



இணைந்தகணிதம்

நியம வடிவம்

$a^2 = 2ab + b^2 = (a+b)^2$
 $\cos \frac{A}{2} = \pm \sqrt{\frac{1+\cos A}{2}}$
 $x^2 - a^2 = (x+a)(x-a)$
 $\cosh^2(x) - \sinh^2(x) = 1$
 $\csc(-x) = -\csc(x)$
 $\lim_{h \rightarrow 0} \frac{f(x_0+h) - f(x_0)}{h} = f'(x_0)$
 $\sinh(x) = \frac{e^x - e^{-x}}{2}$
 $X_{k+1} = (X_k + y/X_k)^{n-1}/2$
 $\sin \frac{A}{2} = \sqrt{\frac{1-\cos A}{2}}$
 $S = \sum_{i=1}^N (x_i - \bar{x})^2$
 $\log_n m = \frac{\log m}{\log n}$
 $\text{sech}(x) = 1/\cosh(x) = 2/(e^x + e^{-x})$
 $C_{n,r} = \binom{n}{r} = \frac{n!}{(n-r)!r!}$
 $x^2 + 2ax + a^2 = (x+a)^2$
 $\cos(-x) = \cos(x)$
 $\text{sech}(z) = \text{Sec}(iz)$
 $\cosh(x) = (e^x + e^{-x})/2$
 $\text{csch}(x) = (e^x - e^{-x})/2$
 $\sim \forall x \forall y [p(x,y)] \equiv \exists x \exists y [\sim p(x,y)]$
 $\text{coth}(z) = i \cot(iz)$
 $\text{arccoth}(z) = 1/2 \ln((z+1)/(z-1))$
 $\text{sinh}(x) = (e^x - e^{-x})/2$
 $a^m \times a^n = a^{m+n}$
 $y_{i+1} = y_i + X_n(b - a y_i)$
 $\tanh(x) = \sinh(x)/\cosh(x) = (e^x - e^{-x})/(e^x + e^{-x})$
 $\text{coth}^2(x) - \text{csch}^2(x) = 1$
 $\text{arcsch}(z) = \ln(1 + \sqrt{1+z^2})/z$
 $\tanh(z) = -i \tan(iz)$
 $\text{arcsech}(z) = \ln(1 \pm \sqrt{1-z^2})/z$
 $\text{sch}(z) = \cos(iz)$
 $b^2 = (a+b)^2$
 $\text{sin}(-x) = -\text{sin}(x)$
 $\frac{P(x)}{Q(x)} = G(x) + \frac{R(x)}{Q(x)}$
 $\vec{u} + \vec{v} = \vec{v} + \vec{u}$
 $x^2 - 2ax + a^2 = (x-a)^2$
 $a_n = a_1 r^{n-1}$



நேர்மாறு சார்புகளின் தொகையீடு

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{-1}{\sqrt{a^2 - x^2}} dx = -\cos^{-1}\left(\frac{x}{a}\right) + C$$

$$= -\sin^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{1}{a^2 + (bx)^2} = \frac{1}{ab} \tan^{-1}\left(\frac{bx}{a}\right) + C$$

உதாரணங்கள்

$$\begin{aligned} 1. \int \frac{1}{\sqrt{9 - x^2}} dx \\ &= \int \frac{1}{\sqrt{3^2 - x^2}} dx \\ &= \sin^{-1}\left(\frac{x}{3}\right) + C \end{aligned}$$

$$\begin{aligned} 2. \int \frac{1}{\sqrt{9 - 16x^2}} dx \\ &= \frac{1}{4} \int \frac{1}{\sqrt{3^2 - (4x)^2}} d(4x) \\ &= \frac{1}{4} \sin^{-1}\left(\frac{4x}{3}\right) + C \end{aligned}$$



$$\begin{aligned}
 3. \quad & \int \frac{1}{\sqrt{16-25x^2}} dx \\
 &= \int \frac{1}{\sqrt{4^2-(5x)^2}} dx \\
 &= \frac{1}{5} \text{Sin}^{-1}\left(\frac{5x}{4}\right) + C
 \end{aligned}$$

$$\begin{aligned}
 4. \quad & \int \frac{e^x}{\sqrt{25-e^{2x}}} dx \\
 &= \int \frac{1}{\sqrt{5^2-(e^x)^2}} d(e^x) \\
 &= \text{Sin}^{-1}\left(\frac{e^x}{5}\right) + C
 \end{aligned}$$

$$\begin{aligned}
 5. \quad & \int \frac{1}{\sqrt{4^2-(x-1)^2}} dx \\
 &= \text{Sin}^{-1}\left(\frac{x-1}{4}\right) + C
 \end{aligned}$$

$$\begin{aligned}
 6. \quad & \int \frac{1}{\sqrt{3^2-(2x-1)^2}} dx \\
 &= \frac{\text{Sin}^{-1}\left(\frac{2x-1}{3}\right)}{2} + C \\
 &= \frac{1}{2} \text{Sin}^{-1}\left(\frac{2x-1}{3}\right) + C
 \end{aligned}$$



$$\begin{aligned}
 7. \quad & \int \frac{1}{16 + 81x^2} dx \\
 &= \int \frac{1}{4^2 + (9x)^2} dx \\
 &= \frac{1}{4 \times 9} \tan^{-1} \left(\frac{9x}{4} \right) + C
 \end{aligned}$$

$$\begin{aligned}
 8. \quad & \int \frac{1}{3^2 + (2x + 1)^2} dx \\
 &= \frac{1}{3 \times 2} \tan^{-1} \left(\frac{2x + 1}{3} \right) + C
 \end{aligned}$$

நியம வடிவம் 9

$$I = \int \frac{1}{\sqrt{ax^2 + bx + C}} dx$$

x^2 இன் குணகம் = $a < 0$ வகை மட்டும்

$$I = \int \frac{px + q}{ax^2 + bx + c} dx$$

$px + q = \lambda(2ax + b) + \mu$ ஆகுமாறு λ, μ மாறிலிகள்

உதாரணங்கள்

$$\begin{aligned}
 01. \quad & I = \int \frac{1}{\sqrt{-x^2 + 2x + 3}} dx \\
 &= -x^2 + 2x + 3 = -\{x^2 - 2x + 1\} + 2 \\
 &= 2 - (x - 1)^2 \\
 & I = \int \frac{1}{\sqrt{(\sqrt{2})^2 - (x - 1)^2}} dx \\
 &= \text{Sin}^{-1} \left\{ \frac{(x - 1)}{\sqrt{2}} \right\} + C
 \end{aligned}$$



$$2. \quad I = \int \frac{3x+1}{\sqrt{-x^2+2x+3}} dx$$

$$3x+1 = \lambda(-2x+2) + \mu$$

$$x; 3 = -2\lambda \quad 1 = 2\lambda + \mu$$

$$\lambda = -\frac{3}{2} \quad \mu = 4$$

$$\begin{aligned} I &= \int \frac{-\frac{3}{2}(-2x+2)+4}{\sqrt{-x^2+2x+3}} dx \\ &= \frac{-3}{2} \int \frac{-2x+2}{\sqrt{-x^2+2x+3}} dx + 4 \int \frac{1}{\sqrt{(\sqrt{2})^2-(x-1)^2}} dx \\ &= \frac{-3}{2} \cdot 2\sqrt{-x^2+2x+3} + 4 \cdot \text{Sin}^{-1}\left(\frac{x-1}{\sqrt{2}}\right) + C \\ &= -3\sqrt{-x^2+2x+3} + 4\text{Sin}^{-1}\left(\frac{x-1}{\sqrt{2}}\right) + C \end{aligned}$$

நியம வடிவம் 10(A)

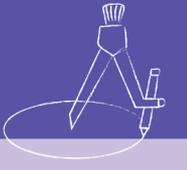
இருபடிக்கோவையும் தொகையீடும்

$$I = \int \frac{1}{ax^2 + bx + c} dx$$

$$\Delta > 0 \Rightarrow I = \ln \text{ வடிவில் அமையும்}$$

$$\Delta = 0 \Rightarrow I = \frac{x^{n+1}}{n+1} \text{ வடிவில் அமையும்}$$

$$\Delta < 0 \Rightarrow I = \tan^{-1}() \text{ வடிவில் அமையும்}$$



உதாரணம் :-

$$\begin{aligned}
 1. \int \frac{1}{x^2 + 3x + 2} dx & \\
 = \int \frac{1}{(x+2)(x+1)} dx & \quad \Delta > 0 \\
 \int \left\{ \frac{-1}{x+2} + \frac{1}{x+1} \right\} dx & \\
 = -\ln|x+2| + \ln|x+1| + C & \\
 = \ln \left| \frac{x+1}{x+2} \right| + C &
 \end{aligned}$$

$$\begin{aligned}
 2. \int \frac{1}{x^2 + 4x + 4} dx & \quad \Delta = 0 \\
 = \int \frac{1}{(x+2)^2} dx & \\
 = \int (x+2)^{-2} dx & \\
 = \frac{(x+2)^{-1}}{-1} dx &
 \end{aligned}$$

$$\begin{aligned}
 3. \int \frac{1}{x^2 + 4x + 13} dx & \quad \Delta < 0 \\
 = \int \frac{1}{x^2 + 4x + 4 + 9} dx & \\
 = \int \frac{1}{(x+2)^2 + 3^2} dx & \\
 = \frac{1}{3} \tan^{-1} \left(\frac{x+2}{3} \right) + C &
 \end{aligned}$$



நியம வடிவம் (10B)

$$I = \int \frac{px+q}{ax^2+bx+c} dx$$

$\Delta > 0 \Rightarrow I = \ln//$ வடிவில் அமையும்

$\Delta = 0 \Rightarrow I = \ln// + \frac{x^{n+1}}{n+1}$ வடிவில் அமையும்

$\Delta < 0 \Rightarrow I = \ln// + \tan^{-1}()$ வடிவில் அமையும்

உதாரணம்

1.

$$\begin{aligned} & \int \frac{5x+2}{x^2+3x+2} dx \\ &= \int \frac{5x+2}{(x+2)(x+1)} dx \\ &= \int \left\{ \frac{8}{x+2} + \frac{-3}{x+1} \right\} dx \\ &= 8 \ln|x+2| - 3 \ln|x+1| + C \\ &= \ln \frac{(x+2)^8}{(x+1)^3} + C \end{aligned}$$

2.

$$\begin{aligned} & \int \frac{2x+1}{x^2+4x+4} dx \\ &= \int \frac{2x+1}{(x+2)^2} dx \\ & \frac{2x+1}{(x+2)^2} = \frac{A}{x+2} + \frac{B}{(x+2)^2} \\ & 2x+1 = A(x+2) + B \\ & x; 2 = A \\ & x; 1 = 2A + B \Rightarrow A = 2, B = -3 \\ & \int \frac{2x+1}{x^2+4x+4} dx = \int \left\{ \frac{2}{x+2} + \frac{-3}{(x+2)^2} \right\} dx \\ &= 2 \ln|x+2| - 3 \frac{(x+2)^{-1}}{-1} + C \\ &= 2 \ln|x+2| + \frac{3}{x+2} + C \end{aligned}$$



3.
$$\int \frac{2x+7}{x^2+4x+13} dx$$

$2x+7 = \lambda(2x+4) + \mu$

$x; 2 = 2\lambda$

$x^0; 7 = 4\lambda + \mu \Rightarrow \lambda = 1, \mu = 3$

$$\int \frac{2x+7}{x^2+4x+13} dx$$

$$= \int \frac{1(2x+4) + 3}{x^2+4x+13} dx$$

$$= \int \frac{2x+4}{x^2+4x+13} dx + 3 \int \frac{1}{(x+2)^2+3^2} dx$$

$$= \ln(x^2+4x+13) + 3 \cdot \frac{1}{3} \tan^{-1}\left(\frac{x+2}{3}\right) + C$$

$$= \ln(x^2+4x+13) + \tan^{-1}\left(\frac{x+2}{3}\right) + C$$

பயிற்சிகள்

1.
$$\int \frac{1}{x^2+7x+10} dx$$

2.
$$\int \frac{1}{x^2+6x+9} dx$$

3.
$$\int \frac{1}{x^2+6x+10} dx$$

4.
$$\int \frac{3x+1}{x^2+7x+12} dx$$

5.
$$\int \frac{3x+5}{x^2+10x+25} dx$$

6.
$$\int \frac{3x+1}{x^2+8x+25} dx$$