1. LHS = RHS = 5 (5)

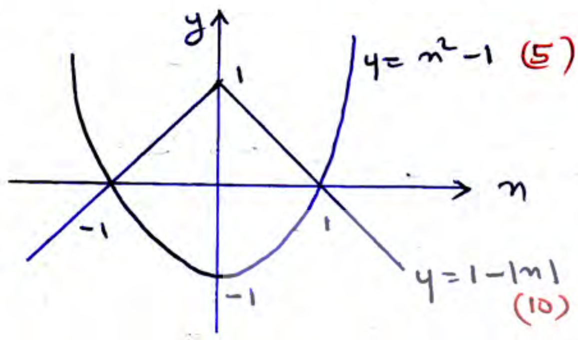
$n = p \Rightarrow \sum_{r=1}^{p} (4r+1) = 2p^2 + 3p$ (5)

$n = p+1 \Rightarrow \sum_{r=1}^{p+1} (4r+1) = \sum_{r=1}^{p} (4r+1) + (4p+5)$ (5)

$= 2(p+1)^2 + 3(p+1)$ (5)

Statement (5) [25]

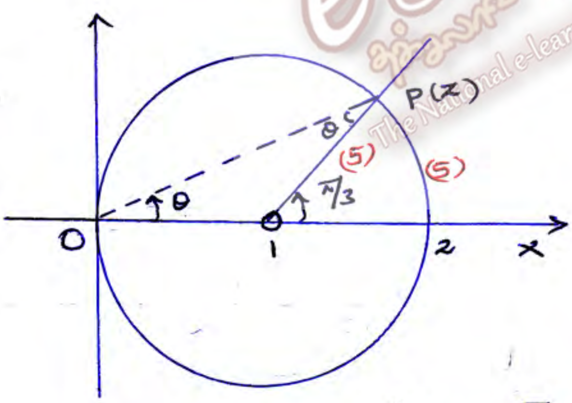
2.




$1 - |x| > x^2 - 1$ (5)

$\therefore$ Solution set $-1 < x < 1$ (5) [25]

3.




$OP = \sqrt{3}$ (5) $\quad P\hat{O}X = \frac{\pi}{6}$ (5)

$z = \sqrt{3}\left\{\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right\}$ (5) [25]

4. (i) $^7C_5 = 21$ (5) (5)

(ii) $^5C_3 \cdot {}^2C_2 + {}^5C_4 \cdot {}^2C_1 + {}^5C_5$ (10)

$= 21$ (5) [25]

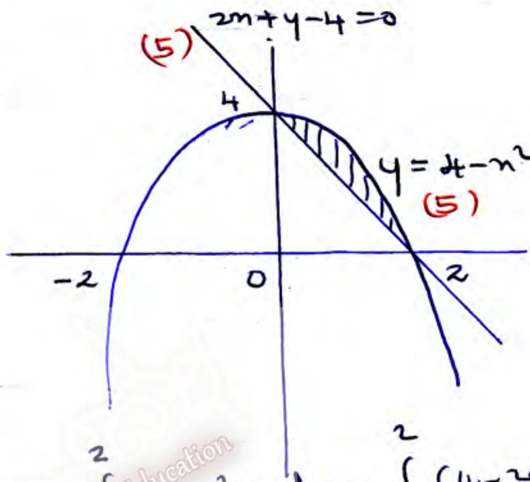
5. $\lim_{x \to 1} \dfrac{\sin\left(\frac{\pi}{2} - \frac{\pi}{2}x\right)}{(1-x)(1+x+x^2)}$ (10)

$= \lim_{x \to 1} \dfrac{\sin \frac{\pi}{2}(1-x)}{\frac{\pi}{2}(1-x)} \cdot \frac{\pi}{2} \cdot \lim_{x \to 1} \dfrac{1}{(1+x+x^2)}$ (5)

$= \frac{\pi}{2}$ (5) $\cdot 1 \cdot \dfrac{1}{(1+1+1)}$ (5)

$= \frac{\pi}{6}$ [25]

6.




$A = \int_0^2 (4 - x^2)\,dx - \int_0^2 (4 - 2x)\,dx$ (5)

$= \int_0^2 (2x - x^2)\,dx = \left[x^2 - \frac{x^3}{3}\right]_0^2$ (5)

$= 4 - \frac{8}{3} = \frac{4}{3}$ (5) [25]

7. $\dfrac{dy}{dx} = \dfrac{3x^2}{2y}$ (5)

$\left(\dfrac{dy}{dx}\right)_{P(4t^2, 8t^3)} = 3t$ (5)

$Q \equiv (4T^2, 8T^3)$, $3t = \dfrac{8T^3 - 8t^3}{4T^2 - 4t^2}$ (5)

$t^2 + Tt - 2T^2 = 0$ (5)

$Q \equiv (t^2, -t^3)$ (5) [25]

8. $y - 2a = \dfrac{2a - 2b}{a^2 - b^2}(x - a^2)$ (5)

$y - 2a = \dfrac{2}{a+b}(x - a^2)$ (10)

$(1, 0) \Rightarrow 0 - 2a = \dfrac{2}{a+b}(1 - a^2)$ (5)

$\Rightarrow ab = -1$ (5) [25]

9. $a = b$ (5)

$a + b - 4 = 0$ (5)

$a = b = 2$ (5)

$x^2 + y^2 - x - y - 10 = 0$ (5)

radius $= \sqrt{21/2}$ (5) [25]

10. $f(\theta) = 3\cos\theta\left(\cos\theta\cos\frac{\pi}{3} - \sin\theta\sin\frac{\pi}{3}\right) + 5\cos\theta + 3$ (5)

$= \frac{3}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta + 5\cos\theta + 3$ (5)

$= 7\left(\frac{13}{14}\cos\theta - \frac{3\sqrt{3}}{14}\sin\theta\right) + 3$ (5)

$= 7\cos(\theta + \alpha) + 3$ (5)

$-4 \le f(\theta) \le 10$ (5) [25]

11. (a) (i) $\Delta = 4(b+c-a)^2 - 4(2bc - a^2)$ (10)

$= 4\{b^2 + c^2 + a^2 + 2bc - 2ca - 2ab - 2bc + a^2\}$ (10)

$= 4\{(a-b)^2 + (c-a)^2\} \ge 0$ (10) [30]

(ii) $(x-\alpha)(x-\beta) < 0$ (10)

$\Rightarrow f(x) < 0$ (5) [15]

(iii) $(\alpha+\beta) + 2a$ (5) $\quad \alpha+\beta = -2(b+c-a)$ (5)

$= 2(2a - b - c)$ (5) $\quad \alpha\beta = 2bc - a^2$ (5)

& $\alpha\beta + a(\alpha+\beta) + a^2$ (5)

$= 2(a^2 + bc - ab - ac)$ (5)

$\therefore$ Equation is

$x^2 - 2(2a - b - c)x + 2(a^2 + bc - ab - ac) = 0$. (5) [35]

(b) $f(x) = (ax+b)\phi(x) + R$ (10)

$f(-b/a) = \left(a(-b/a) + b\right)\phi(-b/a) + R$ (5)

$\therefore$ Reminder is $f(-b/a)$ (5) [20]

$f(1/2) = -\frac{25}{8} \Rightarrow -\frac{1}{2}\left(A(\frac{1}{4}) + B(\frac{1}{2}) + c\right) = -\frac{25}{8}$ (5) (5)

$A + 2B + 4C = 25$ (5)

$f(-\frac{1}{2}) = -\frac{57}{8} \Rightarrow -\frac{3}{2}\left[A(\frac{1}{4}) + B(-\frac{1}{2}) + c\right] = -\frac{57}{8}$ (5) (5)

$A - 2B + 4C = 17$ (5)

$f(0) = -5 \Rightarrow C = 5$ (5) (5)

$A = 1, B = 2$ (5)

$f(x) = x^3 + x^2 + 3x - 5$ (5) [50]

12. (a) Expansion [10]

$\frac{^nC_r}{^nC_{r-1}} = \frac{n-r+1}{r}$ [15]

$^nC_{r-1} : {^nC_r} : {^nC_{r+1}} = 1 : 7 : 42$ (5)

$\frac{n-r+1}{r} = 7$ (5) $\Rightarrow 8r - n = 1$ (5)

$\frac{n-(r+1)+1}{r+1} = 6$ (5) $\Rightarrow -7r + n = 6$ (5)

$r = 7$, (5) $n = 55$ (5)

7th, 8th and 9th terms are in the ratio of $1 : 7 : 42$. (5) [40]

(b) $U_r = \frac{(2r+1)^2 - 2r}{(2r-1)(2r+1)}\left(\frac{1}{3}\right)^r$ (10)

$\frac{4r^2 + 2r + 1}{(2r-1)(2r+1)} = A + \frac{B}{2r-1} + \frac{C}{2r+1}$ (10)

$A = 1$, (5) $B = 3/2$, (5) $C = -1/2$ (5)

$U_r = \left(\frac{1}{3}\right)^r + \frac{1/2}{2r-1}\left(\frac{1}{3}\right)^{r-1} - \frac{1/2}{2r+1}\left(\frac{1}{3}\right)^r$ (5)

$= \left(\frac{1}{3}\right)^r + f(r) - f(r+1)$; where

$f(r) = \frac{1}{2(2r-1)}\left(\frac{1}{3}\right)^r$ (5) [45]

$u_1 = (1/3)^1 + f(1) - f(2)$

$u_2 = (1/3)^2 + f(2) - f(3)$ (5)

$\vdots$

$u_{n-1} = \left(\frac{1}{3}\right)^{n-1} + f(n-1) - f(n)$ (5)

$u_n = \left(\frac{1}{3}\right)^n + f(n) - f(n+1)$

$\sum_{r=1}^{n} u_r = \sum_{r=1}^{n}\left(\frac{1}{3}\right)^r + f(1) - f(n+1)$ (5)

$= \frac{(1/3)(1-(1/3)^n)}{1 - 1/3} + 1/2 - \frac{1}{2(2n+1)}(1/3)^n$ (5) (5)

$= 1 - \left(\frac{n+1}{2n+1}\right)\left(\frac{1}{3}\right)^n$ (5) [30]

$$\lim_{n\to\infty}\left\{1-\left(\frac{1+1/n}{2+1/n}\right)\left(\frac{1}{3}\right)^n\right\}=1 \quad (5)\ (5)$$

$\therefore$ The series is convergent.

Value of convergence is 1. (5)

[15]

13. (a) $2B = \begin{pmatrix} 2 & b \\ 0 & 12 \end{pmatrix}$ (5)

$2B - C = \begin{pmatrix} 1 & 1 \\ 4 & 12-k \end{pmatrix}$ (10)

$A(2B-C) = \begin{pmatrix} 6 & 14-k \\ 0 & k-8 \end{pmatrix}$ (10)

$k = 14$ (5)

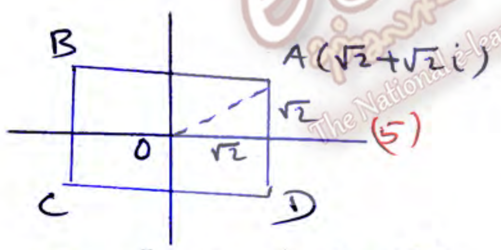
$I = A\left\{\frac{1}{6}(2B-C)\right\}$ (5)

$A^{-1} = \frac{1}{6}(2B-C)$ (5)

$A^{-1} = \frac{1}{6}\left\{\begin{pmatrix} 2 & 6 \\ 0 & 12 \end{pmatrix} - \begin{pmatrix} 1 & 5 \\ -4 & 14 \end{pmatrix}\right\}$ (5)

$= \begin{pmatrix} 1/6 & 1/6 \\ 4/6 & -2/6 \end{pmatrix}$ (5) [50]

(b)



(5)

$z_B = -\sqrt{2}+\sqrt{2}i$, $z_C = -\sqrt{2}-\sqrt{2}i$

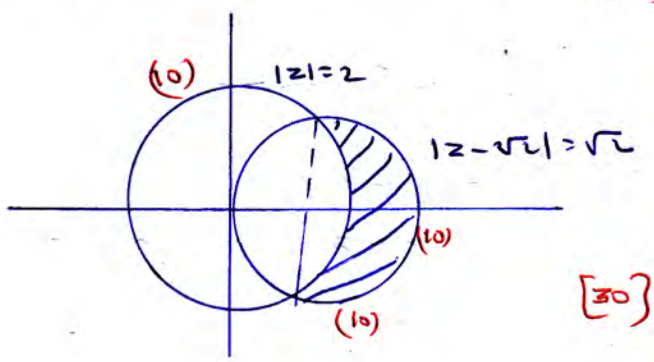
$z_D = \sqrt{2}-\sqrt{2}i$ (15)

radius $= 2$ (5)

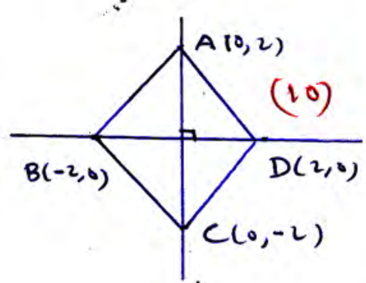
centre $= 0 + i0$ (5)

$|z-(0+i0)| = 2$ (5) $R = 2$ (5)

[40]




[30]




$z_A' = 0+2i$, $z_B' = -2$, $z_C' = -2i$ (20)

14. (a) $f'(x) = \dfrac{(6x-10)6x-(3x^2-3)6}{(6x-10)^2}$ (10)

$= \dfrac{6(3x-1)(x-3)}{(6x-10)^2}$ (5)

$f'(x)=0 \Rightarrow x = 1/3$ or $x = 3$ (5)

$x = 5/3$ is a vertical asymptote. (5) (5)

| Range of $x$ | $x<1/3$ | $1/3<x<5/3$ | $5/3<x<3$ | $x>3$ |
|---|---|---|---|---|
| Sign of $f'(x)$ | (+) | (−) | (−) | (+) |

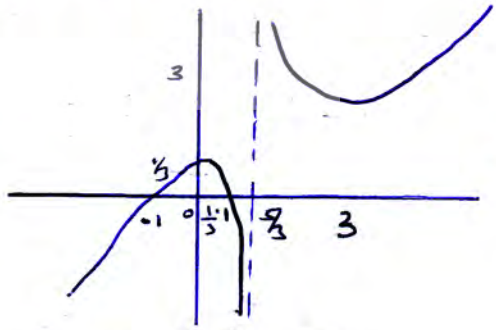
(20)

maximum point $=\left(\frac{1}{3},\frac{1}{3}\right)$ (5)

minimum point $=(3,3)$ (5)

$\left(0,\frac{3}{10}\right)$ (5), $(1,0)$ $(-1,0)$ (5)

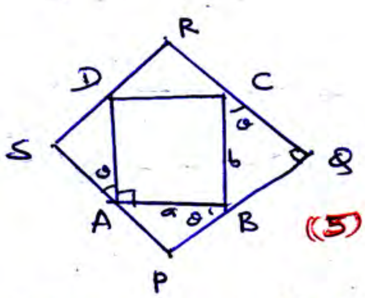
$x\to\pm\infty$, $f(x)\to\pm\infty$ (5)



(15)

[90]

(b)



(5)

$PQ = a\cos\theta + b\sin\theta$ (5) $PS = a\sin\theta + b\cos\theta$ (5)

$S = ab + \left(\dfrac{a^2+b^2}{2}\right)\sin 2\theta$ (10)

$\dfrac{dS}{d\theta} = \left(\dfrac{a^2+b^2}{2}\right)\cos 2\theta \cdot 2$ (5)

$\dfrac{dS}{d\theta} = 0 \Rightarrow \cos 2\theta = 0$ (5)

Range of $\theta$: $0<\theta<\pi/4$ , $\pi/4<\theta<\pi/2$

Sign of $\frac{ds}{d\theta}$: $+$ , $-$ (10)

$PQ = \frac{1}{\sqrt{2}}(a+b)$ , $PS = \frac{1}{\sqrt{2}}(a+b)$ (5)

$S_{max} = \frac{(a+b)^2}{2}$ (5)

{60}

15. (a) $\frac{d}{dx}\{\ln(x+\sqrt{x^2+a^2})\} = \frac{1}{\sqrt{a^2+x^2}}$ (10)

$\int \frac{1}{\sqrt{a^2+x^2}}\,dx = \ln\{x+\sqrt{a^2+x^2}\}+k$ (5)

$\frac{d}{dx}\left(\sqrt{x^2+4x+5}\right) = \frac{x+2}{\sqrt{x^2+4x+5}}$ (10)

$\int \frac{x+2}{\sqrt{x^2+4x+5}}\,dx = \sqrt{x^2+4x+5}+c$ (10)

$\int \frac{2x+5}{\sqrt{x^2+4x+5}}\,dx = 2\int\frac{x+2}{\sqrt{x^2+4x+5}}\,dx + \int\frac{1}{\sqrt{(x+2)^2+1}}\,dx$ (10)

$= 2\sqrt{x^2+4x+5}+\ln\{(x+2)+\sqrt{x^2+4x+5}\}+c_1$ (10)

(b) $\frac{3x-1}{(x^2-1)(x^2+4)} = \frac{A}{x-1}+\frac{B}{x+1}+\frac{Cx+D}{x^2+4}$

$A=1/5$ , $B=2/5$ , $C=-3/5$ , $D=1/5$ (20)

$\int\frac{3x-1}{(x^2-1)(x^2+4)}\,dx = \frac{1}{5}\ln|x-1| + \frac{2}{5}\ln|x+1| - \frac{3}{10}\ln(x^2+4) + \frac{1}{10}\tan^{-1}(x/2)+k$ (5) (5) (5) (5)

{55}

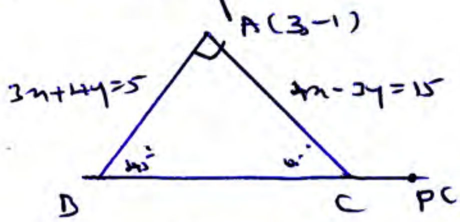
(c) $\int_1^{e^\pi}\cos(\ln x)\,dx = -e^\pi-1+\int_1^{e^\pi}\sin(\ln x)\,dx$ (20)

$= -(e^\pi+1)-\int_1^{e^\pi}\cos(\ln x)\,dx$ (20)

$\Rightarrow \int_1^{e^\pi}\cos(\ln x)\,dx = -\frac{1}{2}(e^\pi+1)$ (5)

{45}

16. (a) Theory (20)




$m_{AB}\times m_{AC} = -1$ (5) $B\hat{A}C = \pi/2$ (5)

$A\equiv(3,-1)$ (5)

$A\hat{B}C = A\hat{C}B = 45°$ (5)

Let the grad. of BC be m

$\tan 45 = \left|\frac{4m+3}{3m-4}\right|$ (10) $\Rightarrow m=-7, 1/7$ (10)

Equations of BC: $7x+y-9=0$ & (5)

$x-7y+13=0$ (5)

[70]

(b) Theory (30)

$2g+4f=c+1$ (10), $3g+4f=c-3/2$ (10), $-2g+6f=c-3$ (10)

$g=-5/2$ (5), $f=-7$ (5), $c=-34$ (5)

Equation of the circle: $x^2+y^2-5x-14y-34=0$ (5)

{80}

17. (a) $y^2+z^2 = 10+6\sin(\theta+\phi)$ (15)

$y^2 = 6+6\sin(\theta+\phi)$ (15)

$y_{max} = 2\sqrt{3}$ (10)

$\theta = 2\phi = \pi/3$ (15)

{50}

(b) Sine rule [05]

$\frac{\sin(C-\theta)}{\sin\theta\sin C} = \frac{\sin C}{\sin A\sin B}$ (10)

$\frac{\sin C\cos\theta-\cos C\sin\theta}{\sin\theta\sin C} = \frac{\sin(A+B)}{\sin A\sin B}$ (10)

$\Rightarrow \cot\theta-\cot C = \cot B+\cot A$ (15)

{35}

(c) (i) $\alpha = \tan^{-1}(5/12)$ , $\beta = \tan^{-1}(7/17)$

$0<\alpha<\pi/4$ , $0<\beta<\pi/4$ (5)

$\tan(\alpha+\beta) = 1$ (15)

$\alpha+\beta = \pi/4$ (5) $(0<\alpha+\beta<\pi/2)$ (5)

[30]

(ii) $2\sin 2x\cos x - 2\cos^2 x = 0$ (10)

$2\cos^2 x(2\sin x-1)=0$ (10)

$x = 2n\pi\pm\pi/2$ , $n\in\mathbb{Z}$ (5)

$x = n\pi+(-1)^n\pi/6$ , $n\in\mathbb{Z}$ (5)

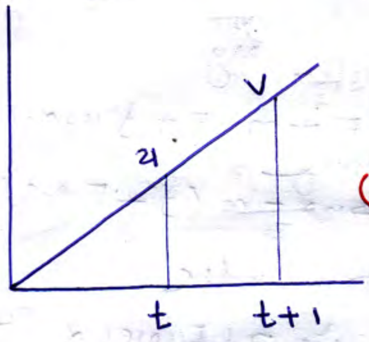
{30}

Field Work Centre

June-2017

Grade: 13(2017) Combined Mathematics II

01.



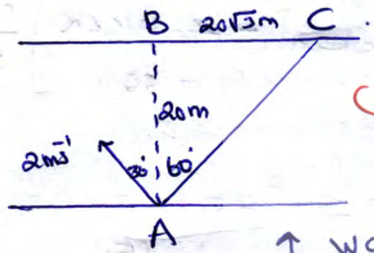

(10)

$v-u=g$ (5) $v+u=h/5$ (5)

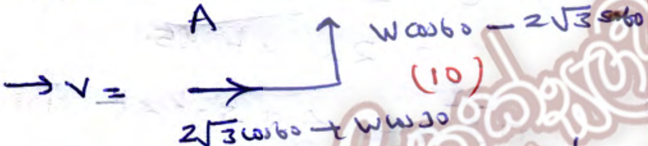
$x=\frac{1}{2}\frac{u^2}{g}$ (5)

[25]

02.




(5)




(10)

$w = 6\ ms^{-1}$ (5)

$v = 4\sqrt{3}\ ms^{-1}$ (5)

[25]

03. $kmv = mu + km \times u/n$ (10)

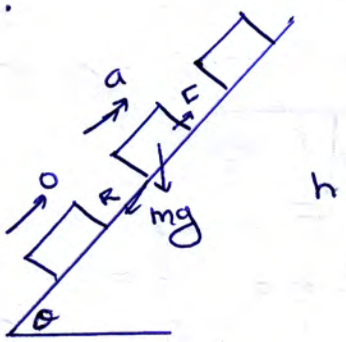
$v = e(u-u/n)$ (5)

$\Rightarrow e = \frac{(n+k)}{k(n-1)}$ (5)

$\frac{n+k}{k(n-1)} < 1 \Rightarrow k > \frac{n}{n-2}$ (5)

[25]

04.




$a = \frac{v^2\sin\theta}{2h}$ (10)

$F - R - mg\sin\theta = ma$ (10)

$P = (R + mg\sin\theta + \frac{mv^2\sin\theta}{2h})v$ (5)

[25]

07. $P(A\cap B) = P(A) + P(B) - P(A\cup B)$ (5)

$= \frac{1}{2}x + \frac{1}{4}x - \frac{5}{8}x$

$= 1/8\, x$ (5)

$P(A\cap B) = P(A)\cdot P(B)$ (10)

$\Rightarrow x = 1$ (5)

[25]

08. $\frac{{}^6C_1 \cdot {}^8C_1}{{}^{14}C_2} = \frac{48}{91}$

(15) (10)

[25]

09. $\frac{1\cdot1 + 2x + 3y + 4\cdot5 + 5\cdot2}{8+x+y} = 3$ (5)

$x = 7$ (5)

$1\cdot2 = \frac{1^2\cdot1 + 2^2\cdot x + 3^2\cdot y + 4^2\cdot 5 + 5^2\cdot 2}{8+x+y} - 3^2$ (10)

$y = 5$ (5)

[25]

10. $x+y = 24$ (5)

$y > 16$ (5)

$x = 4$ or $8$ (5)

$\Downarrow$

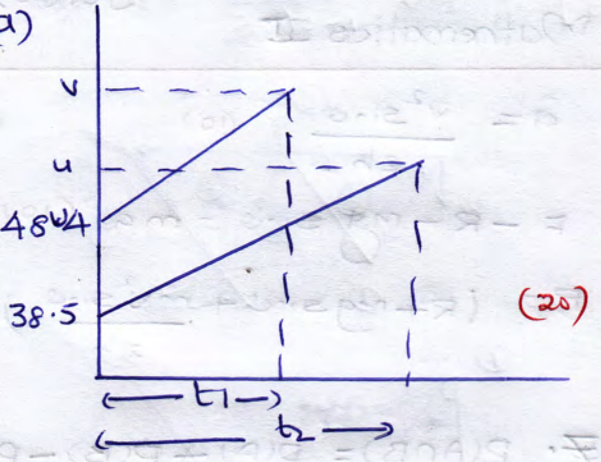
$y = 16$ ∴ impossible (5)

$\Rightarrow x=4,\ y=20$ (5)

[25]

01

11. (a)




(20)

$\dfrac{v - 48.4}{t_1} = 0.55$ (5)  $\dfrac{u - 38.5}{t_2} = 0.44$ (5)

$\left(\dfrac{v + 48.4}{2}\right) t_1 = 1320$ (5)  $\left(\dfrac{u + 38.5}{2}\right) t_2 = 1100$ (5)

$t_1 = 24$ (5)  $t_2 = 25$ (5)

$\therefore$ Y will reach the finishing before 1 s. X will reach the finishing. (5)

$\left(\dfrac{2 \times 48.4 + 0.55t}{2}\right) t - \left(\dfrac{38.5 \times 2 + 0.44t}{2}\right) t = 220$ (10)

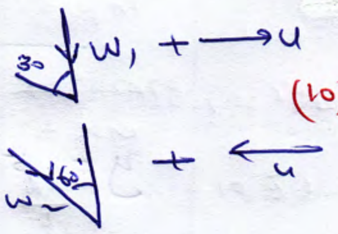
$t = 20$ (10)

$S_x = 858$ (5)

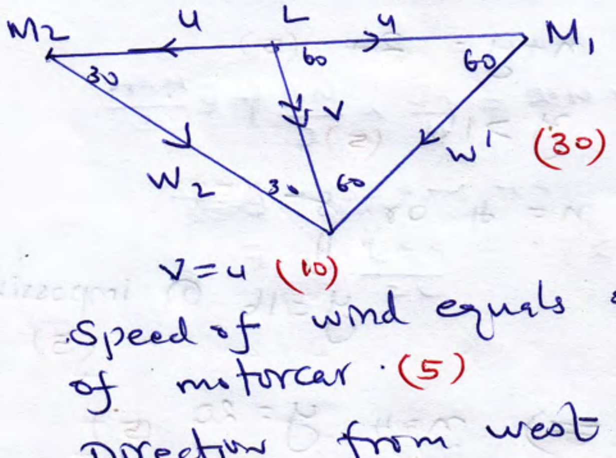
$\therefore$ Before the finishing $= 1100 - 858 = 242$ m. (5)

[85]

(b) $V_{AE} =$




(10)




(30)

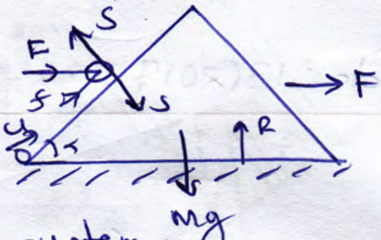
$v = u$ (10)

Speed of wind equals speed of motorcar. (5)

Direction from west 30° of north (10)

[65]

12. (a)




(20)

For the system

$\rightarrow \; 0 = MF + m(F + f\cos\alpha)$ (15)

For the particle

$\nearrow \; -mg\sin\alpha = m(f + F\cos\alpha)$ (15)

Relative to wedge

$\nearrow \; 0 = v^2 + 2fh\,\mathrm{cosec}\,\alpha$ (5)

$\Rightarrow v = \sqrt{\dfrac{2gh(M+m)}{M + m\sin^2\alpha}}$ (10)

$\nearrow \; 0 = v + ft_1$ (5)

Time taken $= 4\,\mathrm{cosec}\,\alpha \sqrt{\dfrac{h(M + m\sin^2\alpha)}{2g(M+m)}}$ (10)

[80]

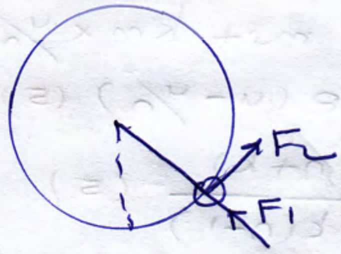
(b) $\theta = \sin^2 2t$

$\dot{\theta} = 2\sin 4t$ (5)

$a\dot{\theta}^2 = 4a\sin^2 4t$ (10)

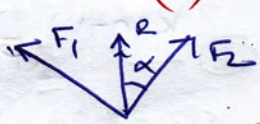
$\ddot{\theta} = 8\cos 4t$ (5)

$a\ddot{\theta} = 0 \Rightarrow t = \pi/8$ (5) (5)




$F_1 = 3am$ (10)  $F_2 = 4am$ (10)

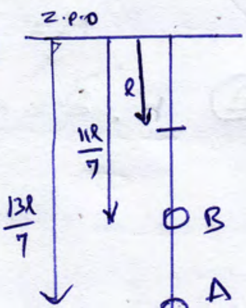
$R = 5am$ (10)




$\tan\alpha = 3/4$ (10)

[70]

13. (a)

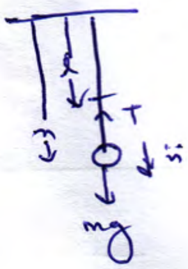



02

Conservation of energy

$$-\frac{13mgl}{7} + \frac{1}{2}\lambda\frac{\left(\frac{6l}{7}\right)^2}{l} = -\frac{11mgl}{7} + \frac{1}{2}\lambda\frac{\left(\frac{4l}{7}\right)^2}{l} \quad (20)$$

$$\Rightarrow \lambda = \frac{7mg}{5} \quad (10)$$



$$mg - T = m\ddot{x} \quad (10)$$

$$mg - \frac{7mg(x-l)}{5l} = m\ddot{x}$$

$$g - \frac{7g}{5l}(x-l) = \ddot{x} \quad (10)$$

$$\ddot{x} = -\frac{7g}{5l}\left(x - \frac{12l}{7}\right) \quad (10)$$

$$x - 12l/7 = A\cos\omega t + B\sin\omega t \quad (10)$$

$$\dot{x} = -A\omega\sin\omega t + B\omega\cos\omega t \quad (10)$$

$$\ddot{x} = -\omega^2(x - 12l/7) \quad (10)$$

$$\omega = \sqrt{7g/5l} \quad (10)$$

$$A = l/7 \ (10), \quad B = 0 \ (10)$$

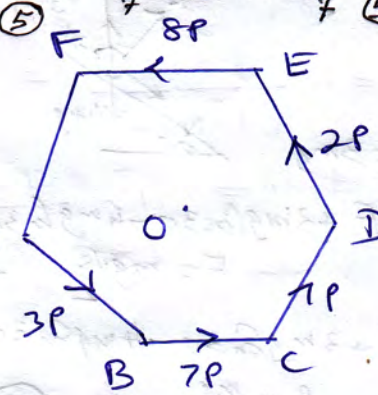
$$-\frac{l\omega}{14} = -\frac{l\omega}{7}\sin\omega t \quad (15)$$

$$\Rightarrow t = \frac{\pi}{6}\sqrt{\frac{5l}{7g}} \quad (05)$$

[150]

$v \cdot \frac{3m}{2} = m\frac{l\omega}{7} \Rightarrow v = \frac{2l\omega}{7}$ (5) (5)

14. (a)




AB = 2l

$$\rightarrow X = 7P + P\cos 60 - 2P\cos 60 - 8P + 3P\cos 60 \quad (10)$$

$$= 0 \quad (15)$$

$$\uparrow Y = P\sin 60 + 2P\sin 60 - 3P\sin 60 \quad (10)$$

$$= 0 \quad (5)$$

$$O\circlearrowleft \ G_0 = \sqrt{3}lP(7+1+2+8+3) \quad (10)$$

$$= 21\sqrt{3}lP \quad (5)$$

03

$X = 0, Y = 0, G_0 \neq 0$. (5)

Reduces to a couple. (5)

If it reduces to a single force along AD,

$G_0 = 0, X \neq 0$ (5)

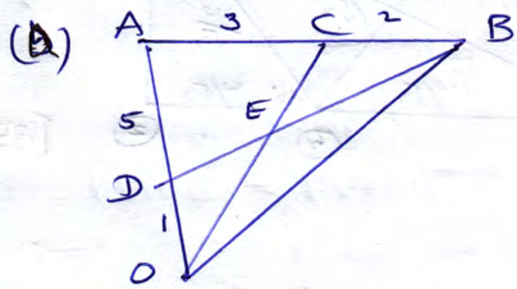
$G_0 = 0$ (5)

$(7+1+2-\lambda+8+3)P = 0$ (10)

$\lambda = 21$ (5)

[80]

(b)




$$\vec{OC} = \vec{OA} + \vec{AC} = a + \frac{3}{5}\vec{AB} \quad (5)\ (5)$$

$$= \frac{2}{5}a + \frac{3}{5}b \quad (10)$$

$\vec{OE} = \lambda\vec{OC}$ (5), $\vec{DB} = \vec{DO} + \vec{OB}$ (5)

$$= -\frac{1}{6}a + b \quad (5)$$

$$\vec{DE} = \vec{DO} + \vec{OE} = \left(\frac{2\lambda}{5} - \frac{1}{6}\right)a + \frac{3\lambda}{5}b \quad (10)$$

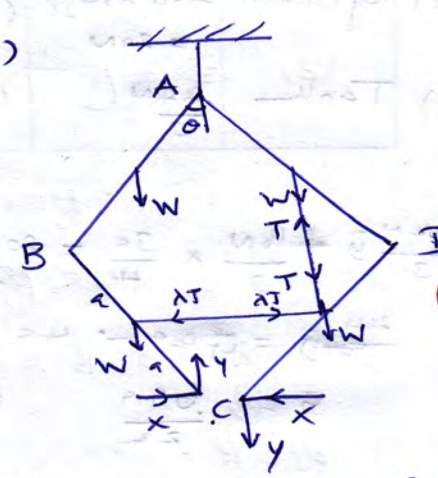
(5) D, E, B straight line.

$$\frac{\frac{2\lambda}{5} - \frac{1}{6}}{-\frac{1}{6}} = \frac{3\lambda/5}{1} \Rightarrow \lambda = \frac{1}{3} \quad (10)\ (5)$$

OE : EC = 1 : 2 (5)

[70]

15. (a)




(10)

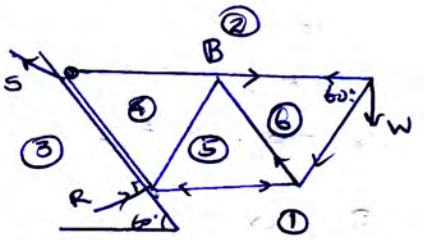
BC: $B\circlearrowleft \ X\,2a\cos\theta + Y\,2a\sin\theta = \lambda T a\cos\theta + W a\sin\theta$ (15)

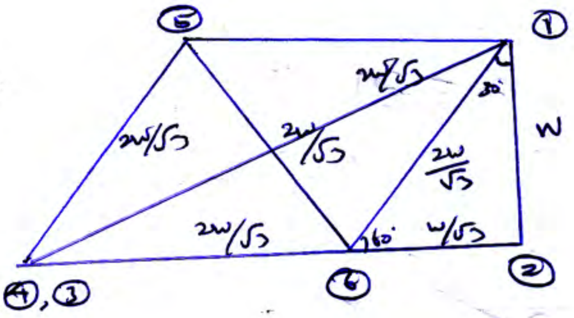
DC: $D\circlearrowleft \ Y\cdot 2a\sin\theta + (W+T)a\sin\theta + \lambda T a\cos\theta = X\,2a\cos\theta$ (15)

AB+BC: $A\circlearrowleft \ X\,4a\cos\theta + 2W a\sin\theta = \lambda T\cdot 3a\cos\theta$ (15)

$\Rightarrow T = \dfrac{4W}{2\cot\theta - 1}$ (10) [65]

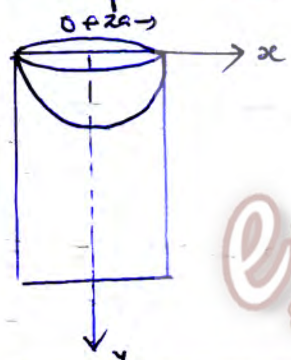
(b)




[85]

16. Theory (40)




| Body | weight | cent. Gr. |
|---|---|---|
| (10) hemisphere | $\frac{2}{3}\pi(2a)^3 \rho g = \frac{16}{3}M$ | $(0, \frac{3a}{4})$ |
| (10) cylinder | $4\pi a^2 \times 5a\rho g = 20M$ | $(0, \frac{5a}{2})$ |
| (10) Tank | $\frac{44M}{3}$ | $(0, \bar{y})$ |

$$\frac{44M}{3}\bar{y} + \frac{16M}{3} \times \frac{3a}{4} = 20M \times \frac{5a}{2}$$

$$\frac{44}{3}\bar{y} = 50a - 4a$$

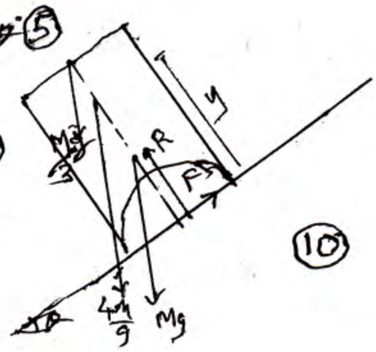
$$\bar{y} = \frac{69a}{22} \quad (20)$$

$\rightarrow F = \frac{4mg}{3}\sin\theta$ (5)

$\uparrow R = \frac{4mg}{3}\cos\theta$ (5)

$\frac{F}{R} \le \mu$

$\tan\theta \le \mu$. (5)




(10)

$\frac{4mg}{3} y = Mg\bar{x} + \frac{Mg}{3} 5a$ (10)

$y = \frac{317}{88}a$ (5)

$y\tan\theta \le 2a$ (10)

$\tan\theta \le \frac{2a}{y}$ (5)

$\tan\theta \le \frac{176}{317}$. (5)

(5) $A \equiv (4,5) \quad B \equiv (x,y)$

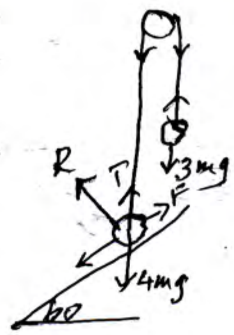
$\vec{AB} = (x-4)\underline{i} + (y-5)\underline{j} = 2\underline{i} + 6\underline{j}$ (10)

$x - 4 = 2 \quad y - 5 = 6$ (5)

$x = 6 \quad y = 11$ (5)

$\vec{OB} = 6\underline{i} + 11\underline{j}$. (5)

(6)




(5)

(5) $T = 3mg$

$\rightarrow F + 3mg\cos 30 - 4mg\sin 30 = 0$

$F = mg\frac{\sqrt{3}}{2}$ — (5)

$\nwarrow R + 3mg\sin 30 - 4mg\sin 30 = 0$

$R = \frac{mg}{2}$ — (5)

$\frac{F}{R} = \mu$

$\mu = \sqrt{3}$. (5)

(17) (a)

| $x$ | 1 | 2 | 3 |
|---|---|---|---|
| $P(x)$ | $\frac{1}{4}$ | $\frac{1}{2}$ | $\frac{1}{4}$ |

(5)

$x_1+x_2=4$

$P(x_1+x_2=4) = P(1,3)+P(3,1)+P(2,2)$

$= \frac{1}{4}\cdot\frac{1}{4}+\frac{1}{4}\cdot\frac{1}{4}+\frac{1}{2}\cdot\frac{1}{2}$ (15)

$= \frac{3}{8}$ (5)

| $y$ | 1 | 2 | 3 |
|---|---|---|---|
| $P(y)$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{2}$ |

(5)

$y_1+y_2=4$

$P(y_1+y_2=4) = P(1,3)+P(3,1)+P(2,2)$

$= \frac{1}{4}\cdot\frac{1}{2}+\frac{1}{2}\cdot\frac{1}{4}+\frac{1}{4}\cdot\frac{1}{4}$ (15)

$= \frac{5}{16}$ (5)

$P(B/4) = \dfrac{P(4/B)\cdot P(B)}{P(4/B)\cdot P(B)+P(4/A)\cdot P(A)}$ (10)

$= \dfrac{\frac{5}{16}\cdot\frac{1}{2}}{\frac{5}{16}\cdot\frac{1}{2}+\frac{3}{8}\cdot\frac{1}{2}}$ (5) (10)

$= 5/11$ (5)

(b) Theory (30)

$y = ax+b$

$\bar{y} = a\bar{x}+b$

$50 = 40a+b$ (10)

$64 = a^2\cdot 25$

$a = \frac{8}{5}$ (10)

$50 = 40\cdot\frac{8}{5}+b$

$b = -14$ (10)

$x=40 \Rightarrow y = 50$ (10)