අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2018 අගෝස්තු
கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2018 ஓகஸ்ற்
General Certificate of Education (Adv. Level) Examination, August 2018

උසස් ගණිතය I
உயர் கணிதம் I
Higher Mathematics I

11 E I

29.08.2018 / 0830 - 1140

පැය තුනයි
மூன்று மணித்தியாலம்
Three hours

අමතර කියවීම් කාලය - මිනිත්තු 10 යි
மேலதிக வாசிப்பு நேரம் - 10 நிமிடங்கள்
Additional Reading Time - 10 minutes

Use **additional reading time** to go through the question paper, select the questions and decide on the questions that you give priority in answering.

Index Number | | | | | | | |

Instructions:

* *This question paper consists of two parts;*
  Part A (Questions **1 – 10**) and **Part B** (Questions **11 – 17**).
* **Part A:**
  *Answer **all** questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.*
* **Part B:**
  *Answer **five** questions only. Write your answers on the sheets provided.*
* *At the end of the time allotted, tie the answer scripts of the two parts together so that* **Part A** *is on top of* **Part B** *and hand them over to the supervisor.*
* *You are permitted to remove **only** **Part B** of the question paper from the Examination Hall.*

For Examiners' Use only

(11) Higher Mathematics I

Part	Question No.	Marks
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
	Total	
	Percentage	

Paper I	
Paper II	
Total	
Final Marks	

Final Marks

In Numbers	
In Words	

Code Numbers

Marking Examiner		
Checked by:	1	
	2	
Supervised by:		

[see page two

Part A

1. Factorize: $8(a+b+c)^3-(a+b)^3-(b+c)^3-(c+a)^3$.

2. A relation R is defined on the set of all positive rational numbers $\mathbb{Q}^+$ by aRb if $a=3^k b$ for some $k \in \mathbb{Z}$. Show that R is an equivalence relation on $\mathbb{Q}^+$ and write down the equivalence class of 1.

[see page three

0112

3. Let $f(x)=(x-1)^2+2$ for $x>1$. Show that f is one-to-one and that $f^{-1}(2f(2))=3$.

4. Show that

$$\begin{vmatrix} a-b-c & 2b & 2c \\ 2a & b-c-a & 2c \\ 2a & 2b & c-a-b \end{vmatrix} = (a+b+c)^3.$$

[*see page four*

5. The chord of the parabola $y^2 = 4ax$ joining the points $(at^2, 2at)$ and $(aT^2, 2aT)$ passes through the point $(4a, 0)$. Show that $tT = -4$.

6. Let $f : \mathbb{R} \to \mathbb{R}$ be the function defined by

$$f(x) = \begin{cases} \dfrac{\sin 2x}{x} & \text{if} \quad x < 0, \\ p & \text{if} \quad x = 0, \\ (x^2 + q)e^{-(x+1)} & \text{if} \quad x > 0. \end{cases}$$

It is given that f is continuous at $x = 0$. Find the values of p and q.

[see page five

7. Let $f(x) = (x-1)^{\frac{1}{3}}|x-1|$ for $x \in \mathbb{R}$. Show that $f(x)$ is differentiable at $x = 1$ and write down its derivative $f'(x)$ for all $x \in \mathbb{R}$.

8. Solve $\dfrac{dy}{dx} + 2y\tan x = \sin x$.

[see page six

9. Let $T > 0$ and let f be a real-valued continuous function on $\mathbb{R}$ such that $f(x+T) = f(x)$ for all $x \in \mathbb{R}$.

Show that $\int_{a}^{b} f(x)\,dx = \int_{a+T}^{b+T} f(x)\,dx$.

10. Sketch the curves $r = 2\sin\theta$ and $r\cos\left(\theta - \frac{\pi}{4}\right) = \sqrt{2}$ in the same diagram, and find the polar coordinates of their points of intersection.

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கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2018 ஓகஸ்ற்
General Certificate of Education (Adv. Level) Examination, August 2018

උසස් ගණිතය I
உயர் கணிதம் I
Higher Mathematics I

11 E I

Part B

※ Answer **five** questions only.

11. (*a*) Let X, Y and Z be subsets of a universal set S. Stating clearly any result in Algebra of Sets that you use, show that

(i) $(X-Y) - Z \subseteq X - Z$,

(ii) $(X-Y)-(Y-Z)=X-Y$,

where $X-Y$ is defined by $X-Y = X \cap Y'$.

(*b*) A survey was carried out using 100 customers in a restaurant to determine which food they like to have for Breakfast, from among string hoppers, hoppers and bread. The following data were collected from this survey:

44 like string hoppers,
15 like **only** bread,
10 like string hoppers and hoppers but not bread,
78 like bread or hoppers,
12 like bread and hoppers but not string hoppers.
27 like all three and 19 did not like any of the three.

Find the number of customers who

(i) liked string hoppers but not hoppers,

(ii) liked **only** hoppers,

(iii) liked string hoppers and bread but not hoppers.

12. (*a*) Let a, b and c be positive real numbers such that $a + b + c = 1$. Using the Arithmetic Mean - Geometric Mean inequality show that $\dfrac{1}{abc} \geq 27$.

Hence, show that (i) $\dfrac{1}{a}+\dfrac{1}{b}+\dfrac{1}{c} \geq 9$ and (ii) $\dfrac{1}{ab}+\dfrac{1}{bc}+\dfrac{1}{ca} \geq 27$.

Deduce that $\left(1+\dfrac{1}{a}\right)\left(1+\dfrac{1}{b}\right)\left(1+\dfrac{1}{c}\right) \geq 64$.

(*b*) The transformation $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$, maps points in the xy-plane into points in the $x'y'$-plane. Find the equation of the line in the $x'y'$-plane to which the line $y = ax + b$, where a and b are constants, gets mapped onto under this transformation.

Let $A \equiv (2, 3)$ and $B \equiv (3, 2)$ be two points in the xy-plane. Find the equation of the line in the $x'y'$-plane to which the line AB gets mapped onto.

[see page eight

13. State **De Moivre's Theorem** for a positive integral index.

Using the **De Moivre's Theorem**, show that

(i) $\sin n\theta = {}^{n}C_1 \cos^{n-1}\theta \sin\theta - {}^{n}C_3 \cos^{n-3}\theta \sin^3\theta + \cdots + (-1)^{\frac{n-1}{2}} \sin^n\theta$ for odd n.

(ii) $\sin n\theta = {}^{n}C_1 \cos^{n-1}\theta \sin\theta - {}^{n}C_3 \cos^{n-3}\theta \sin^3\theta + \cdots + (-1)^{\frac{n-2}{2}}\, {}^{n}C_{n-1} \cos\theta \sin^{n-1}\theta$ for even n.

Deduce that $\dfrac{\sin 5\theta - \sin 4\theta}{\sin\theta} = 16\cos^4\theta - 8\cos^3\theta - 12\cos^2\theta + 4\cos\theta + 1$ for $\sin\theta \neq 0$.

By considering the roots of the equation $x^4 - x^3 - 3x^2 + 2x + 1 = 0$, show that

$\cos\frac{\pi}{9} + \cos\frac{3\pi}{9} + \cos\frac{5\pi}{9} + \cos\frac{7\pi}{9} = \frac{1}{2}$ and $\cos\frac{\pi}{9} \cdot \cos\frac{3\pi}{9} \cdot \cos\frac{5\pi}{9} \cdot \cos\frac{7\pi}{9} = \frac{1}{8}$

14. (*a*) Sketch the curves $y = e^{2x}$ and $y = 2x - x^2$ in the same diagram.

Let R be the region bounded by the above two curves and the lines $x = 0$ and $x = 2$. Find the area of R.

Also, find the volume of the solid generated by rotating the region R through four right angles about the x-axis.

(*b*) A family of curves satisfies the differential equation $\dfrac{dy}{dx} = \dfrac{y^2 - x^2}{xy}$.

By substituting $y = vx$, solve this differential equation.

Also, obtain the differential equation satisfied by the orthogonal trajectories of this family of curves and solve it.

15. (*a*) Let $I_n = \displaystyle\int_0^1 x^n \sqrt{1 - x^2}\, dx$ for $n \in \mathbb{Z}^+$.

Show that $I_n = \left(\dfrac{n-1}{n+2}\right) I_{n-2}$.

Hence, find the value of $\displaystyle\int_0^1 x^4 \sqrt{1 - x^2}\, dx$.

(*b*) Write down the **Maclaurin** series expansions of e^x and $\sin x$.

Hence, find the Maclaurin series expansion of $e^{\sin x}$ up to and including the term involving x^4.

Using this, find an approximate value for $\displaystyle\int_0^1 e^{\sin x}\, dx$.

[see page nine

16. Find the equation of the tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point $P(a \sec\theta, b \tan\theta)$.

The tangent at P meets the tangents at the ends of the major axis of the hyperbola at Q and R. Show that the line segment QR subtends a right angle at each focus.

Let the coordinates of point P lying on the hyperbola $\frac{x^2}{9} - y^2 = 1$ with focuses S_1 and S_2 be $\left(5, \frac{4}{3}\right)$. Show that points Q, R, S_1 and S_2 defined as above are concyclic and find the equation of the circle through these points.

17. (*a*) Let $f(x) = \frac{3\cos x - 4\sin x}{4\cos x + 3\sin x + 10}$.

(i) State the domain of $f(x)$.

(ii) Find the maximum value and the minimum value of $f(x)$, and find the x-coordinates of the points at which these values are attained.

(iii) Solve the equation $f(x) = 0$.

(*b*) **Using Simpson's rule** with values of $\ln(1+x^2)$ given in the following table, find an approximate value for $\int_0^1 \ln(1+x^2)\, dx$.

x	0	0.25	0.50	0.75	1.0
$\ln(1+x^2)$	0	0.0606	0.2231	0.4463	0.6931

Deduce an approximate value for $\int_0^1 \ln\left(\frac{1+x^2}{2}\right) dx$.

* * *

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இலங்கைப் பரீட்சைத் திணைக்களம்
Department of Examinations, Sri Lanka

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2018 අගෝස්තු
கல்விப் பொதுத் தராதரப் பத்திர (உயர் தரப் பரீட்சை, 2018 ஓகஸ்ற்
General Certificate of Education (Adv. Level) Examination, August 2018

උසස් ගණිතය II
உயர் கணிதம் II
Higher Mathematics II

11 E II

01.09.2018 / 1300 - 1610

පැය තුනයි
மூன்று மணித்தியாலம்
Three hours

අමතර කියවීම් කාලය - මිනිත්තු 10 යි
மேலதிக வாசிப்பு நேரம் - 10 நிமிடங்கள்
Additional Reading Time - 10 minutes

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Index Number							

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  *Answer **five** questions only. Write your answers on the sheets provided.*
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- *You are permitted to remove **only** **Part B** of the question paper from the Examination Hall.*
- *Statistical Tables will be provided.*
- *g denotes the acceleration due to gravity.*

For Examiners' Use only

(11) Higher Mathematics II		
Part	**Question No.**	**Marks**
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
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B	11	
	12	
	13	
	14	
	15	
	16	
	17	
	Total	
	Percentage	

Paper I	
Paper II	
Total	
Final Marks	

Final Marks

In Numbers	
In Words	

Code Numbers

Marking Examiner		
Checked by:	1	
	2	
Supervised by:		

[see page two

Part A

1. The position vectors of three points A, B and C with respect to an origin O are $a\mathbf{i} + 2\mathbf{j} - \mathbf{k}$, $4\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ and $\mathbf{i} - 2\mathbf{j} + c\mathbf{k}$, respectively. Find the values of the constants a and c such that $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are perpendicular to each other and $\overrightarrow{OA} \times \overrightarrow{OB} = 3\,\overrightarrow{OC}$. Show further that, with these values for a and c, the vector $\overrightarrow{AC}$ is perpendicular to the vector $\overrightarrow{OB}$.

2. A force $\mathbf{F}$ is represented in magnitude, direction and line of action by $\lambda\,\overrightarrow{AB}$, where λ is a scalar, $\overrightarrow{OA} = -\mathbf{i} + \mathbf{j}$ and $\overrightarrow{OB} = \mathbf{k}$. Show that the moment vector of $\mathbf{F}$ about the origin O is $\lambda(\mathbf{i} + \mathbf{j})$. Further, if $\mathbf{F}$ is of unit magnitude find possible values of λ.

[see page three

0112

3. A uniform solid spherical shell of inner radius a, outer radius $2a$ and of density σ floats, partially immersed in a homogeneous liquid of density ρ. Show that $\frac{\sigma}{\rho} < \frac{8}{7}$ and find the smallest weight of the particle that can be attached to the highest point of the shell to make the shell float, totally immersed in the liquid.

4. Linear momentum at time t of a particle P of mass m moving in the Oxy-plane and passing through the point with position vector $a\mathbf{i}$ at time $t = 0$, is $ma\omega\,(-\mathbf{i}\sin\omega t + \mathbf{j}\cos\omega t)$, where a and ω are positive constants. Show that the position vector $\mathbf{r}$ of P is given by $\mathbf{r} = a(\mathbf{i}\cos\omega t + \mathbf{j}\sin\omega t)$, the force $\mathbf{F}$ acting on it is $\mathbf{F} = -\,m\omega^2\mathbf{r}$ and that its angular momentum about the origin O is $ma^2\omega\mathbf{k}$, where $\mathbf{k}$ is the vector $\mathbf{i} \times \mathbf{j}$.

[see page four

5. A smooth uniform sphere A falling vertically, without rotation, collides with an equal sphere B hanging at rest by a light inextensible string. Just before impact the speed of A is u and the line joining the centres of the spheres makes an angle 45° with the vertical. Just after impact the speed of B is $\frac{u}{2}$. Show that the coefficient of restitution between the two spheres is $\frac{1}{2}$.

6. A uniform square lamina $ABCD$ of mass M and side $2a$ performs small oscillations about a fixed smooth horizontal axis through AB. Assuming that the moment of inertia of the lamina about AB is $\frac{4}{3}Ma^2$, show that the period of small oscillations is $4\pi\sqrt{\frac{a}{3g}}$.

[see page five

7. A tetrahedral die with its faces marked 1, 2, 3, 4 is biased such that, when it is tossed the probability that it lands with the downward face has number r marked, is pr, where p is a positive constant and $r = 1, 2, 3, 4$. Let X be the random variable defined to be "the number marked on the downward face of the die". Show that $p = \frac{1}{10}$ and find the expectation of X. Show that $Var(X) = 1$.

8. An unbiased coin is tossed 8 times. Find the probability of getting more heads than tails.

[see page six

9. The probability density function $f(x)$ of a continuous random variable X is given by

$$f(x) = \begin{cases} \dfrac{2}{3k}x(k-x) & , \text{ for } 0 \le x \le k, \\ 0 & , \text{ otherwise,} \end{cases}$$

where k is a constant. Show that $k = 3$ and find the expectation of X.

10. The cumulative distribution function $F(x)$ of a continuous random variable X is given by

$$F(x) = \begin{cases} 0 & \text{if } \quad x < 0, \\ kx(4-x) & \text{if } \quad 0 \le x \le 1, \\ 1 & \text{if } \quad x > 1. \end{cases}$$

where k is a constant.
Find
(i) the value of k,
(ii) $P\left(X < \frac{1}{4}\right)$ and
(iii) $P\left(\frac{1}{4} < X < \frac{1}{2}\right)$.

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2018 අගෝස්තු
கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2018 ஓகஸ்ற்
General Certificate of Education (Adv. Level) Examination, August 2018

උසස් ගණිතය II
உயர் கணிதம் II
Higher Mathematics II

11 E II

Part B

※ Answer **five** questions only.

11. Forces $\mathbf{F}_s$, act at points A_s with the position vectors $\mathbf{r}_s$ with respect to the origin O, where $s = 1, 2, \ldots, n$.

Show that this system can be reduced to a single force $\mathbf{R} = \sum_{s=1}^{n} \mathbf{F}_s$ acting at O, together with a couple of moment vector $\mathbf{G} = \sum_{s=1}^{n} \mathbf{r}_s \times \mathbf{F}_s$. Obtain the conditions for the system to be equivalent to a single resultant force.

A system consisting four forces is given below:

Point of action	Position vector	Force
A	$3\mathbf{i}$	$4\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$
B	$2\mathbf{i} - 2\mathbf{k}$	$3\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$
C	$-5\mathbf{i} + 11\mathbf{j}$	$2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$
D	$\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$	$3\mathbf{i} + 7\mathbf{j} + 5\mathbf{k}$

Show that this system reduces to a single force $\mathbf{R}$ at the origin O together with a couple of moment vector $\mathbf{G} = 4\mathbf{i} - 12\mathbf{j} + 4\mathbf{k}$, and find $\mathbf{R}$.

Hence, show that the system is equivalent to a single resultant force of magnitude $4\sqrt{22}$.

Obtain a vector equation of the line of action of the resultant force, indicating the position vector of a point which lies on this line.

12. A circular lamina of radius a is immersed vertically in a liquid of constant density ρ, with its centre O at a depth a below the surface of the liquid. Show, by integration, that

(i) the magnitude of the liquid thrust on the lamina is $\pi a^3 \rho g$, and

(ii) the centre of pressure of the lamina is on its vertical diameter, at a distance $\frac{a}{4}$ below the centre O.

A solid hemisphere of radius a is immersed in a liquid of constant density ρ, with its highest point just touching the liquid surface and its plane face vertical.

Find the upthrust on the hemisphere, and write down the thrust on the plane face.

Hence, find the magnitude, direction and the line of action of the thrust on the curved surface of the hemisphere.

[Assume that the centre of gravity of a uniform solid hemisphere of radius a, lies on its axis of symmetry, at a distance $\frac{3a}{8}$ from the centre.]

[see page eight

13. A particle P of mass m is projected from the origin O, with initial velocity $\mathbf{u} = u(\mathbf{i}\cos\alpha + \mathbf{j}\sin\alpha)$, where u and α are constants, $\mathbf{i}$ and $\mathbf{j}$ being unit vectors in the horizontal and vertically upward directions, respectively. There is a resisting force, $-mk\mathbf{v}$ to the motion of the particle when its velocity is $\mathbf{v}$, where k is a positive constant. Obtain the equation of motion for the particle in the vector form $(\ddot{x} + k\dot{x})\mathbf{i} + (\ddot{y} + k\dot{y} + g)\mathbf{j} = \mathbf{0}$, where $\mathbf{r} = x\mathbf{i} + y\mathbf{j}$ is the position vector of the particle at time t.

Assuming the solutions $x = A + Be^{-kt}$ and $y = C + De^{-kt} - \frac{g}{k}t$ for the above equation in component form, find the values of the constants A, B, C and D, in terms of u and α.

Deduce the limiting value of the horizontal distance the particle can move.

If the constant k is negligible, **deduce** also the Cartesian equation of the path of the particle.

14. With the usual notation, show that the radial and transverse components of acceleration of a particle moving on a plane in terms of polar coordinates (r, θ) are $\ddot{r} - r\dot{\theta}^2$ and $\frac{1}{r}\frac{d}{dt}(r^2\dot{\theta})$, respectively.

A particle P of mass m placed on a smooth horizontal table is connected to an equal particle Q by a light inextensible string which passes through a small smooth hole O on the table, and P is held so that Q hangs freely. Initially, OP is of length a and particle P is projected horizontally at right angles to the string with speed V. Suppose OP is of length $r(\geq a)$ and OP has turned through an angle θ from its initial position, at time t. Show that

(i) $r^2\dot{\theta} = aV$, and

(ii) $2\ddot{r} - \dfrac{a^2V^2}{r^3} + g = 0$.

Hence, show that $\dot{r}^2 = \dfrac{V^2}{2}\left(1 - \dfrac{a^2}{r^2}\right) - g(r - a)$.

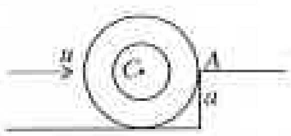
For this motion to be possible such that $a \leq r \leq 2a$ show that $V = \sqrt{\dfrac{8ga}{3}}$, given that the length of the string is greater than $2a$.

Find the tension in the string in the extreme position $r = 2a$, and show that the acceleration of Q in this position is $\dfrac{2g}{3}$ vertically downwards.

15. A wheel R of mass M and centre C is made from a uniform circular disc of radius $2a$ by removing a concentric circular disc of radius a. Show that the moment of inertia of the wheel R about an axis through a point of its outer circular edge perpendicular to its plane is $\dfrac{13}{2}Ma^2$.

[You may assume that the moment of inertia of a uniform circular disc of mass m and radius r, about an axis through its centre perpendicular to its plane is $\dfrac{1}{2}mr^2$.]

The wheel R is rolling without slipping on a rough horizontal floor. The plane of the wheel is vertical and perpendicular to a vertical step of height a on the floor, and the speed of the centre C is u towards the step (see the adjoining figure).



The impact of the wheel and the step is inelastic, and the wheel begins to rotate in its own plane with angular speed ω about the point A of contact with the step after collision. Show that $a\omega = \dfrac{9u}{26}$ and find the kinetic energy retained in the wheel, just after collision.

Hence, show that, for the wheel to mount the step $u \geq \dfrac{4}{9}\sqrt{13ga}$.

[see page nine

16. (*a*) The sentry on duty at the entrance to a building has n number of identical-looking keys, just one of which opens the front door. On a request by an authorized person, the sentry selects one key after another, at random, without replacement in order to open the door. Let X be the random variable "the number of keys he tries before opening the door".

Show that $P(X = r) = \frac{1}{n}$, for $r = 1, 2, \ldots, n$.

Find the expected number of keys $E(X)$ and show that the variance of X is $\frac{n^2 - 1}{12}$.

If the standard deviation of X is 2, find the number of keys.

(*b*) A sewing machine, within first year of its purchase requires X number of inspection visits by a maintenance technician, and X follows a Poisson distribution defined by

$$P(X = r) = \begin{cases} e^{-\mu}\dfrac{\mu^r}{r!}, & r = 0, 1, 2, \ldots \quad (\mu > 0) \\ 0, & \text{otherwise.} \end{cases}$$

State the mean and the variance of X.

Further, it is given that $\mu = 4$. Find the probability that more than 4 visits are required.

First visit is free of charge and subsequent visits cost Rs. 1000 each. Find the mean cost of maintenance in the first year of purchase of the machine.

17. (*a*) The probability density function $f(x)$ of a random variable X is given by

$$f(x) = \begin{cases} \dfrac{1}{15}e^{-\frac{x}{15}}, & \text{if } x \geq 0, \\ 0, & \text{otherwise.} \end{cases}$$

(i) Show that $E(X) = 15$ and find $Var(X)$.

(ii) Find the distribution function of X and **hence** find $P(X \geq 20)$.

(*b*) The weights of packets of milk powder are normally distributed with mean 405 g and standard deviation 20 g.

(i) Find the probability that the weight of a randomly selected packet of milk powder will be between 395 g and 420 g.

(ii) Five packets of milk powder are selected at random. Find the probability that at least two of these packets have weight between 395 g and 420 g.

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