0200

ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව
இலங்கைப் பரீட்சைத் திணைக்களம்
Department of Examinations, Sri Lanka

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2018 අගෝස්තු
கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2018 ஓகஸ்ற்
General Certificate of Education (Adv. Level) Examination, August 2018

ගණිතය I
கணிதம் I
Mathematics I

07 E I

29.08.2018 / 0830 - 1140

පැය තුනයි
மூன்று மணித்தியாலம்
Three hours

අමතර කියවීම් කාලය - මිනිත්තු 10 යි
மேலதிக வாசிப்பு நேரம் - 10 நிமிடங்கள்
Additional Reading Time - 10 minutes

Use **additional reading time** to go through the question paper, select the questions and decide on the questions that you give priority in answering.

Index Number							

Instructions:

* *This question paper consists of two parts;*
  Part A (Questions **1 – 10**) and **Part B** (Questions **11 – 17**).
* **Part A:**
  *Answer **all** questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.*
* **Part B:**
  *Answer **five** questions only. Write your answers on the sheets provided.*
* *At the end of the time allotted, tie the answers of the two parts together so that* **Part A** *is on top of* **Part B** *before handing them over to the supervisor.*
* *You are permitted to remove **only** **Part B** of the question paper from the Examination Hall.*

For Examiners' Use only

(07) Mathematics I		
Part	**Question No.**	**Marks**
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
	Total	
	Percentage	

Paper I	
Paper II	
Total	
Final Marks	

Final Marks

In Numbers	
In Words	

Code Numbers

Marking Examiner		
Checked by:	1	
	2	
Supervised by:		

[see page two

Part A

1. Let A, B and C be subsets of a universal set S. Show that $(A \cup C) \cap [(A \cap B) \cup (C' \cap B)] = A \cap B$.

2. Let $S = \{n \in \mathbb{Z} : 1 \leq n \leq 20\}$ be the universal set, and let A be the set of odd numbers in S, B be the set of factors of 36 in S and $C = \{9, 10\}$.

   Find $(A \cup B)'$, $(A \cup C)'$ and $(A \cup B \cup C)'$.

[see page three

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3. Let $f:[-\pi,\pi]\rightarrow\mathbb{R}$ be the function defined by $f(x) = \sin x - \cos x$. A relation R is defined by aRb, if $f(a) = f(b)$ for $a, b \in [-\pi,\pi]$. Show that R is an equivalence relation on $[-\pi,\pi]$, and find the equivalence class of $\frac{\pi}{4}$.

4. The linear functions $f(x) = ax + b$ and $g(x) = px + q$ are inverses of each other. Show that $ap = 1$ and $pb + q = 0$. If $f(0) = 2$ and $f(1) = 1$, find $f^{-1}(x)$.

[see page four

5. Solve the simultaneous equations $\log_2(x+2y)=3$ and $\log_3 x = 2\log_3 y$ for x and y.

6. Let $f(x)=\begin{vmatrix} 1 & 1 & 1 \\ a & x & b \\ a^2 & x^2 & bx \end{vmatrix}$, where $a, b \in \mathbb{R}$ and $ab \neq 0$.

Without expanding the determinant, show that $(x-a)$ is a factor of $f(x)$.
Hence or otherwise, solve $f(x)=0$ for x.

[see page five

7. Let $A \equiv (-1, -1)$ and $B \equiv (5, -7)$. Show that the line AB is perpendicular to the line $x - y = 4$.
If $C \equiv (1, -3)$ is the point of intersection of the above lines, find $AC : CB$.

8. Find the equation of the circle with radius 5 units and the centre having a positive y-coordinate and passing through the points $(-4, 0)$ and $(4, 0)$.

[see page six

9. The area of a rectangle increases at a rate of $48\ \text{cm}^2\ \text{s}^{-1}$, always maintaining its shape with the length as twice as the breadth. Find the rate at which the length increases when the breadth is 4 cm.

10. The gradient of the tangent drawn to the curve $y = 2(x-a)^2 + b$ at the point $P \equiv (0, c)$ on it is 4, where a, b and c are real constants. The equation of the normal drawn to the curve at P is given to be $x + 4y = 4$. Find the values of a, b and c.

* *

[see page seven

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இலங்கைப் பரீட்சைத் திணைக்களம்
Department of Examinations, Sri Lanka

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2018 අගෝස්තු
கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2018 ஓகஸ்ற்
General Certificate of Education (Adv. Level) Examination, August 2018

ගණිතය I
கணிதம் I
Mathematics I

07 E I

Part B

※ Answer **five** questions only.

11. (*a*) Let α and β be the roots of the equation $2x^2 - 8x + 9 = 0$.

Find the quadratic equation with integer coefficients that has $\alpha^2 - 1$ and $\beta^2 - 1$ as its roots.

(*b*) Let $P(x)$ be a polynomial. It is given that when $P(x)$ is divided by $(x - 1)$ and $(x - 2)$, the respective remainders are 2 and 1. Find the remainder when $P(x)$ is divided by $(x - 1)(x - 2)$.

If $P(x)$ is a cubic polynomial with the coefficient of x^3 equals 1 and $P(0) = -3$, determine $P(x)$.

12. (*a*) Using the **Principle of Mathematical Induction**, prove that

$$\sum_{r=1}^{n} \frac{1}{r(r+1)} = \frac{n}{n+1} \text{ for all } n \in \mathbb{Z}^+.$$

(*b*) Verify that $\frac{2}{r} - \frac{1}{r+1} = \frac{(r+2)}{r(r+1)}$ for $r \in \mathbb{Z}^+$

Let $U_r = \frac{(r+2)}{r(r+1)} \cdot \frac{1}{2^r}$ for $r \in \mathbb{Z}^+$.

Using the above result, find $f(r)$ such that $U_r = f(r) - f(r+1)$ for $r \in \mathbb{Z}^+$.

Hence, find $\sum_{r=1}^{n} U_r$ and show that $\sum_{r=1}^{\infty} U_r$ is convergent.

Is the series $\sum_{r=1}^{\infty} (U_r + 1)$ convergent? Justify your answer.

13. (*a*) How many three-digit numbers starting with a non-zero digit can be formed from the 10 digits $0, 1, 2, \ldots, 9$ in which

(i) no number contains 7 ?

(ii) numbers that contain 7 at least once?

(iii) numbers that contain 7 at most once?

(*b*) A school has a squad of 10 sportsmen consisting of 2 wrestlers, 3 boxers and 5 runners. The school needs to form a 6 member team to participate in a competition in such a manner that the team will comprise at least one representation from each of the above mentioned sports.

Find the number of different ways in which such a team can be formed.

[see page eight

14. Let $\mathbf{A} = \begin{pmatrix} 0 & -2 & 1 \\ 2 & 1 & 0 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 0 & 1 \\ 1 & 1 \\ 1 & 0 \end{pmatrix}$.

Find the product matrix $\mathbf{C} = \mathbf{AB}$. Write down $\mathbf{C}^{-1}$.

Find the matrix $\mathbf{D}$ such that $\mathbf{CDC}^{-1} = 2\mathbf{C}^2 + 3\mathbf{C}$.

Verify that $(\mathbf{CD})^{-1} = \mathbf{D}^{-1}\mathbf{C}^{-1}$.

Find the product matrix $\mathbf{P} = \mathbf{BA}$.

Let $\mathbf{X} = \begin{pmatrix} a \\ 2 \\ b \end{pmatrix}$, where $a, b \in \mathbb{R}$.

Find the values of a and b such that $\mathbf{PX} = \begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix}$.

15. (*a*) Show that the binomial expansion of $\left(2x + \dfrac{1}{x^3}\right)^6$ has no constant term and find the coefficient of x^{-10} in that expansion.

(*b*) A person opens a bank account which pays an interest of 1% per month compounded monthly by depositing Rs. 50 000 at the beginning of a month. He then deposits Rs. 10 000 at the beginning of each month afterwards for 5 years. Assuming that he makes no other transaction during this period, find the balance in his account after 5 years. He then withdraws Rs. 25 000 at the beginning of each month for the next one year. Find the balance in his account after this 6-year period.

16. Find the centre and the radius of the circle C given by $x^2 + y^2 - 4x - 8y - 5 = 0$.

Show that the line l given by $3x - 4y = 15$ touches the circle C.

Verify that the point $P \equiv (1, -3)$ lies on l, and find the equation of the other tangent drawn from P to C.

Find the length of the chord joining the points of contact of the above tangents drawn to C.

17. (*a*) Find $\lim\limits_{x \to 1} \dfrac{x^3 - 1}{x^2 - 1}$.

(*b*) Differentiate each of the following with respect to x:

(i) $x^5 \ln x + 2e^{-x}$ (ii) $\sqrt{\dfrac{1 + e^x}{1 - e^x}}$ (iii) $\ln\left(\dfrac{\sin x}{1 + \cos x}\right)$

(*c*) An open box with a square base is to be made out of a given quantity of cardboard of area 9 m^2. Show that the maximum volume of the box is $\dfrac{3\sqrt{3}}{2}$ m^3.

* * *

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இலங்கைப் பரீட்சைத் திணைக்களம்
Department of Examinations, Sri Lanka

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2018 අගෝස්තු
கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2018 ஓகஸ்ற்
General Certificate of Education (Adv. Level) Examination, August 2018

ගණිතය II
கணிதம் II
Mathematics II

07 E II

30.08.2018 / 0830 - 1140

පැය තුනයි
மூன்று மணித்தியாலம்
Three hours

අමතර කියවීම් කාලය - මිනිත්තු 10 යි
மேலதிக வாசிப்பு நேரம் - 10 நிமிடங்கள்
Additional Reading Time - 10 minutes

Use **additional reading time** to go through the question paper, select the questions and decide on the questions that you give priority in answering.

Index Number							

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* **Part B:** *Answer **five** questions only. Write your answers on the sheets provided.*
* *At the end of the time allotted, tie the answers of the two parts together so that* **Part A** *is on top of* **Part B** *before handing them over to the supervisor.*
* *You are permitted to remove **only** **Part B** of the question paper from the Examination Hall.*
* *Statistical tables will be provided.*

For Examiners' Use only

(07) Mathematics II		
Part	**Question No.**	**Marks**
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
	Total	
	Percentage	

Paper **I**	
Paper **II**	
Total	
Final Marks	

Final Marks

In Numbers	
In Words	

Code Numbers

Marking Examiner		
Checked by:	1	
	2	
Supervised by:		

[see page two

Part A

1. Find all real values of x satisfying the inequality $\frac{x}{x-2} \geq \frac{3x-4}{x}$.

2. Shade the region in the xy-plane satisfying the inequalities $y \geq x^2$, $y \leq x+2$ and $-1 \leq x \leq 1$.

[see page three

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3. Express $\cos x + \sqrt{3}\sin x - 2\sin\left(x + \frac{\pi}{3}\right)$ in the form $R\cos(x+\alpha)$, where R (>0) and $\alpha\left(0 < \alpha < \frac{\pi}{2}\right)$ are real constants to be determined.

4. Express $\frac{3x}{(x-1)(x+2)}$ in partial fractions. **Hence**, find $\int \frac{x}{(x-1)(x+2)}\,\mathrm{d}x$.

[*see page four*

5. Using the method of **integration by parts**, show that $\int_0^{\pi} e^x \sin x \, dx = \frac{1}{2}\left(e^{\pi} + 1\right)$.

6. The mean of a random variable X is 6. If the mean of the random variable Y defined by the transformation $Y = X(X-3)$ is 54, find the variance of X.

[see page five

7. A game is played by repeatedly rolling a six-sided fair die with the six digits 1, 2, 3, 4, 5 and 5 marked on the faces, until a face marked with a 5 turns up. Let the score obtained in the game be the number of times the die was rolled including the turn that had a face marked with the digit 5 turned up. Outcome of each rolling is independent of the others.

Find the probability that

(i) the score obtained is 1,

(ii) the score obtained is 2, given that it is greater than 1.

8. Let A and B be two events defined on the same sample space S and let B' be the complement of the event B. If $P(A \cap B) = \frac{1}{3}$ and $P(A \cap B') = \frac{1}{5}$, find $P(A)$ and $P(B|A)$.

[see page six

9. In a class of 40 children, 60% are girls. In this class, 80% of the boys and 40% of the girls prefer to play cricket.

(i) Find the probability that a randomly selected child from this class prefers to play cricket.

(ii) If three children are randomly selected from this class, find the probability that at least one of them prefers to play cricket.

10. The continuous random variable X can assume only positive values not exceeding the value $k(> 0)$. If the probability density function $f(x)$ of X is of the form $f(x) = \frac{4}{3}(2x+1)$, find the value of k. Also, find the probability that the random variable takes a value less than 0.3.

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[see page seven

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இலங்கைப் பரீட்சைத் திணைக்களம்
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අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2018 අගෝස්තු
கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2018 ஓகஸ்ற்
General Certificate of Education (Adv. Level) Examination, August 2018

ගණිතය II
கணிதம் II
Mathematics II

07 E II

Part B

※ *Answer five questions only.*

11. A manufacturer wishes to determine the number of units from the products A and B that has to be produced per week to maximize his total profit. Two types of processes I and II are required to manufacture each of these products.

The following table gives the hours needed at each process to produce one unit of product A and product B, and the number of hours of work that can be handled by each process per week.

Process Type	Hours needed to produce one unit		Number of hours of work that can be handled by the process per week
	Product A	Product B	
I	2	1	1000
II	1	1	800

The profit per unit for products A and B are 3000 rupees and 2000 rupees respectively. There is no constraint on the supply of raw materials for the manufacturing. Demand for the product B is unlimited, but the weekly demand for the product A is at most 350 units.

(i) Formulate this as a linear programming problem.

(ii) Sketch the feasible region.

(iii) Using the graphical method, find the number of units of each of the products A and B that need to be manufactured per week to maximize the total profit.

If the demand on the product B has dropped to 500 units per week, find the drop in the total profit, if the manufacturer still wishes to maximize the profit.

12. (a) Solve the equation $4\cos x(2+\cos x)=5$ for $0 \le x \le \frac{\pi}{2}$.

(b) Solve: $\sin^{-1}\left(\frac{4}{5}\right)+\sin^{-1}x = 2\sin^{-1}\left(\frac{1}{2}\right)$.

(c) In the usual notation, state the **Sine Rule**, for a triangle ABC.

For the triangle ABC, it is given that $\frac{a+b}{b+c}=2$.

Show that $\sin(A+B)=\frac{1}{2}[\sin A-\sin B]$.

[See page eight

13. (a) Find the area enclosed by the circle $x^2 + y^2 = 16$ and the curve $y^2 = 6x$.

(b) The following table gives the values of the function $f(x) = \dfrac{x}{x^2 + 1}$, correct to four decimal places, for values of x between 0 and 2 at intervals of length 0.5.

x	0.00	0.50	1.00	1.50	2.00
$f(x)$	0.0000	0.4000	0.5000	0.4615	0.4000

Using **Simpson's Rule** find an approximate value for $I = \displaystyle\int_0^2 \frac{x}{x^2+1}\,dx$, correct to three decimal places.

Hence, find an approximate value for ln 5.

14. (a) The mean weight (kg) of a group of persons is 61.4. Also, the mean weights (kg) of all males in the group and all females in the group are 65.3 and 60.1 respectively. Find the percentage of males in this group.

If 20 persons are randomly selected from this group, find the expected number of males among the 20 selected persons.

If 5 more males are added to the group of 20 persons to form a new group of 25 persons, estimate the mean weight of the new group.

(b) The following frequency distribution gives the times (in minutes) taken by 130 workers to perform a specific task.

Time (minutes)	**Number of workers**
30 - 39	10
40 - 49	35
50 - 59	44
60 - 69	27
70 - 79	14

Using an appropriate coding method, find the mean and the variance of the frequency distribution.

15. Suppose 40%, 30% and 30% of the kettles sold by a store are of brand A, B and C respectively. Further, suppose that 1%, 2% and 1% of kettles sold of the brands A, B and C respectively get returned during the warranty period.

Find the probability that, among the kettles sold, a randomly selected kettle gets returned during the warranty period.

(i) If a kettle is returned during the warranty period, find the probability that it was of brand A.

(ii) Given that a kettle is not of brand A, find the probability that it will be returned during the warranty period.

(iii) Given that a kettle is not returned during the warranty period, find the probability that it is not of brand A.

(iv) If 2 kettles are randomly selected, find the probability that exactly one of these kettles will be returned during the warranty period.

[See page nine

16. The heights of students in a school are normally distributed with a mean of 62.8 inches and a standard deviation of σ inches. If 33% of the students are below 60.6 inches in height, find σ.

(i) If 71.9% of the students are shorter than Amal, calculate Amal's height to the nearest inch.

(ii) Find the percentage of students who are taller than 66 inches.

(iii) If a randomly selected student is taller than the mean height of 62.8 inches, find the probability that this student is taller than 66 inches.

(iv) If three students are randomly selected from the school, find the probability that the shortest student among them will be taller than 66 inches. (You need not simplify the answer).

17. A street has identical lamps at one of its sides fixed at 50 metres apart. If a lamp is working on a day, the probability that it will work on the following day is 0.80. If a lamp is non-working on a day, the probability that it will be repaired to work on the following day is 0.60.

Consider the condition of a lamp as a two-state Markov chain with 'working (W)' and 'non-working (NW)' as the two states.

Write down the one-step transition probability matrix P and obtain the two-step transition probability matrix.

Suppose all lamps were in working condition on the 1^{st} of January 2018.

Find the probability that a randomly selected lamp was in working condition on the 3^{rd} of January 2018.

On the 3^{rd} of January 2018, a person walks starting from the first lamp in one end of the street towards the other end of the street.

Find the probability that the person has to walk more than 500 metres to reach the first non-working lamp. (You need not simplify the answer.)

Given that the person has already walked 100 metres, find the probability that he has to walk exactly 200 metres to reach the first non-working lamp.

* * *