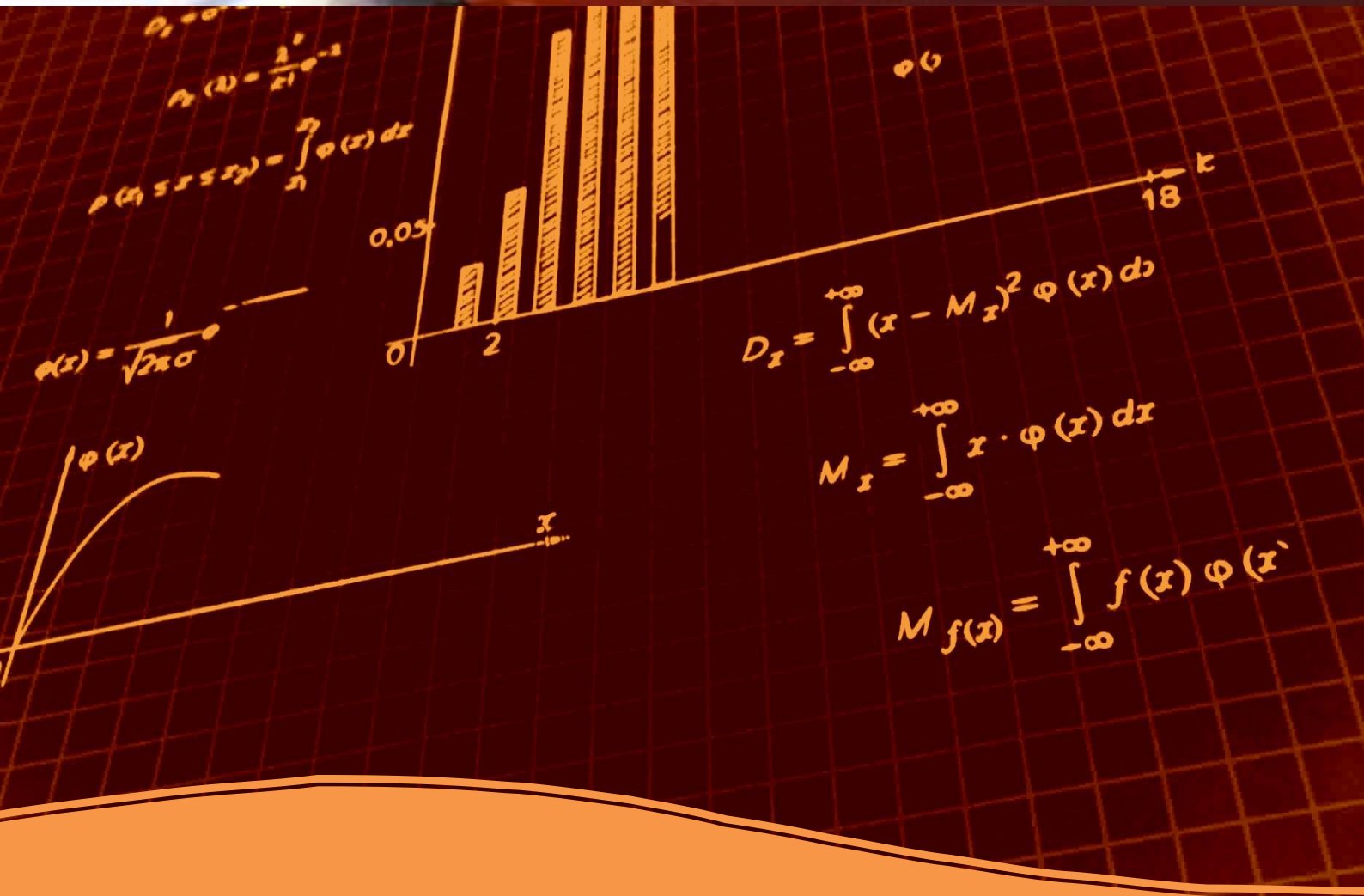


# අ.පො.ස. උසස්පෙළ සංයුක්ත ගණිතය



ඒකකය 8

8.1

1.  $\tan \theta + \cot \theta = \sec \theta \cdot \operatorname{cosec} \theta$  බව පෙන්වන්න.

$$\text{ච.පැ} \quad \tan \theta + \cot \theta$$

$$\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta}$$

$$\frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta \cdot \sin \theta}$$

$$\frac{1}{\cos \theta \cdot \sin \theta}$$

$$\frac{1}{\cos \theta} \times \frac{1}{\sin \theta}$$

$$\sec \theta \cdot \operatorname{cosec} \theta = \text{ද.පැ}$$

2.  $\cot^2 \theta - \cos^2 \theta = \cot^2 \theta \cdot \cos^2 \theta$  බව පෙන්වන්න.

$$\text{ච.පැ} \quad \cot^2 \theta - \cos^2 \theta$$

$$\frac{\cos^2 \theta}{\sin^2 \theta} - \cos^2 \theta$$

$$\frac{\cos^2 \theta - \cos^2 \theta \cdot \sin^2 \theta}{\sin^2 \theta}$$

$$\frac{\cos^2 \theta (1 - \sin^2 \theta)}{\sin^2 \theta}$$

$$\frac{\cos^2 \theta}{\sin^2 \theta} \cdot \cos^2 \theta$$

$$\cot^2 \theta - \cos^2 \theta = \text{ද.පැ}$$

3.  $\tan^4 \theta + \sec^2 \theta = \sec^4 \theta - \tan^2 \theta$  බව පෙන්වන්න.

$$\text{ච.පැ} \quad \tan^4 \theta + \sec^2 \theta$$

$$(\tan^2 \theta)^2 + \sec^2 \theta$$

$$(\sec^2 \theta - 1)^2 + (1 + \tan^2 \theta)$$

$$\sec^4 \theta - 2 \sec^2 \theta + 1 + 1 + \tan^2 \theta$$

$$\sec^4 \theta - 2(\sec^2 \theta - 1) \tan^2 \theta$$

$$\sec^4 \theta - 2 \tan^2 \theta + \tan^2 \theta$$

$$\sec^4 \theta - \tan^2 \theta = \text{ද.පැ}$$

4.  $(\sec \theta - \cos \theta)(\tan \theta + \cot \theta)(\operatorname{cosec} \theta - \sin \theta) = 1$  බව පෙන්වන්න

ච.පැ  $(\sec \theta - \cos \theta)(\tan \theta + \cot \theta)(\operatorname{cosec} \theta - \sin \theta)$

$$\left(\frac{1}{\cos \theta} - \cos \theta\right) \left(\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta}\right) \left(\frac{1}{\cos \theta} - \sin \theta\right)$$

$$\frac{(1 - \cos^2 \theta)}{\cos \theta} \frac{(\sin^2 \theta + \cos^2 \theta)}{\cos \theta \sin \theta} \frac{(1 - \sin^2 \theta)}{\sin \theta}$$

$$\frac{\sin^2 \theta}{\cos^2 \theta} \times \frac{1}{\cos \theta \sin \theta} \times \frac{\cos^2 \theta}{\sin \theta}$$

1 ද.පැ

5.  $(1 + \cot \theta - \operatorname{cosec} \theta)(1 + \tan \theta + \sec \theta) = 2$  බව පෙන්වන්න.

ච.පැ  $(1 + \cot \theta - \operatorname{cosec} \theta)(1 + \tan \theta + \sec \theta)$

$$\left(1 + \frac{\cos \theta}{\sin \theta} - \frac{1}{\sin \theta}\right) \left(1 + \frac{\sin \theta}{\cos \theta} + \frac{1}{\cos \theta}\right)$$

$$\left(\frac{\sin \theta + \cos \theta - 1}{\sin \theta}\right) \left(\frac{\cos \theta + \sin \theta + 1}{\cos \theta}\right)$$

$$\frac{\{(\sin \theta + \cos \theta) - 1\} \{(\sin \theta + \cos \theta) + 1\}}{\sin \theta \cos \theta}$$

$$\frac{(\sin \theta + \cos \theta)^2 - 1}{\sin \theta \cos \theta}$$

$$\frac{\sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta - 1}{\sin \theta \cos \theta}$$

$$\frac{1 + 2 \sin \theta \cos \theta - 1}{\sin \theta \cos \theta}$$

$$\frac{2 \sin \theta \cos \theta}{\sin \theta \cos \theta}$$

2 ද.පැ

6.  $\sin^6 \theta + \cos^6 \theta = 1 - 3 \sin^2 \theta \cdot \cos^2 \theta$  බව සාධනය කරන්න.

ච.පැ  $\sin^6 \theta + \cos^6 \theta$

$$(\sin^2 \theta)^3 + (\cos^2 \theta)^3$$

$$(\sin^2 \theta + \cos^2 \theta)((\sin^2 \theta)^2 - \sin^2 \theta \cos^2 \theta + (\cos^2 \theta)^2)$$

$$1 \times \{(\sin^2 \theta + \cos^2 \theta)^2 - 2 \sin^2 \theta \cos^2 \theta - \sin^2 \theta \cos^2 \theta\}$$

$$1^2 - 3 \sin^2 \theta \cos^2 \theta$$

$$1 - 3 \sin^2 \theta \cos^2 \theta \text{ ද.පැ}$$

8 සටහන්

7.  $\frac{\sin \theta}{(1+\cos \theta)} + \frac{(1+\cos \theta)}{\sin \theta} = 2 \operatorname{cosec} \theta$  බව සාධනය කරන්න.

ච.පැ  $\frac{\sin \theta}{(1+\cos \theta)} + \frac{(1+\cos \theta)}{\sin \theta}$   
 $\frac{\sin^2 \theta + (1+\cos \theta)^2}{(1+\cos \theta) \sin \theta}$   
 $\frac{\sin^2 \theta + 1 + 2 \cos \theta + \cos^2 \theta}{(1+\cos \theta) \sin \theta}$   
 $\frac{2(1+\cos \theta)}{(1+\cos \theta) \sin \theta}$   
 $\frac{2}{\sin \theta}$   
 $2 \operatorname{cosec} \theta$  ද.පැ

8.  $\frac{\tan \theta + \sec \theta - 1}{\tan \theta - \sec \theta + 1} = \frac{1 + \sin \theta}{\cos \theta}$

ච.පැ  $\frac{\tan \theta + \sec \theta - 1}{\tan \theta - \sec \theta + 1}$   
 $\frac{\tan \theta + \sec \theta - (\sec^2 \theta - \tan^2 \theta)}{\tan \theta - \sec \theta + 1}$   
 $\frac{(\tan \theta + \sec \theta) - (\sec \theta - \tan \theta)(\sec \theta + \tan \theta)}{\tan \theta - \sec \theta + 1}$   
 $\frac{(\sec \theta + \tan \theta)[1 - \sec \theta + \tan \theta]}{[1 - \sec \theta + \tan \theta]}$   
 $\sec \theta + \tan \theta$   
 $\frac{1}{\cos \theta} + \frac{\sin \theta}{\cos \theta}$   
 $\frac{1 + \sin \theta}{\cos \theta} =$  ද.පැ

8 සටහන්

9.  $\sqrt{\frac{1+\sin \theta}{1-\sin \theta}} = \sec \theta + \tan \theta$  බව පෙන්වන්න.

ච.පැ  $\sqrt{\frac{1+\sin \theta}{1-\sin \theta}}$

$$\sqrt{\frac{(1+\sin \theta)(1+\sin \theta)}{(1-\sin \theta)(1+\sin \theta)}}$$

$$\sqrt{\frac{(1+\sin \theta)^2}{1-\sin^2 \theta}}$$

$$\sqrt{\frac{(1+\sin \theta)^2}{\cos^2 \theta}}$$

$$\frac{1+\sin \theta}{\cos \theta}$$

$$\frac{1}{\cos \theta} + \frac{\sin \theta}{\cos \theta}$$

$\sec \theta + \tan \theta =$  ද.පැ

10.  $(1 + \tan \alpha \cdot \tan \beta)^2 + (\tan \alpha - \tan \beta)^2 = \sec^2 \alpha \cdot \sec^2 \beta$

බව සාධනය කරන්න.

ච.පැ  $(1 + \tan \alpha \cdot \tan \beta)^2 + (\tan \alpha - \tan \beta)^2$

$$1 + 2 \tan \alpha \cdot \tan \beta + \tan^2 \alpha \cdot \tan^2 \beta + \tan^2 \alpha - 2 \tan \alpha \cdot \tan \beta + \tan^2 \beta$$

$$(1 + \tan^2 \alpha) + \tan^2 \beta (1 + \tan^2 \alpha)$$

$$(1 + \tan^2 \alpha)(1 + \tan^2 \beta)$$

$\sec^2 \alpha \cdot \sec^2 \beta =$  ද.පැ

8 සටහන්

11.  $0 < \theta < \frac{\pi}{2}$  නම්  $\operatorname{cosec} \theta + \cot \theta = a$  නම්  $\operatorname{cosec} \theta, \tan \theta$  සහ  $\cos \theta$  හි අගය

සොයන්න

$$\operatorname{cosec} \theta + \cot \theta = a \quad \text{--- (1)}$$

$$\operatorname{cosec}^2 \theta - \cot^2 \theta = 1$$

$$(\operatorname{cosec} \theta - \cot \theta)(\operatorname{cosec} \theta + \cot \theta) = 1$$

$$\operatorname{cosec} \theta - \cot \theta = \frac{1}{(\operatorname{cosec} \theta + \cot \theta)}$$

$$\operatorname{cosec} \theta - \cot \theta = \frac{1}{a} \quad \text{--- (2)}$$

(1) + (2)

$$2 \operatorname{cosec} \theta = a + \frac{1}{a}$$

$$\frac{2}{\sin \theta} = \frac{a^2 + 1}{a}$$

$$\sin \theta = \frac{2a}{(a^2 + 1)}$$

(1) - (2)

$$2 \cot \theta = a - \frac{1}{a}$$

$$\frac{2}{\tan \theta} = \frac{a^2 - 1}{a}$$

$$\tan \theta = \frac{2a}{(a^2 - 1)}$$

$$\cos \theta = \cot \theta \cdot \sin \theta$$

$$= \frac{(a^2 - 1)}{2a} \times \frac{2a}{(a^2 + 1)}$$

$$\cos \theta = \frac{a^2 - 1}{a^2 + 1}$$

12.  $0 < \theta < \frac{\pi}{2}$ ,  $m > n > k$  ද  $m \cos \theta - n \sin \theta = k$  නම්,

$m \sin \theta + n \cos \theta = \sqrt{m^2 + n^2 - k^2}$  බව සාධනය කරන්න.

$$m \cos \theta - n \sin \theta = k \text{ නම්}$$

දෙපසම වර්ග කිරීමෙන්

$$(m \cos \theta - n \sin \theta)^2 = k^2$$

$$m^2 \cos^2 \theta + n^2 \sin^2 \theta - 2mn \cos \theta \cdot \sin \theta = k^2$$

$$m^2(1 - \sin^2 \theta) + n^2(1 - \cos^2 \theta) - 2mn \cos \theta \cdot \sin \theta = k^2$$

$$m^2 - m^2 \sin^2 \theta + n^2 - n^2 \cos^2 \theta - 2mn \cos \theta \cdot \sin \theta = k^2$$

$$(m \sin \theta)^2 + 2mn \sin \theta \cdot \cos \theta + (n \cos \theta)^2 = m^2 + n^2 - k^2$$

$$(m \sin \theta + n \cos \theta)^2 = m^2 + n^2 - k^2$$

$$m \sin \theta + n \cos \theta = \sqrt{m^2 + n^2 - k^2}$$

13.  $0 < \theta < \frac{\pi}{2}$  දී  $a, b > 0$   $\operatorname{cosec} \theta - \sin \theta = a^3$  දී

$\sec \theta - \cos \theta = b^3$  දී නම්  $a^2 b^2 (a^2 + b^2) = 1$  බව පෙන්වන්න.

$$\operatorname{cosec} \theta - \sin \theta = a^3$$

$$\frac{1}{\sin \theta} - \sin \theta = a^3$$

$$\frac{(1 - \sin^2 \theta)}{\sin \theta} = a^3$$

$$\cos^2 \theta = a^3 \sin \theta \quad \text{1} \text{---} \text{○}$$

$$\sec \theta - \cos \theta = b^3$$

$$\frac{1}{\cos \theta} - \cos \theta = b^3$$

$$\frac{1 - \cos^2 \theta}{\cos \theta} = b^3$$

$$\sin^2 \theta = b^3 \cos \theta \quad \text{2} \text{---} \text{○}$$

2

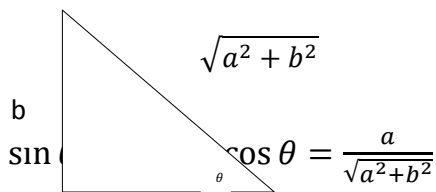
1

$$\tan^2 \theta = \frac{b^3}{a^3} \cot \theta$$

$$\tan^3 \theta = \frac{b^3}{a^3} \tan \theta \Rightarrow \frac{b}{a}$$

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$\theta$  සුළු කෝණයක් නිසා



a



8 සටහන්

①න්

$$\cos^2 \theta = a^3 \sin \theta$$

$$\frac{a^2}{(a^2+b^2)} = a^3 \times \frac{b}{\sqrt{a^2+b^2}}$$

$$\sqrt{a^2 + b^2} = ab(a^2 + b^2)$$

$$(a^2 + b^2) = a^2 b^2 (a^2 + b^2)^2$$

$$\therefore a^2 b^2 (a^2 + b^2) = 1$$

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