අ.පො.ස. උසස්පෙළ
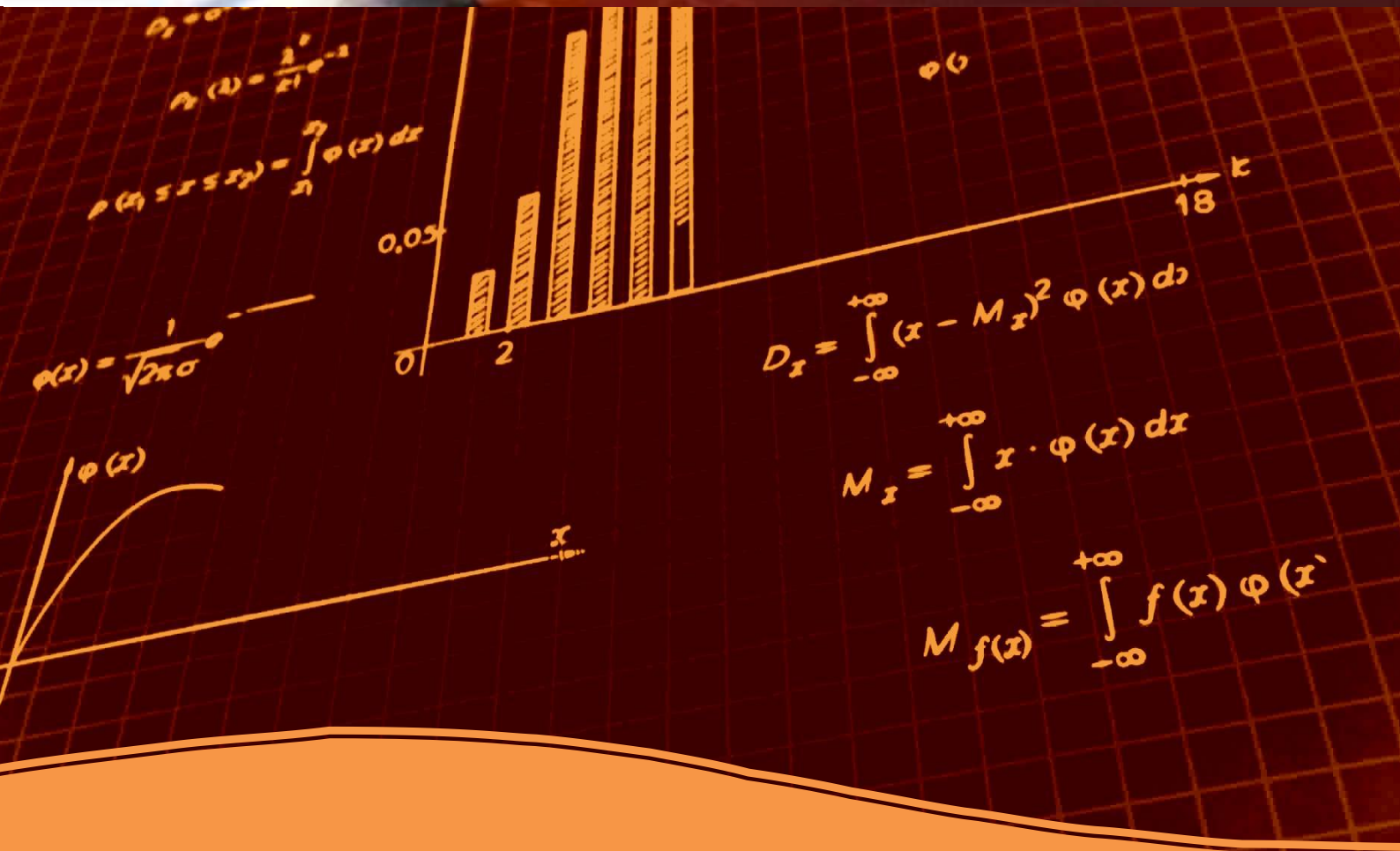# සංයුක්ත ගණිතය

Combined Maths
සංයුක්ත ගණිතය

ඒකකය 12

12.1

1. පහත සමීකරණ විසඳන්න.
   - I. $\cos 4\theta = 0$
   - II. $\tan\left(\frac{5\theta}{2}\right) = 0$
   - III. $\sin^2\left(\frac{7\theta}{2}\right) = 0$

I. $\cos 4\theta = 0$

$$4\theta = (2n + 1)\frac{\pi}{2} \quad n\epsilon\mathbb{z}$$

$$\theta = (2n + 1)\frac{\pi}{8}$$

II. $\tan\left(\frac{5\theta}{2}\right) = 0$

$$\frac{5\theta}{2} = n\pi \quad n\epsilon\mathbb{z}$$

$$\theta = \frac{2n\pi}{5}$$

III. $\sin^2\left(\frac{7\theta}{2}\right) = 0$

$$\sin\left(\frac{7\theta}{2}\right) = 0$$

$$\frac{7\theta}{2} = n\pi \quad n\epsilon\mathbb{z}$$

$$\theta = \frac{2n}{7}\pi$$

2. පහත සමීකරණවල සාධාරණ විසඳුම් ලියන්න.

    I. $\cos\theta = \frac{\sqrt{3}}{2}$

   II. $\tan\theta = 1$

  III. $\text{coses}\,\theta = 2$

I. $\cos\theta = \frac{\sqrt{3}}{2}$

$$\cos\theta = \cos\left(\frac{\pi}{6}\right)$$

$$\theta = 2n\pi \pm \frac{\pi}{6} \quad n\epsilon\mathbb{z}$$

II. $\tan\theta = 1$

$$\tan\theta = \tan\left(\frac{\pi}{4}\right)$$

$$\theta = n\pi + \frac{\pi}{4} \quad n\epsilon\mathbb{z}$$

III. $\csc\theta = 2$

$$\sin\theta = \frac{1}{2}$$

$$\sin\theta = \sin\left(\frac{\pi}{6}\right)$$

$$\theta = n\pi + (-1)^n\left(\frac{\pi}{6}\right) \quad n\epsilon\mathbb{z}$$

3. පහත ත්‍රිකෝණමිතික සමීකරණ විසඳන්න.

    I. $\cos 3\theta = -\frac{1}{2}$

    II. $\tan\frac{\theta}{2} = \sqrt{3}$

    III. $\sin\left(\frac{\theta}{3}\right) = -\frac{\sqrt{3}}{2}$

    IV. $\sec\left(\frac{3\theta}{2}\right) = 2$

I. $\cos 3\theta = -\frac{1}{2}$

$$\cos 3\theta = \cos\left(\pi - \frac{\pi}{3}\right)$$

$$\cos(3\theta) = \cos\left(\frac{2\pi}{3}\right)$$

$$3\theta = 2n\pi \pm \frac{2\pi}{3} \quad n\epsilon \mathbf{z}$$

$$\theta = \frac{2n}{3}\pi \pm \frac{2\pi}{9}$$

II. $\tan\left(\frac{\theta}{2}\right) = \sqrt{3}$

$$\tan\left(\frac{\theta}{2}\right) = \tan\left(\frac{\pi}{3}\right)$$

$$\frac{\theta}{2} = n\pi + \frac{\pi}{3} \quad n\epsilon \mathbf{z}$$

$$\theta = 2n\pi + \frac{2\pi}{3}$$

III. $\sin\left(\frac{\theta}{3}\right) = -\frac{\sqrt{3}}{2}$

$$\sin\left(\frac{\theta}{3}\right) = -\sin\left(\frac{\pi}{3}\right)$$

$$\sin\left(\frac{\theta}{3}\right) = \sin\left(-\frac{\pi}{3}\right)$$

$$\frac{\theta}{3} = n\pi + (-1)^n\left(-\frac{\pi}{3}\right) \quad n\epsilon z$$

$$\theta = 3n\pi - (-1)^n\pi$$

IV. $\sec\left(\frac{3\theta}{2}\right) = 2$

$$\cos\left(\frac{3\theta}{2}\right) = \frac{1}{2}$$

$$\cos\left(\frac{3\theta}{2}\right) = \cos\left(\frac{\pi}{3}\right)$$

$$\frac{3\theta}{2} = 2n\pi \pm \frac{\pi}{3} \quad n\epsilon z$$

$$\theta = \frac{4n}{3}\pi \pm \frac{2\pi}{9}$$

4. විසඳන්න.

    I. $4\sin^2\theta = 3$

    II. $4\cos^2(2\theta) = 1$

    III. $\tan^2\left(\frac{\theta}{2}\right) = 1$

අන්තර්ගතය : H.D. වලංග ශෂිමාල් ප්‍රනාන්දුමයා - කො/ විවේකානන්ද ජාතික පාසල - කොළඹ - 13

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I. $4\sin^2\theta = 3$

$\sin^2\theta = \dfrac{3}{4}$

$\sin^2\theta = \left(\dfrac{\sqrt{3}}{2}\right)^2$

$\sin^2\theta = \sin^2\left(\dfrac{\pi}{3}\right)$

$\underline{\underline{\theta = n\pi \pm \dfrac{\pi}{3} \; n\epsilon z}}$

II. $4\cos^2(2\theta) = 1$

$\cos^2(2\theta) = \dfrac{1}{4}$

$\cos^2(2\theta) = \left(\dfrac{1}{2}\right)^2$

$\cos^2(2\theta) = \cos^2\left(\dfrac{\pi}{3}\right)$

$2\theta = n\pi \pm \dfrac{2}{3} \; n\epsilon z$

$\underline{\underline{\theta = n\dfrac{\pi}{2} \pm \dfrac{\pi}{6}}}$

III. $\tan^2\left(\dfrac{\theta}{2}\right) = 1$

$\tan^2\left(\dfrac{\theta}{2}\right) = \tan^2\left(\dfrac{\pi}{4}\right)$

$\dfrac{\theta}{2} = n\pi \pm \dfrac{\pi}{4} \; n\epsilon z$

$\underline{\underline{\theta = 2n\pi \pm \dfrac{\pi}{2}}}$

5. පහත ත්‍රිකෝණමිතික සමීකරණ වල සාධාරණ විසඳුම් සොයන්න.

    I. $\sin 3\theta = \sin \theta$

    II. $\sin 2\theta + \cos \theta = 0$

    III. $\tan(3\theta) = \cot \theta$

I. $\sin 3\theta = \sin \theta$

$$3\theta = n\pi + (-1)^n \theta$$

$$\theta(3 - (-1)^n) = n\pi \qquad n\epsilon\mathbb{z}$$

$$\theta = \frac{n\pi}{[3 - (-1)^n]}$$

II. $\sin 2\theta + \cos \theta = 0$

$$\cos \theta = -\sin 2\theta$$

$$\cos \theta = \cos\left(\frac{\pi}{2} + 2\theta\right)$$

$$\theta = 2n\pi \pm \left(\frac{\pi}{2} + 2\theta\right)$$

$$\theta = 2n\pi \pm \frac{\pi}{2} \pm 2\theta$$

$$\theta(1 \pm 2) = 2n\pi \pm \frac{\pi}{2} \, n\epsilon\mathbb{z}$$

$$\theta = \frac{4n\pi \pm \pi}{(2 \pm 4)}$$

III. $\tan(3\theta) = \cot \theta$

$$\tan(3\theta) = \tan\left(\frac{\pi}{2} - \theta\right)$$

$$3\theta = n\pi + \left(\frac{\pi}{2} - \theta\right) n\epsilon\mathbb{z}$$

$$4\theta = n\pi = \frac{\pi}{2}$$

$$\theta = n\frac{\pi}{4} + \frac{\pi}{8}$$

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6. පහත ත්‍රිකෝණමිතික සමීකරණ විසඳන්න.

   I. $\sin(2\theta) + \sin(4\theta) + \sin(6\theta) = 0$
   II. $\sin(\theta) + \sin(2\theta) + \sin(3\theta) = \cos(\theta) + \cos(2\theta) + \cos(3\theta)$
   III. $2\cos^2\theta + 3\sin\theta = 0$
   IV. $\sin^2\theta - \cos 2\theta = 2 - \sin 2\theta$
   V. $3\sin^2\theta - 2\sin\theta.\cos\theta + 3\cos^2\theta = 2$

I. $\sin(2\theta) + \sin(4\theta) + \sin(6\theta) = 0$

$\sin(2\theta) + \sin(6\theta) + \sin(4\theta) = 0$

$2\sin\left(\dfrac{2\theta + 6\theta}{2}\right).\cos\left(\dfrac{2\theta - 6\theta}{2}\right) + \sin(4\theta) = 0$

$2\sin(4\theta).\cos(-2\theta) + \sin(4\theta) = 0$

$\sin(4\theta)[2\cos(2\theta) + 1] = 0$

$\sin(4\theta) = 0$ හෝ

$4\theta = n\pi$ $n\epsilon\mathbb{z}$

$\theta = n\dfrac{\pi}{4}$

$\cos(2\theta) + 1 = 0$

$\cos(2\theta) = -\dfrac{1}{2}$

$\cos(2\theta) = \cos\left(\pi - \dfrac{\pi}{3}\right)$

$\cos(2\theta) = \cos\left(\dfrac{2\pi}{3}\right)$

$2\theta = 2n\pi \pm \dfrac{2\pi}{3}$ $n\epsilon\mathbb{z}$

$\theta = n\pi \pm \dfrac{\pi}{3}$

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සැකසුම : A.M.I.U. අරම්පොළ මෙය - අ/ දිගන්හල්මිල්ලවැව්විද්‍යාලය - කහටගස්දිගිලිය.

II. $\sin(\theta) + \sin(2\theta) + \sin(3\theta) = \cos(\theta) + \cos(2\theta) + \cos(3\theta)$

$\sin(\theta) + \sin(3\theta) + \sin(2\theta) = \cos(\theta) + \cos(3\theta) + \cos(2\theta)$

$2\sin\left(\frac{\theta+3\theta}{2}\right).\cos\left(\frac{\theta-3\theta}{2}\right) + \sin(2\theta) = 2\cos\left(\frac{\theta+3\theta}{2}\right).\cos\left(\frac{\theta-3\theta}{2}\right) + \cos(2\theta)$

$2\sin(2\theta).\cos(-\theta) + \sin(2\theta) = 2\cos(2\theta).\cos(-\theta) + \cos(2\theta)$

$\sin 2\theta[2\cos\theta + 1] = \cos 2\theta[2\cos\theta + 1]$

$[2\cos\theta + 1][\sin 2\theta - \cos 2\theta] = 0$

$2\cos\theta + 1 = 0$ හෝ $\sin 2\theta - \cos 2\theta = 0$

$\cos\theta = -\frac{1}{2}$ $\cos 2\theta = \sin 2\theta$

$\cos\theta = \cos\left(\frac{2\pi}{3}\right)$ $\tan(2\theta) = 1$

$\theta = 2n\pi \pm \frac{2\pi}{3} \quad n\epsilon\mathbb{z}$ $\tan(2\theta) = \tan\left(\frac{\pi}{4}\right)$

                                       $2\theta = n\pi + \frac{\pi}{4} \quad n\epsilon\mathbb{z}$

                                       $\theta = n\frac{\pi}{2} + \frac{\pi}{8}$

III. $2\cos^2\theta + 3\sin\theta = 0$

$2(1 - \sin^2\theta) + 3\sin\theta = 0$

$2 - 2\sin^2\theta + 3\sin\theta = 0$

$2\sin^2\theta - 3\sin\theta - 2 = 0$

$(2\sin\theta + 1).(\sin\theta - 2) = 0$

$2\sin\theta + 1 = 0$ හෝ $\sin\theta = 2$

$\sin\theta = -\frac{1}{2}$

$\sin\theta = \sin\left(-\frac{\pi}{6}\right)$

$\theta = n\pi + (-1)^n\left(-\pi/6\right)$

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IV. $\sin^2 \theta - \cos 2\theta = 2 - \sin 2\theta$

$\sin^2 \theta - (1 - 2\sin^2 \theta) = 2 - 2\sin \theta . \cos \theta$

$3\sin^2 \theta + 2\sin \theta . \cos \theta = 3$

$3\sin^2 \theta + 2\sin \theta . \cos \theta - 3(\sin^2 \theta + \cos^2 \theta) = 0$

$3\cancel{\sin^2 \theta} + 2\sin \theta . \cancel{\cos \theta} - 3\sin^2 \theta - 3\cos^2 \theta = 0$

$\cos[2\sin \theta - 3\cos \theta] = 0$

$$\cos \theta = 0$$
$$\theta = (2n + 1)\frac{\pi}{2} \, n\epsilon \mathbf{z}$$

හෝ

$$2\sin \theta - 3\cos \theta = 0$$
$$\tan \theta = \frac{3}{2}$$
$$\tan \theta = \tan \alpha = \frac{3}{2} \, n\epsilon \mathbf{z}$$
$$\theta = n\pi + \alpha$$

V. $3\sin^2 \theta - 2\sin \theta . \cos \theta + 3\cos^2 \theta = 2$

$3\sin^2 \theta - 2\sin \theta . \cos \theta + 3\cos^2 \theta = 2(\sin^2 \theta + \cos^2 \theta)$

$3\sin^2 \theta - 2\sin \theta . \cos \theta + 3\cos^2 \theta = 2\sin^2 \theta + 2\cos^2 \theta$

$\sin^2 \theta - 2\sin \theta . \cos \theta + \cos^2 \theta = 0$

$\tan^2 \theta - 2\tan \theta + 1 = 0$ $\hspace{3cm}$ $\cos \theta \neq 0$

$(\tan \theta - 1)^2 = 0$

$\tan \theta = 1$

$\tan \theta = \tan \left(\frac{\pi}{4}\right)$

$\theta = n\pi + \frac{\pi}{4} \, n\epsilon \mathbf{z}$

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7. විසඳන්න.

    I. $\sqrt{3}(\tan\theta + \tan 2\theta) + \tan\theta.\tan 2\theta = 1$

    II. $\tan(2\theta) + \tan(3\theta) + \tan(5) = \tan 2\theta.\tan 3\theta.\tan 5\theta$

I. $\sqrt{3}(\tan\theta + \tan 2\theta) + \tan\theta.\tan 2\theta = 1$

    $\sqrt{3}(\tan\theta + \tan 2\theta) = 1 - \tan\theta.\tan 2\theta$

    $\dfrac{\tan\theta + \tan 2\theta}{1 - \tan\theta.\tan 2\theta} = \dfrac{1}{\sqrt{3}}$

    $\tan(\theta + 2\theta) = \dfrac{1}{\sqrt{3}}$

    $\tan(3\theta) = \tan\left(\dfrac{\pi}{6}\right)$

    $3\theta = n\pi + \dfrac{\pi}{6} \quad n\epsilon \mathbf{z}$

    $\theta = n\dfrac{\pi}{3} + \dfrac{\pi}{18}$

II. $\tan(2\theta) + \tan(3\theta) + \tan(5\theta) = \tan(2\theta).\tan(3\theta).\tan(5\theta)$

    $\tan(2\theta) + \tan(3\theta) = -\tan(5\theta)[1 - \tan(3\theta).\tan(5\theta)]$

    $\dfrac{\tan(2\theta) + \tan(3\theta)}{1 - \tan(3\theta).\tan(5\theta)} = -\tan(5\theta)$

    $\tan(2\theta + 3\theta) = -\tan(5\theta)$

    $2\tan(5\theta) = 0$

    $\tan(5\theta) = 0$

    $5\theta = n$

    $\theta = n\dfrac{\pi}{5} \quad n\epsilon \mathbf{z}$

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