (Adv. Level) Examination, August 2018

සංයුක්ත ගණිතය I
இணைந்த கணிதம் I
Combined Mathematics I

10 E I

Part B

* Answer **five** questions only.

11. (*a*) Let $a, b \in \mathbb{R}$. Write down the discriminant of the equation $3x^2 - 2(a+b)x + ab = 0$ in terms of a and b, and **hence**, show that the roots of this equation are real.

Let α and β be these roots. Write down $\alpha + \beta$ and $\alpha\beta$ in terms of a and b.

Now, let $\beta = \alpha + 2$. Show that $a^2 - ab + b^2 = 9$ and **deduce** that $|a| \le \sqrt{12}$, and find b in terms of a.

(*b*) Let $c (\neq 0)$ and d be real numbers, and let $f(x) = x^3 + 4x^2 + cx + d$. The remainder when $f(x)$ is divided by $(x + c)$ is $-c^3$. Also, $(x - c)$ is a factor of $f(x)$. Show that $c = -2$ and $d = -12$.

For these values of c and d, find the remainder when $f(x)$ is divided by $(x^2 - 4)$.

12. (*a*) A committee of six members has to be selected from among the members of two groups, each having three boys and two girls, such that the number of girls in the committee is at most two. Find the number of different such committees that can be formed if,

(i) even number of members from each group are to be selected for the committee,

(ii) only one girl is to be selected for the committee.

(*b*) Let $f(r) = \dfrac{1}{(r+1)^2}$ and $U_r = \dfrac{(r+2)}{(r+1)^2\,(r+3)^2}$ for $r \in \mathbb{Z}^+$.

Show that $f(r) - f(r+2) = 4\,U_r$ for $r \in \mathbb{Z}^+$.

Hence, show that $\displaystyle\sum_{r=1}^{n} U_r = \frac{13}{144} - \frac{1}{4(n+2)^2} - \frac{1}{4(n+3)^2}$ for $n \in \mathbb{Z}^+$.

Deduce that the infinite series $\displaystyle\sum_{r=1}^{\infty} U_r$ is convergent and find its sum.

Let $t_n = \displaystyle\sum_{r=n}^{2n} U_r$ for $n \in \mathbb{Z}^+$.

Show that $\displaystyle\lim_{n \to \infty} t_n = 0$.

[see page eight

13.(*a*) Let $\mathbf{A}=\begin{pmatrix} 1 & 1 & 0 \\ 2 & 4 & -1 \end{pmatrix}$ and $\mathbf{B}=\begin{pmatrix} 3 & 2a \\ -1 & 0 \\ 1 & 3a \end{pmatrix}$, where $a \in \mathbb{R}$.

Find the matrix $\mathbf{P}$ defined by $\mathbf{P} = \mathbf{AB}$ and show that $\mathbf{P}^{-1}$ **does not exist** for any value of a.

Show that if $\mathbf{P}\begin{pmatrix} 1 \\ 2 \end{pmatrix} = 5\begin{pmatrix} 2 \\ 1 \end{pmatrix}$, then $a = 2$.

With this value for a, let $\mathbf{Q} = \mathbf{P} + \mathbf{I}$, where $\mathbf{I}$ is the identity matrix of order 2.

Write down $\mathbf{Q}^{-1}$ and find the matrix $\mathbf{R}$ such that $\mathbf{AA}^{\mathrm{T}} - \frac{1}{2}\mathbf{R} = \left(\frac{1}{5}\mathbf{Q}\right)^{-1}$.

(*b*) Let $z = x + iy$, where $x, y \in \mathbb{R}$. Define the modulus $|z|$ and the conjugate $\bar{z}$ of z.

Show that (i) $z\bar{z} = |z|^2$,

(ii) $z + \bar{z} = 2\,\mathrm{Re}\,z$ and $z - \bar{z} = 2i\,\mathrm{Im}\,z$.

Let $z \neq 1$ and $w = \dfrac{1+z}{1-z}$. Show that $\mathrm{Re}\,w = \dfrac{1-|z|^2}{|1-z|^2}$ and $\mathrm{Im}\,w = \dfrac{2\,\mathrm{Im}\,z}{|1-z|^2}$.

Show further that if $z = \cos\alpha + i\sin\alpha$ $(0 < \alpha < 2\pi)$, then $w = i\cot\dfrac{\alpha}{2}$.

(*c*) In an Argand diagram, the points A and B represent the complex numbers $-3i$ and 4 respectively. The points C and D lie in the first quadrant such that $ABCD$ is a rhombus and $B\hat{A}D = \theta$, where $\theta = \sin^{-1}\left(\frac{7}{25}\right)$. Find the complex numbers represented by the points C and D.

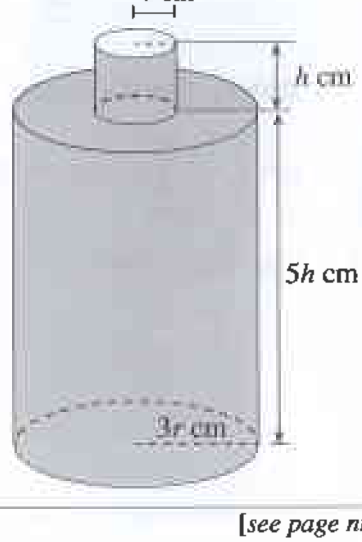
14.(*a*) Let $f(x) = \dfrac{16(x-1)}{(x+1)^2(3x-1)}$ for $x \neq -1, \frac{1}{3}$.

Show that $f'(x)$, the derivative of $f(x)$, is given by $f'(x) = \dfrac{-32x(3x-5)}{(x+1)^3(3x-1)^2}$ for $x \neq -1, \frac{1}{3}$.

Sketch the graph of $y=f(x)$ indicating the asymptotes and the turning points.

Using the graph, find the values of $k \in \mathbb{R}$ such that the equation $k(x+1)^2(3x-1) = 16(x-1)$ has exactly one root.

(*b*) A bottle with a volume of $391\pi\,\mathrm{cm}^3$ is to be made by removing a disc of radius r cm from the top face of a closed hollow right circular cylinder of radius $3r$ cm and height $5h$ cm, and fixing an open hollow right circular cylinder of radius r cm and height h cm, as shown in the figure. It is **given that** the total surface area $S\,\mathrm{cm}^2$ of the bottle is $S = \pi r(32h + 17r)$. Find the value of r such that S is minimum.



[see page nine

15.(*a*) (i) Comparing the coefficients of x^2, x^1 and x^0, find the values of the constants A, B and C such that $Ax^2(x-1)+Bx(x-1)+C(x-1)-Ax^3=1$ for all $x\in\mathbb{R}$.

Hence, write down $\dfrac{1}{x^3(x-1)}$ in partial fractions and find $\displaystyle\int\frac{1}{x^3(x-1)}\,dx$.

(ii) Using integration by parts, find $\displaystyle\int x^2\cos 2x\,dx$.

(*b*) Using the substitution $\theta=\tan^{-1}(\cos x)$, show that $\displaystyle\int_0^{\pi}\frac{\sin x}{\sqrt{1+\cos^2 x}}\,dx=2\ln\left(1+\sqrt{2}\right)$.

Using the formula $\displaystyle\int_0^a f(x)\,dx=\int_0^a f(a-x)\,dx$, where a is a constant, find $\displaystyle\int_0^{\pi}\frac{x\sin x}{\sqrt{1+\cos^2 x}}\,dx$.

16. Let $A\equiv(-2,-3)$ and $B\equiv(4,5)$. Find the equations of the lines l_1 and l_2 through the point A such that the acute angle made by each of the lines l_1 and l_2 with the line AB is $\frac{\pi}{4}$.

The points P and Q are taken on l_1 and l_2 respectively such that $APBQ$ is a square.

Find the equation of PQ, and the coordinates of P and Q.

Also, find the equation of the circle S through the points A, P, B and Q.

Let $\lambda>1$. Show that the point $R\equiv(4\lambda,5\lambda)$ lies outside the circle S.

Find the equation of the chord of contact of the tangents drawn from the point R to the circle S.

As $\lambda\,(>1)$ varies, show that these chords of contact pass through a fixed point.

17. (*a*) Solve $\cos 2\theta+\cos 3\theta=0$ for $0\le\theta\le\pi$.

Write down $\cos 2\theta$ and $\cos 3\theta$ in terms of $\cos\theta$, and show that

$$\cos 2\theta+\cos 3\theta=4t^3+2t^2-3t-1,\ \text{where } t=\cos\theta.$$

Hence, write down the three roots of the equation $4t^3+2t^2-3t-1=0$ and show that the roots of the equation $4t^2-2t-1=0$ are $\cos\frac{\pi}{5}$ and $\cos\frac{3\pi}{5}$.

Deduce that $\cos\frac{3\pi}{5}=\frac{1-\sqrt{5}}{4}$.

(*b*) Let ABC be a triangle and let D be the point on BC such that $BD:DC=m:n$, where $m,n>0$. It is given that $B\hat{A}D=\alpha$ and $D\hat{A}C=\beta$. Using the Sine Rule for the triangles BAD and DAC, show that $\dfrac{mb}{nc}=\dfrac{\sin\alpha}{\sin\beta}$, where $b=AC$ and $c=AB$.

Hence, show that $\dfrac{mb-nc}{mb+nc}=\tan\left(\dfrac{\alpha-\beta}{2}\right)\cot\left(\dfrac{\alpha+\beta}{2}\right)$.

(*c*) Show that $2\tan^{-1}\left(\dfrac{1}{3}\right)+\tan^{-1}\left(\dfrac{4}{3}\right)=\dfrac{\pi}{2}$.

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இலங்கைப் பரீட்சைத் திணைக்களம்
Department of Examinations, Sri Lanka

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2018 අගෝස්තු
கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2018 ஓகஸ்ற்
General Certificate of Education (Adv. Level) Examination, August 2018

සංයුක්ත ගණිතය II
இணைந்த கணிதம் II
Combined Mathematics II

10 E II

08.08.2018 / 0830 – 1140

පැය තුනයි
மூன்று மணித்தியாலம்
Three hours

අමතර කියවීම් කාලය - මිනිත්තු 10 යි
மேலதிக வாசிப்பு நேரம் - 10 நிமிடங்கள்
Additional Reading Time - 10 minutes

Use **additional reading time** to go through the question paper, select the questions and decide on the questions that you give priority in answering.

Index Number							

Instructions:

* *This question paper consists of two parts;*
  Part A (Questions 1 – 10) *and* **Part B** (Questions 11 – 17)
* **Part A:**
  *Answer **all** questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.*
* **Part B:**
  *Answer **five** questions only. Write your answers on the sheets provided.*
* *At the end of the time allotted, tie the answer scripts of the two parts together so that* **Part A** *is on top of* **Part B** *and hand them over to the supervisor.*
* *You are permitted to remove **only Part B** of the question paper from the Examination Hall.*

* In this question paper, g denotes the acceleration due to gravity.

For Examiners' Use only

(10) Combined Mathematics II		
Part	**Question No.**	**Marks**
A	1	
	2	
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	5	
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	9	
	10	
B	11	
	12	
	13	
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	15	
	16	
	17	
	Total	
	Percentage	

Paper I	
Paper II	
Total	
Final Marks	

Final Marks

In Numbers	
In Words	

Code Numbers

Marking Examiner		
Checked by:	1	
	2	
Supervised by:		

[see page two

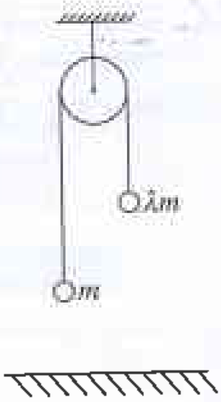
Part A

1. Two particles A and B of masses $2m$ and m respectively, moving on a smooth horizontal table along the same straight line towards each other with the same speed u, collide directly. Just after collision, the particle A comes to rest. Show that the coefficient of restitution is $\frac{1}{2}$, and that the magnitude of the impulse exerted on B due to collision is $2mu$.

2. A particle is projected from a point on the horizontal ground, at an angle $\alpha\left(0 < \alpha < \frac{\pi}{2}\right)$ to the horizontal, with initial speed $u = \sqrt{2gR}$, where R is the horizontal range of the projectile on the ground. Show that the angle between the two possible initial directions of projection is $\frac{\pi}{3}$.

[see page three

3. A particle P of mass m and a particle Q of mass λm are attached to the two ends of a light inextensible string which passes over a smooth fixed pulley. The system is released from rest, with the string taut, as shown in the figure. The particle P moves downwards with acceleration $\frac{g}{2}$. Show that $\lambda = \frac{1}{3}$.
If the particle P strikes a horizontal **inelastic** floor with speed v and the particle Q never reaches the pulley, find the time taken by the particle Q to reach the maximum height from the instant when the particle P struck the floor.



4. A car of mass 1200 kg moves with the engine shut off, down a straight road of inclination α to the horizontal, where $\sin\alpha = \frac{1}{30}$, at a certain constant speed. Taking the acceleration due to gravity $g = 10\,\mathrm{m\,s^{-2}}$, find in newtons, the resistance to the motion of the car.
Find the power of the engine in kilowatts, when the car ascends the same road, under the same resistance with an acceleration $\frac{1}{6}\,\mathrm{m\,s^{-2}}$, at the instant when its speed is $15\,\mathrm{m\,s^{-1}}$.

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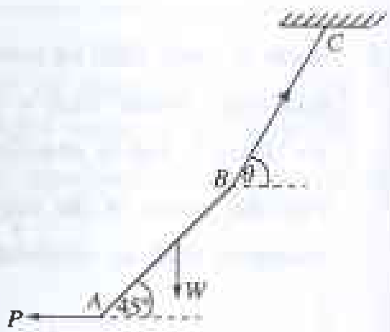
5. In the usual notation, let $3\mathbf{i}$ and $2\mathbf{i}+3\mathbf{j}$ be the position vectors of two points A and B respectively, with respect to a fixed origin O. Let C be the point on the straight line OB such that $O\hat{C}A = \frac{\pi}{2}$. Find $\overrightarrow{OC}$ in terms of $\mathbf{i}$ and $\mathbf{j}$.

6. A uniform rod AB of length $2a$ and weight W, is held in equilibrium by a light inextensible string BC, and by a horizontal force P applied at the end A, as shown in the figure. Given that the rod makes an angle $45°$ with the horizontal, show that angle θ that the string BC makes with the horizontal is given by $\tan\theta = 2$.



Find the tension in the string, in this position, in terms of W.

[see page five

7. Let A and B are two events in a sample space S. With the usual notation, $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$ and $P(A \cap B) = \frac{1}{6}$. Find $P(A|B')$, $P(A' \cap B')$ and $P(B'|A')$, where A' and B' denote complementary events of A and B respectively.

8. A bag contains 4 red balls and 3 black balls which are identical in all aspects except for their colour. Four balls are drawn from the bag at random, one at a time, without replacement. Find the probability that
   (i) the balls drawn are of the same colour,
   (ii) the balls drawn, on any two consecutive draws are of different colours.

[see page six

9. Five positive integers each of which is less than 8, have only one mode. Their mean, mode and median are in the ratios $6:10:5$. Find these five integers.

10. Temperature in a certain city was recorded daily for 20 days. For this set of data the mean μ and the standard deviation σ were calculated as $28\,°C$ and $4\,°C$ respectively. However, after detecting that two of the above temperatures have been mistakenly entered as $35\,°C$ and $21\,°C$, they were later corrected as $25\,°C$ and $31\,°C$. Find the correct values of μ and σ.

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see page seven

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இலங்கைப் பரீட்சைத் திணைக்களம்
Department of Examinations, Sri Lanka

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2018 අගෝස්තු
கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2018 ஓகஸ்ட்
General Certificate of Education (Adv. Level) Examination, August 2018

සංයුක්ත ගණිතය II
இணைந்த கணிதம் II
Combined Mathematics II

10 E II

Part B

* Answer **five** questions only.

(In this question paper, g denotes the acceleration due to gravity.)

11. (*a*) A lift moving in a mine pit of depth $4d$ metres begins to move vertically downwards from rest from a point A, at time $t = 0$. First, it moves a distance d metres with constant acceleration $\frac{g}{2}$ m s^{-2}, and then it moves with the velocity attained at the end of that motion, for another distance of d metres. The lift then moves the remaining distance with constant retardation so as to come to rest exactly at the point B at a distance $4d$ metres below A.

Sketch the velocity-time graph for the motion of the lift.

Hence, find the total time taken by the lift to move down from A to B.

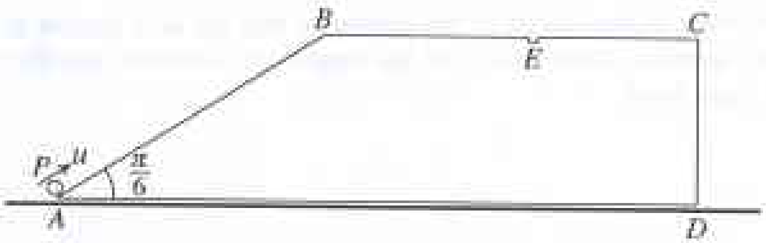
(*b*) A ship is sailing due North with uniform speed u km h^{-1}, relative to earth. At a certain instant a boat B_1 is observed at an angle β East of South, from the ship at a distance p km **from the path of the ship**. At the same instant, a boat B_2 is observed Westward at a distance q km from the ship. Both boats sail in straight line paths, with uniform speed $v\,(> u)$ km h^{-1} relative to earth, intending to intercept the ship. Sketch in the same diagram, velocity triangles to determine the paths of the boats relative to earth.

Show that the path of the boat B_1 relative to earth makes an angle $\beta - \sin^{-1}\left(\frac{u \sin\beta}{v}\right)$ West of North, and find the path of the boat B_2 relative to earth.

Let $\beta = \frac{\pi}{3}$ and $v = \sqrt{3}u$. Show that if $3q^2 > 8p^2$, then the boat B_1 will intercept the ship before the boat B_2.

12. (*a*) The trapezium $ABCD$ such that $AB = a$ and $B\hat{A}D = \frac{\pi}{6}$, shown in the figure, is a vertical cross-section through the centre of gravity of a smooth uniform block of mass $2m$. The lines AD and BC are parallel, and the line AB is a line of greatest slope of the face containing it. The block is placed with the face containing AD on a smooth horizontal floor. A particle P of mass m is placed at the point A and it is given a velocity u along $\overrightarrow{AB}$, where $u^2 = \frac{7ga}{3}$, as shown in the figure. Show that the retardation of P relative to the block is $\frac{2g}{3}$ and find the velocity of the particle P relative to the block when the particle P reaches B.

Also, there is a small hole on the upper face of the block at the point E on BC such that $BE = \frac{\sqrt{3}a}{2}$. By considering the motion relative to the block, show that the particle P will fall into the hole at E.



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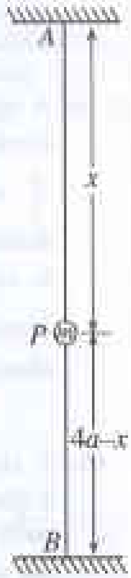
(*b*) One end of a light inextensible string of length a is attached to a fixed point O and the other end to a particle P of mass m. The particle hangs at rest vertically below O, and it is given a horizontal velocity of magnitude $u = \sqrt{kag}$, where $2 < k < 5$. Show that, when the string has turned through an angle θ and still taut, the speed v of the particle is given by $v^2 = (k-2)\,ag + 2ag\cos\theta$.

Find the tension in the string, in this position.

Deduce that the string becomes slack when $\theta = \alpha$, where $\cos\alpha = \dfrac{2-k}{3}$.

13. A particle P of mass m is attached to two ends of two equal light elastic strings, each of natural length a and modulus mg. The free end of one string is attached to a fixed point A, and the free end of the other string is attached to a fixed point B which is at a distance $4a$ vertically below A. (See the figure.) Show that, with both strings taut, the particle will be in equilibrium at a distance $\dfrac{5a}{2}$ below A.



The particle P is now raised to the mid-point of AB and is gently released from rest in that position. When both strings are taut and the length of the string AP is x, show that $\ddot{x} + \dfrac{2g}{a}\left(x - \dfrac{5a}{2}\right) = 0$.

Re-write this equation in the form $\ddot{X} + \omega^2 X = 0$, where $X = x - \dfrac{5a}{2}$ and $\omega^2 = \dfrac{2g}{a}$.

Using the formula $\dot{X}^2 = \omega^2\left(c^2 - X^2\right)$, find the amplitude c of this motion.

At the instant when the particle P reaches its lowest position, the string PB is cut.

Show that the particle reaches its highest position in the new motion when $x = a$.

Show further that the total time taken by the particle P to move from its initial position at $x = 2a$, a distance a downwards, and then a distance $\dfrac{a}{2}$ upwards is $\dfrac{\pi}{3}\sqrt{\dfrac{a}{2g}}\left(3+\sqrt{2}\right)$.

14.(*a*) Let OAB be a triangle, D be the mid-point of AB and E be the mid-point of OD. The point F lies on OA such that $OF : FA = 1:2$. The position vectors of A and B with respect to O are $\mathbf{a}$ and $\mathbf{b}$ respectively. Express the vectors $\overrightarrow{BE}$ and $\overrightarrow{BF}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

Deduce that B, E and F are collinear and find the ratio $BE : EF$.

Find the scalar product $\overrightarrow{BF}\cdot\overrightarrow{DF}$ in terms of $|\mathbf{a}|$ and $|\mathbf{b}|$ and show that if $|\mathbf{a}| = 3|\mathbf{b}|$, then $\overrightarrow{BF}$ is perpendicular to $\overrightarrow{DF}$.

(*b*) A system of forces in the Oxy–plane consists of three forces $3P\mathbf{i} + 2P\mathbf{j}$, $2P\mathbf{i} - P\mathbf{j}$ and $-P\mathbf{i} + 2P\mathbf{j}$ acting at the points $(-a, 2a)$, $(0, a)$ and $(-a, 0)$ respectively, where P and a are positive quantities measured in newtons and metres, respectively. Show that the clockwise moment of the system about the origin O is $12Pa$ N m.

Also, show that the system is equivalent to a single resultant force of magnitude $5P$ N and find its direction and the equation of the line of action.

Now, an extra force is introduced to the system so that the new system is equivalent to a couple with clockwise moment $24Pa$ N m. Find the magnitude, direction and the equation of the line of action of the extra force.

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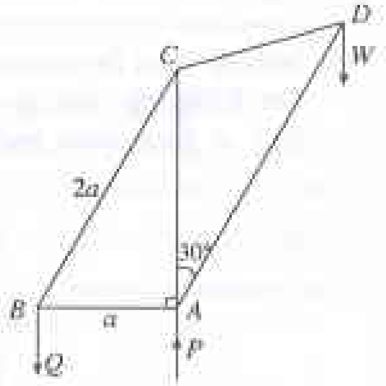
15. (*a*) A uniform rod AB of weight W and length $2a$, has its end A placed on a rough horizontal ground and the end B against a smooth vertical wall. The rod lies in a vertical plane perpendicular to the wall and makes an angle θ with the horizontal, where $\tan\theta = \frac{3}{4}$. A particle of weight W is attached to the point C on the rod with $AC = x$. The rod with the particle is in equilibrium. The coefficient of friction between the rod and the ground is $\frac{5}{6}$. Show that $x \leq \frac{3a}{2}$.

(*b*) The framework shown in the adjoining figure is made of five light rods AB, BC, AC, CD and AD freely jointed at their ends. It is given that $AB = a$, $BC = 2a$, $AC = CD$ and $C\hat{A}D = 30°$. A load of weight W hangs at D and the framework is in equilibrium in a vertical plane with AB horizontal and AC vertical, supported by vertical forces P and Q acting respectively at A and B in the **directions indicated in the figure**. Find the value of Q, in terms of W.



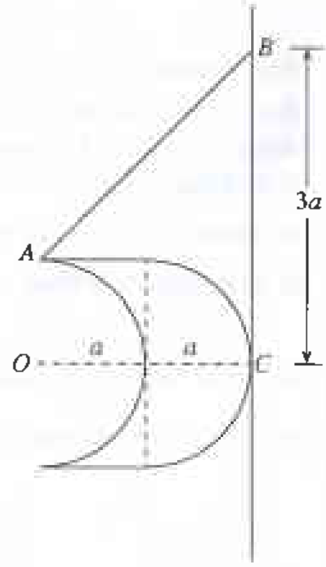
Draw a stress diagram using Bow's notation and **hence**, find the stresses in the five rods and state whether these stresses are tensions or thrusts.

16. Show that the centre of mass of a uniform solid hemisphere of radius a is at a distance $\frac{3}{8}a$ from its centre.

A hemispherical portion of radius a is carved out from a uniform solid right circular cylinder of radius a, height a and density ρ. Now, a uniform solid hemisphere of radius a and density $\lambda\rho$ is fastened at its circular face to the circular face of the remaining portion of the cylinder, so that the two axes of symmetry coincide, as shown in the adjoining figure. Show that the centre of mass of the body S thus formed, lies on its axis of symmetry at a distance $\frac{(11\lambda+3)a}{4(2\lambda+1)}$ from O, the centre of the rim.

Let $\lambda = 2$ and let A be a point on the circular rim of the body S.

This body S is kept in equilibrium against a rough vertical wall by means of a light inextensible string with one end attached to the point A and the other end to a fixed point B on the vertical wall. In this equilibrium position, the axis of symmetry of S is perpendicular to the wall and the hemispherical surface of S touches the wall at the point C at a distance $3a$ vertically below the point B. (See the adjoining figure.) The points O, A, B and C lie on a vertical plane perpendicular to the wall.



If μ is the coefficient of friction between the hemispherical surface of S and the wall, show that $\mu \geq 3$.

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17.(*a*) All applicants who apply for a certain job in an institute are required to sit for an aptitude test. Those who obtain A grades in this aptitude test are selected for the job, and the rest of the applicants have to face an interview. In a survey, it was found that 60% of the applicants get A grades, and of these, 40% are females. From the applicants who face the interview, only 10% get selected and 70% of them are females.

Find the probability that

(i) a male is selected for this job,

(ii) a male who has been selected for the job has obtained an A grade for the aptitude test.

(*b*) Waiting times (in minutes) before being treated of 100 patients at a certain hospital are collected. The following table gives the distribution of values obtained by subtracting 20 minutes from each of these times and dividing each of the resulting differences by 10.

Range of values	**Number of patients**
−2 — 0	30
0 — 2	40
2 — 4	15
4 — 6	10
6 — 8	5

Estimate the mean and the standard deviation of the distribution given in this table.

Hence, estimate the mean μ and the standard deviation σ of the waiting times of the 100 patients.

Also, estimate the coefficient of skewness κ defined by $\kappa = \dfrac{\mu - M}{\sigma}$, where M is the mode of the waiting times of the 100 patients.

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