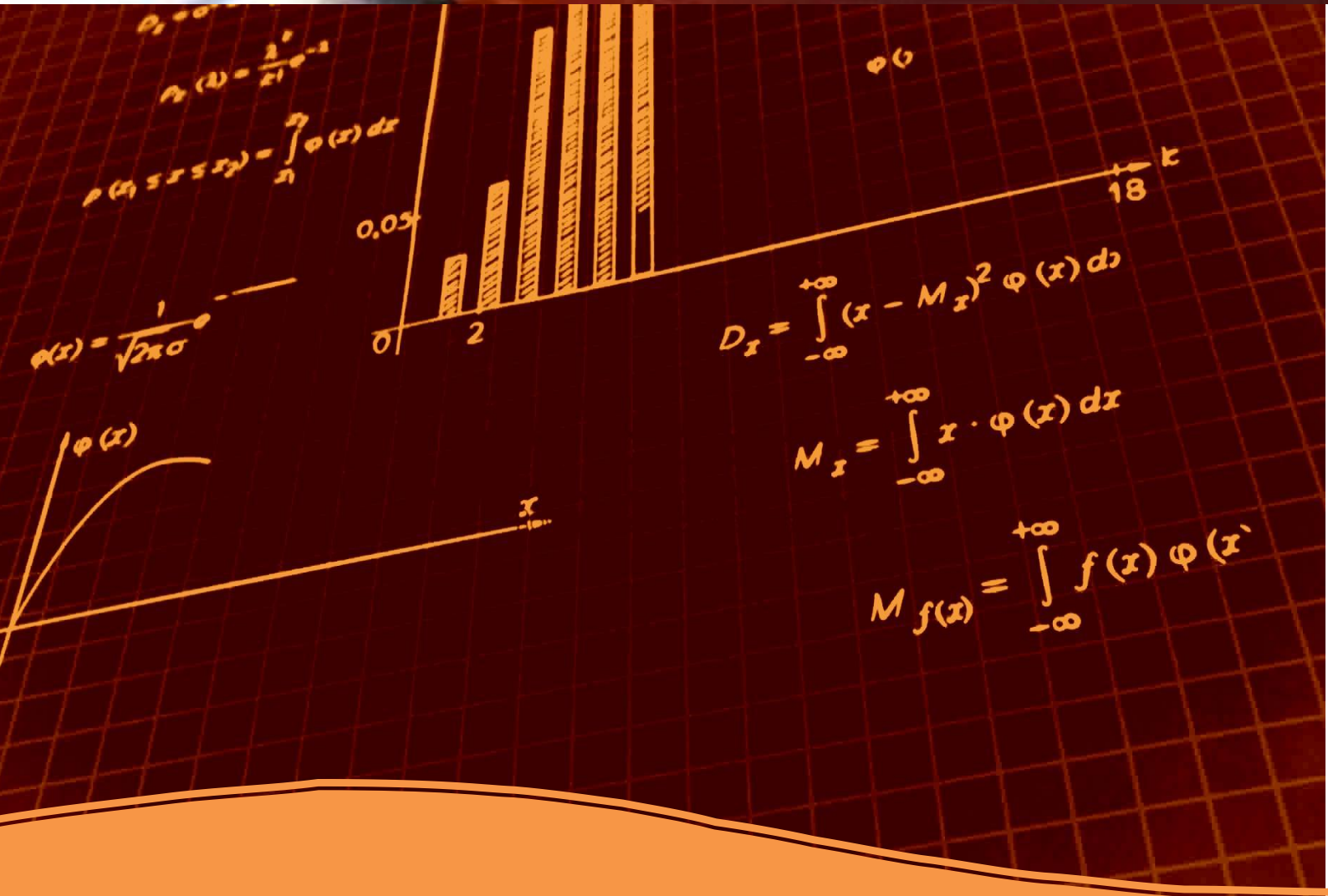


අ.පො.ස. උසස්පෙළ සංයුක්ත ගණිතය



ඒකකය 8

8.4

◆ අගය සොයන්න.

- I. $\sin\left(22\frac{1}{2}^\circ\right)$
- II. $\cos\left(22\frac{1}{2}^\circ\right)$
- III. $\sin\left(7\frac{1}{2}^\circ\right)$
- IV. $\cos\left(7\frac{1}{2}^\circ\right)$
- V. $\tan\left(22\frac{1}{2}^\circ\right)$
- VI. $\tan\left(11\frac{1}{4}^\circ\right)$
- VII. $\cot\left(7\frac{1}{2}^\circ\right)$

✓ පිළිතුර

$$I. \quad \sin\left(22\frac{1}{2}^\circ\right) = \sin\left(\frac{45^\circ}{2}\right)$$

$$\sin^2(A) = \frac{1 - \cos(2A)}{2}$$

$$A = \left(\frac{45^\circ}{2}\right) \text{ නම්}$$

$$\sin^2\left(\frac{45^\circ}{2}\right) = \frac{1 - \cos\left(2 \times \frac{45^\circ}{2}\right)}{2}$$

$$\frac{1}{2}[1 - \cos(45^\circ)]$$

$$\frac{1}{2}\left[1 - \frac{1}{\sqrt{2}}\right]$$

$$\frac{1}{2}\left[1 - \frac{1}{\sqrt{2}}\right]$$

$$\frac{\sqrt{2} - 1}{2\sqrt{2}} = \frac{2 - \sqrt{2}}{4}$$

$$\sin\left(\frac{45^\circ}{2}\right) = \underline{\underline{\frac{\sqrt{2} - \sqrt{2}}{2}}}$$

$$\begin{aligned} \text{II. } \cos\left(22\frac{1}{2}^\circ\right) &= \cos\left(\frac{45^\circ}{2}\right) \\ \cos^2(A) &= \frac{1 + \cos 2A}{2} \\ \cos\left(22\frac{1}{2}^\circ\right) &= \cos\left(\frac{45^\circ}{2}\right) \\ A &= \left(\frac{45^\circ}{2}\right) \text{ නම්} \\ \cos^2\left(\frac{45^\circ}{2}\right) &= \frac{1 + \cos\left(2 \times \frac{45^\circ}{2}\right)}{2} \end{aligned}$$

$$\frac{1}{2}[1 + \cos(45^\circ)]$$

$$\begin{aligned} \cos^2\left(\frac{45^\circ}{2}\right) &= \frac{1}{2} \left[1 + \frac{1}{\sqrt{2}}\right] \\ &= \frac{(\sqrt{2}+1)}{2\sqrt{2}} = \frac{2+\sqrt{2}}{4} \\ \cos\left(\frac{45^\circ}{2}\right) &= \frac{\sqrt{2+\sqrt{2}}}{2} \end{aligned}$$

$$\begin{aligned}
 \text{III. } \sin\left(7\frac{1}{2}^\circ\right) &= \sin\left(\frac{15^\circ}{2}\right) \\
 \cos 15^\circ &= \cos(45^\circ - 30^\circ) = \cos 45^\circ \cdot \cos 30^\circ + \sin 45^\circ \cdot \sin 30^\circ \\
 &= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2} \\
 &= \frac{\sqrt{3} + 1}{2\sqrt{2}}
 \end{aligned}$$

$$\cos(15^\circ) = \frac{\sqrt{6} + \sqrt{2}}{4}$$

$$\sin^2\left(\frac{15^\circ}{2}\right) = \frac{1 - \cos\left(2 \times \frac{15^\circ}{2}\right)}{2}$$

$$= \frac{1}{2}[1 - \cos(15^\circ)]$$

$$= \frac{1}{2}\left[1 - \frac{(\sqrt{3} + 1)}{2\sqrt{2}}\right]$$

$$= \frac{2\sqrt{2} - \sqrt{3} - 1}{4\sqrt{2}}$$

$$\sin^2\left(\frac{15^\circ}{2}\right) = \frac{4 - \sqrt{6} - \sqrt{2}}{8}$$

$$\sin\left(\frac{15^\circ}{2}\right) = \frac{\sqrt{4 - \sqrt{6} - \sqrt{2}}}{2\sqrt{2}}$$

$$\begin{aligned}
 \text{IV. } \cos\left(7\frac{1}{2}^\circ\right) &= \cos\left(\frac{15^\circ}{2}\right) \\
 \text{ඉහත ප්‍රතිඵලයේ සාධනය අනුව } \cos(15^\circ) &= \frac{\sqrt{6}+\sqrt{2}}{4} \\
 \cos^2\left(\frac{15^\circ}{2}\right) &= \left[\frac{1 + \cos\left(2 \times \frac{15^\circ}{2}\right)}{2}\right] \\
 &= \frac{1}{2}[1 + \cos(15^\circ)] \\
 &= \frac{1}{2}\left[1 + \frac{(\sqrt{6} + \sqrt{2})}{4}\right] \\
 &= \frac{[4 + \sqrt{6} + \sqrt{2}]}{8} \\
 \cos\left(\frac{15^\circ}{2}\right) &= \frac{\sqrt{4 + \sqrt{6} + \sqrt{2}}}{2\sqrt{2}}
 \end{aligned}$$

$$\begin{aligned}
 \text{V. } \tan\left(22\frac{1}{2}^\circ\right) &= \tan\left(\frac{45^\circ}{2}\right) \\
 \tan^2\left(\frac{A}{2}\right) &= \frac{2 \sin^2\left(\frac{A}{2}\right)}{2 \cos^2\left(\frac{A}{2}\right)} = \frac{1 - \cos(A)}{1 + \cos(A)} \\
 \tan^2\left(\frac{45^\circ}{2}\right) &= \frac{1 - \cos(45^\circ)}{1 + \cos(45^\circ)} + \frac{1 - \cos(45^\circ)}{1 + \cos(45^\circ)} + \frac{\sqrt{2} - 1}{\sqrt{2} + 1} \\
 \tan^2\left(\frac{45^\circ}{2}\right) &= \frac{(\sqrt{2} - 1)^2}{(\sqrt{2} + 1)(\sqrt{2} - 1)} + \frac{(\sqrt{2} - 1)^2}{2 + 1} \\
 \tan^2\left(\frac{45^\circ}{2}\right) &= (\sqrt{2} - 1)^2 \\
 \tan^2\left(\frac{45^\circ}{2}\right) &= \underline{\underline{\sqrt{2} - 1}}
 \end{aligned}$$

$$VI. \quad \tan\left(11\frac{1}{4}^\circ\right) = \tan\left(\frac{45^\circ}{2}\right)$$

$$\tan(A) = \frac{2 \tan\left(\frac{A}{2}\right)}{1 - \tan^2\left(\frac{A}{2}\right)}$$

$$A = \frac{45^\circ}{2} \text{ නම්}$$

$$\tan\frac{45^\circ}{2} = \frac{2 \tan\left(\frac{45^\circ}{4}\right)}{1 - \tan^2\left(\frac{45^\circ}{4}\right)}$$

$$\tan\left(\frac{45^\circ}{4}\right) = X \text{ නම්}$$

$$(\sqrt{2} - 1) = \frac{2X}{1 - X^2}$$

$$(\sqrt{2} - 1)(1 - X^2) = 2X$$

$$(\sqrt{2} - 1)X^2 + (\sqrt{2} - 1) = 2X$$

$$(\sqrt{2} - 1)X^2 + (\sqrt{2} - 1) 2X$$

$$(\sqrt{2} - 1)X^2 + 2X - (\sqrt{2} - 1) = 0$$

මෙය X වල වර්ග සමීකරණයකි. $X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ භාවිතයෙන්

$$X = \frac{-2 \pm \sqrt{2^2 - 4 \times (\sqrt{2} - 1) (\sqrt{2} - 1)}}{2 \times (\sqrt{2} - 1)}$$

$$X = \frac{-2 \pm \sqrt{4 - 4(\sqrt{2} - 1)^2}}{2 \times (\sqrt{2} - 1)} = \frac{-2 \pm 2\sqrt{1 + (\sqrt{2} - 1)^2}}{2(\sqrt{2} - 1)}$$

$$= \frac{-1 \pm \sqrt{1 + 2 - 2\sqrt{2}} + 1}{(\sqrt{2} - 1)} = \frac{[-1 \pm \sqrt{4 - 2\sqrt{2}}] (\sqrt{2} + 1)}{(\sqrt{2} - 1) (\sqrt{2} + 1)}$$

$$X = \frac{-(\sqrt{2} + 1) \pm \sqrt{(4 - 2\sqrt{2})(\sqrt{2} + 1)^2}}{2 - 1} = \frac{-(\sqrt{2} + 1) \pm \sqrt{(4 - 2\sqrt{2})(3 + 2\sqrt{2})}}{1}$$

$$\tan\left(\frac{45^\circ}{4}\right) = \sqrt{\frac{(4 - 2\sqrt{2})(3 - 2\sqrt{2}) - (\sqrt{2} + 1)}{2}}$$

$$\text{VII. } \cot\left(7\frac{1}{2}^\circ\right) = \cot\left(\frac{15^\circ}{2}\right)$$

$$\begin{aligned} \cot\left(\frac{15^\circ}{2}\right) &= \frac{\cot\left(\frac{15^\circ}{2}\right)}{\sin\left(\frac{15^\circ}{2}\right)} = \frac{2 \cos^2 \frac{15^\circ}{2}}{2 \sin \frac{15^\circ}{2} \cos \frac{15^\circ}{2}} \\ &= \frac{1 + \cos 15^\circ}{\sin 15^\circ} \end{aligned}$$

$$= \frac{1 + \cos(45^\circ - 30^\circ)}{\sin(45^\circ - 30^\circ)}$$

$$= \frac{1 + \cos(45^\circ) \cos(30^\circ) + \sin(45^\circ) \sin(30^\circ)}{\sin(45^\circ) \cos(30^\circ) - \cos(45^\circ) \sin(30^\circ)}$$

$$= \frac{1 + \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2}}{\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2}}$$

$$\cot\left(\frac{15^\circ}{2}\right) = \frac{(2\sqrt{2} + \sqrt{3} + 1)(\sqrt{3} + 1)}{(\sqrt{3} - 1)(\sqrt{3} + 1)}$$

$$\cot\left(\frac{15^\circ}{2}\right) = \frac{2\sqrt{6} + 3 + \sqrt{3} + 2\sqrt{2} + \sqrt{3} + 1}{(3 - 1)}$$

$$= \frac{2\sqrt{6} + 4 + \sqrt{3} + 2\sqrt{3} + 2\sqrt{2}}{2}$$

$$= \cot\left(\frac{15^\circ}{2}\right) = \sqrt{6} + 4 + \sqrt{3} + \sqrt{2}$$

$$= \cot\left(\frac{15^\circ}{2}\right) = \underline{\underline{\sqrt{2} + \sqrt{3} + \sqrt{4} + \sqrt{6}}}$$

02. $(\cos A + \cos B)^2 + (\sin A + \sin B)^2 = 4 \cos^2 \left(\frac{A-B}{2}\right)$ බව සාධනය කරන්න.

එනමින් $\cos\left(\frac{15^\circ}{2}\right)$ හි අගය සොයන්න.

$$\text{ච.පැ.}(\cos A + \cos B)^2 + (\sin A + \sin B)^2$$

$$\cos^2 A + \cos^2 B + 2 \cos A \cdot \cos B + \sin^2 A + \sin^2 B + 2 \sin A \cdot \sin B$$

$$(\cos^2 A + \sin^2 B) + (\cos^2 B + \sin^2 B) + 2(\cos A \cdot \cos B + \sin A \cdot \sin B)$$

$$1 + 1 + 2 \cos(A - B)$$

$$2 \cdot 2 \cos^2 \left(\frac{A - B}{2}\right)$$

$$4 \cos^2 \left(\frac{A - B}{2}\right) = \text{උ.පැ}$$

$$4 \cos^2 \left(\frac{A - B}{2}\right) = (\cos A + \cos B)^2 + (\sin A + \sin B)^2$$

$$= A = 45^\circ + B = 30^\circ \text{ නම්,}$$

$$= 4 \cos^2 \left(\frac{45^\circ + 30^\circ}{2}\right) = [\cos 45^\circ + \cos 30^\circ]^2 + [\sin 45^\circ + \sin 30^\circ]^2$$

$$= \left[\frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2}\right]^2 + \left[\frac{1}{\sqrt{2}} + \frac{1}{2}\right]^2$$

$$= \frac{1}{2} + \frac{3}{4} + 2 \cdot \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} + \frac{1}{4} + 2 \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$$

$$1 + 1 + \frac{\sqrt{3}}{\sqrt{2}} + \frac{1}{\sqrt{2}}$$

$$= 4 \cos^2 \left(\frac{15^\circ}{2}\right) = \frac{2\sqrt{2} + \sqrt{3} + 1}{\sqrt{2}}$$

$$= 4 \cos^2 \left(\frac{15^\circ}{2}\right) = \frac{4 + \sqrt{6} + \sqrt{2}}{8}$$

$$= \cos^2 \left(\frac{15^\circ}{2}\right) = \frac{\sqrt{4 + \sqrt{6} + \sqrt{2}}}{2\sqrt{2}}$$

03.

I. $\frac{1+\cos 2\theta}{\sin 2\theta} = \cot \theta$

II. $\frac{1+\sin 2\theta-\cos 2\theta}{1+\sin 2\theta+\cos 2\theta} = \tan \theta$ බව සාධනය කරන්න.

I. $\frac{1+\cos 2\theta}{\sin 2\theta} = \cot \theta$

ච.පැ
 $\frac{1+\cos 2\theta}{\sin 2\theta}$

$$\frac{2(\cos 2\theta)}{2 \sin 2\theta \cdot \cos \theta}$$

$$\frac{\cos \theta}{\sin \theta}$$

$\cot \theta =$ ද.පැ

II. $\frac{1+\sin(2\theta)-\cos(2\theta)}{1+\sin(2\theta)+\cos(2\theta)} = \tan \theta$

ච.පැ $\frac{1+\sin(2\theta)-\cos(2\theta)}{1+\sin(2\theta)+\cos(2\theta)}$

$$\frac{(1-\cos 2\theta)+(2 \sin \theta \cdot \cos \theta)}{(1+\cos 2\theta)+(2 \sin \theta \cdot \cos \theta)}$$

$$\frac{(1-\cos 2\theta)+2 \sin \theta \cdot \cos \theta}{(1+\cos 2\theta)+2 \sin \theta \cdot \cos \theta}$$

$$\frac{2 \sin^2 \theta + 2 \sin \theta \cdot \cos \theta}{2 \cos^2 \theta + 2 \sin \theta \cdot \cos \theta}$$

$$\frac{\sin \theta(\sin \theta + \cos \theta)}{\cos \theta(\cos \theta + \sin \theta)} = \tan \theta \text{ ද.පැ}$$

04. $18^\circ, 36^\circ, 54^\circ$ සහ 72° කෝණවල අනුපාතවල අගය සොයන්න.

$$A = 18^\circ$$

$$5A = 90^\circ$$

$$2A = 90^\circ - 3A$$

$$\sin(2A) = \sin(90^\circ - 3A)$$

$$\sin(2A) = \cos(3A)$$

$$2 \sin(2A) \cdot \cos A = 4 \sin^3 A - 3 \cos A$$

$$\cos(A) (2 \sin A - 4 \cos^2 A + 3) = 0$$

$$\cos A = \cos 18^\circ \neq 0$$

$$2 \sin A - 4(1 - \sin^2 A) + 3 = 0$$

$$2 \sin A - 4(1 - \sin^2 A) + 3 = 0$$

$$2 \sin^2 A + 2 \sin A - 1 = 0$$

$$\sin A = \frac{-2 \pm \sqrt{2^2 - 4 \times 4 \times (-1)}}{2 \times 4}$$

$$\sin A = \frac{-2 \pm \sqrt{4 + 16}}{8}$$

$$\sin A = \frac{-2 \pm 2\sqrt{5}}{8} = \frac{-1 \pm \sqrt{5}}{4}$$

$$\sin A = \frac{\sqrt{5} - 1}{4} \quad (A = 18^\circ \text{ නිසා})$$

$$\therefore \sin(18^\circ) = \frac{\sqrt{5} - 1}{4}$$

$$\cos(18^\circ) = \sqrt{1 - \sin^2 18^\circ} = \sqrt{1 - \left(\frac{\sqrt{5}-1}{4}\right)^2}$$

$$= \sqrt{1 - \frac{(5-2\sqrt{5}-1)}{16}} = \sqrt{\frac{10+2\sqrt{5}}{16}}$$

$$\cos(18^\circ) = \frac{\sqrt{10 + 2\sqrt{5}}}{4}$$

$$\sin(54^\circ) = \sin(90^\circ - 36^\circ) = \cos(36^\circ) = \frac{1 + \sqrt{5}}{4}$$

$$\cos 54^\circ = \cos(90^\circ - 36^\circ) = \sin(36^\circ) = \frac{\sqrt{10 - 2\sqrt{5}}}{4}$$

05.

I. $\sin\left(\frac{\pi}{10}\right) + \sin\frac{13\pi}{10} = \frac{-1}{2}$

II. $\sin^2(24^\circ) - \sin^2(6^\circ) = \frac{\sqrt{5}-1}{8}$ බව සාධනය කරන්න.

I. $\sin\left(\frac{\pi}{10}\right) + \sin\frac{13\pi}{10} = \frac{-1}{2}$

$$\sin\left(\frac{\pi}{10}\right) + \sin\left(\frac{13\pi}{10}\right)$$

$$\sin\left(\frac{\pi}{10}\right) + \sin\left(\pi + \frac{3\pi}{10}\right)$$

$$\sin\left(\frac{\pi}{10}\right) - \sin\left(\frac{3\pi}{10}\right)$$

$$\sin(18^\circ) - \sin(54^\circ)$$

$$\sin(18^\circ) - \sin(36^\circ)$$

$$\frac{(\sqrt{5}) - 1}{4} - \frac{(\sqrt{5}) + 1}{4}$$

$$\frac{(\sqrt{5}) - 1}{4} - \frac{(\sqrt{5}) + 1}{4}$$

$$\frac{(\sqrt{5}) - 1 - \sqrt{5} - 1}{4} = -\frac{2}{4} = -\frac{1}{2} \quad \text{ද. පැ}$$

8 සටහන්

$$\begin{aligned}
 \text{II. } \sin^2(24^\circ) - \sin^2(6^\circ) &= \frac{\sqrt{5}-1}{8} \\
 \sin^2(24^\circ) - \sin^2(60^\circ) & \\
 \frac{1}{2} [2 \sin^2(24^\circ) - 2 \sin^2(6^\circ)] & \\
 \frac{1}{2} [(X - \cos 48^\circ) - (X - \cos 12^\circ)] & \\
 \frac{1}{2} [(\cos 48^\circ) - (\cos 12^\circ)] &
 \end{aligned}$$

$$1/2 \sin \sin$$

$$\cos\left(\frac{4\pi}{7}\right)$$

$$\frac{1}{2 \sin\left(\frac{\pi}{7}\right)} \left[2 \sin\left(\frac{\pi}{7}\right) \cdot \cos\left(\frac{\pi}{7}\right) \right] \cos\left(\frac{2\pi}{7}\right) \cdot \cos\left(\frac{4\pi}{7}\right)$$

$$\frac{1}{2 \sin\left(\frac{\pi}{7}\right)} \left(2 \sin\left(\frac{2\pi}{7}\right) \cdot \cos\left(\frac{2\pi}{7}\right) \right) \cos\left(\frac{4\pi}{7}\right)$$

$$\frac{1}{4 \sin\left(\frac{\pi}{7}\right)} \left(2 \sin\left(\frac{2\pi}{7}\right) \cdot \cos\left(\frac{2\pi}{7}\right) \right) \cos\left(\frac{4\pi}{7}\right)$$

$$\frac{1}{4 \sin\left(\frac{\pi}{7}\right)} \sin\left(\frac{4\pi}{7}\right) \cos\left(\frac{4\pi}{7}\right)$$

$$\frac{1}{8 \sin\left(\frac{\pi}{7}\right)} \left[2 \sin\left(\frac{4\pi}{7}\right) \cos\left(\frac{4\pi}{7}\right) \right]$$

$$\frac{1}{8 \sin\left(\frac{\pi}{7}\right)} \sin\left(\frac{8\pi}{7}\right)$$

$$\frac{1}{8 \sin\left(\frac{\pi}{7}\right)} \sin\left(\pi + \frac{\pi}{7}\right)$$

$$\frac{-\sin\left(\frac{\pi}{7}\right)}{8 \sin\left(\frac{\pi}{7}\right)}$$

8 සටහන්

$$\frac{-1}{8} = \quad \text{ද. පැ}$$

$$07. \left[1 + \cos\left(\frac{\pi}{8}\right)\right] \left[1 + \cos\left(\frac{3\pi}{8}\right)\right] \left[1 + \cos\left(\frac{5\pi}{8}\right)\right] \left[1 + \cos\left(\frac{7\pi}{8}\right)\right] = \frac{1}{8} \left[1 + \cos\left(\frac{\pi}{8}\right)\right] \left[1 + \cos\left(\frac{3\pi}{8}\right)\right] \left[1 + \cos\left(\frac{5\pi}{8}\right)\right] \left[1 + \cos\left(\frac{7\pi}{8}\right)\right]$$

$$\left[1 + \cos\left(\frac{\pi}{8}\right)\right] \left[1 + \cos\left(\frac{3\pi}{8}\right)\right] \left[1 + \cos\left(\pi - \frac{3\pi}{8}\right)\right] \left[1 + \cos\left(\pi - \frac{\pi}{8}\right)\right]$$

$$\left[1 + \cos\left(\frac{\pi}{8}\right)\right] \left[1 + \cos\left(\frac{3\pi}{8}\right)\right] \left[1 - \cos\left(\frac{3\pi}{8}\right)\right] \left[1 - \cos\left(\frac{\pi}{8}\right)\right]$$

$$\left[1 + \cos^2\left(\frac{\pi}{8}\right)\right] \left[1 - \cos^2\left(\frac{3\pi}{8}\right)\right]$$

$$\sin^2\left(\frac{\pi}{8}\right) - \sin^2\left(\frac{3\pi}{8}\right)$$

$$\frac{1}{4} \left(2\sin^2\left(\frac{\pi}{8}\right)\right) - \left(2\sin^2\left(\frac{3\pi}{8}\right)\right)$$

$$\frac{1}{4} \left[1 - \cos\left(\frac{\pi}{4}\right)\right] \left[1 + \cos\left(\frac{3\pi}{4}\right)\right]$$

$$\frac{1}{4} \left[1 - \frac{1}{\sqrt{2}}\right] \left[1 + \frac{1}{\sqrt{2}}\right]$$

$$\frac{1}{4} \left(1 - \frac{1}{2}\right)$$

$$\frac{1}{4} \times \frac{1}{2} = \frac{1}{8} = \quad \text{ද. පැ}$$

08. $\cos^4\left(\frac{5\pi}{12}\right) + \cos^4\left(\frac{7\pi}{12}\right) = \frac{25-4\sqrt{3}}{16}$ බව සාධනය කරන්න.

$$\left[\cos^2\left(\frac{5\pi}{12}\right)\right]^2 + \left[\cos^2\left(\frac{7\pi}{12}\right)\right]^2$$

$$+ \left[\frac{1 + \cos\left(\frac{7\pi}{6}\right)}{2}\right]^2$$

$$\left\{\frac{25 - 4\sqrt{3}}{16}\right\} = \text{ද.පැ}$$

09.

I. $\sin A \cdot \sin(60^\circ - A) \cdot \sin(60^\circ + A) = \frac{1}{4} \sin 3A$

II. $\sin^3 A + \sin^3\left(\frac{2\pi}{3} + A\right) + \sin^3\left(\frac{4\pi}{3} + A\right) = -\frac{3}{4} \sin 3A$ බව සාධනය කරන්න.

I. $\sin A \cdot \sin(60^\circ - A) \cdot \sin(60^\circ + A) = \frac{1}{4} \sin 3A$

එ.පැ $\sin A \cdot \sin(60^\circ - A) \cdot \sin(60^\circ + A)$

$\sin A \cdot \{\sin 60^\circ \cdot \cos A - \cos 60^\circ \cdot \sin A\} \{\sin 60^\circ \cdot \cos A - \cos 60^\circ \cdot \sin A\}$

$\{\sin 60^\circ \cdot \cos A - \sin 60^\circ \cdot \sin A\}$

$\left\{\frac{\sqrt{3}}{2} \cdot \cos A + \frac{1}{2} \cdot \sin A\right\}$

$\left\{\sqrt{3} \cos A + \frac{1}{2} \cdot \sin A\right\}$

$\frac{1}{4} \sin 3A = \text{ද.පැ}$

$$\text{II. } \sin^3 A + \sin^3 \left(\frac{2\pi}{3} + A \right) + \sin^3 \left(\frac{4\pi}{3} + A \right) = -\frac{3}{4} \sin 3A$$

$$\text{ච. පැ} \quad \sin^3 A + \sin^3 \left(\frac{2\pi}{3} + A \right) + \sin^3 \left(\frac{4\pi}{3} + A \right)$$

$$+ \frac{1}{4} [3 \sin \left(\frac{2\pi}{3} + A \right) - \sin(4\pi + 3A)]$$

$$\frac{1}{4} [3 \sin A - \sin 3A + 3 \sin(120^\circ + A) - \sin(3A) + 3 \sin(240^\circ + A)]$$

$$- \frac{3}{4} \sin(3A) + \frac{3}{4} [\sin A + 2 \sin(180^\circ + A) \cos(60^\circ)]$$

$$- \frac{3}{4} \sin(3A) + \frac{3}{4} \left[\sin A + (-\sin A) \times \frac{1}{2} \right]$$

$$- \frac{3}{4} \sin(3A) + \frac{3}{4} [\sin A - \sin A]$$

$$- \frac{3}{4} \sin(3A) = \quad \text{ද. පැ}$$