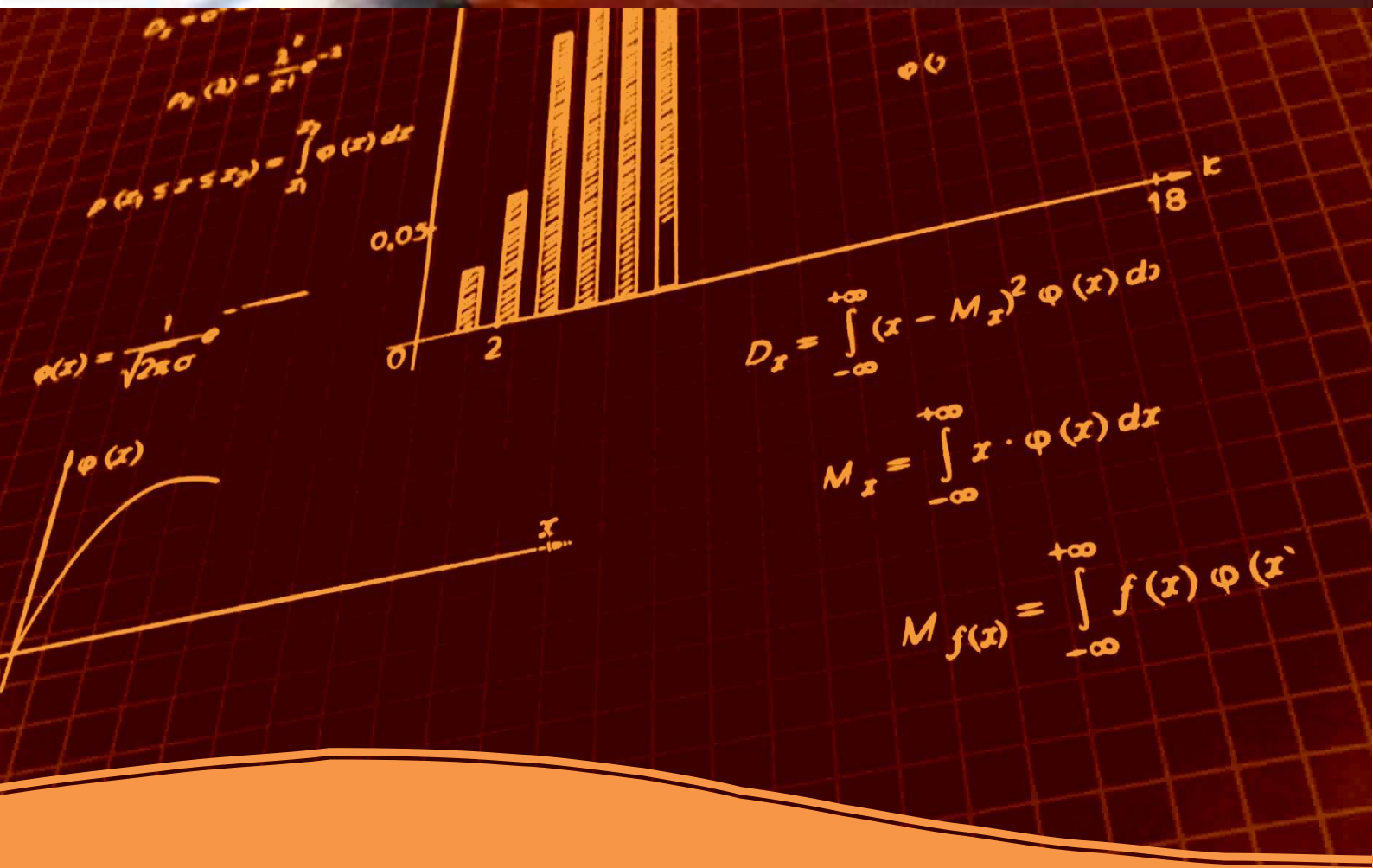
අ.පො.ස. උසස්පෙළ

සංයුක්ත ගණිතය



ඒකකය 10

10.1

01. $\log_{16} 256 - \log_{49} 7$ අගය සොයන්න.

$\log_{16} 256 - \log_{49} 7$

$\log_{16} 16^2 - \log_{49} 49^{1/2}$

$2 \log_{16} 16 - \frac{1}{2}\log_{49} 49$

$2 \times 1 - \frac{1}{2} \times 1$

$2 - \frac{1}{2}$

$\frac{3}{2}$

02. $\dfrac{\log_4 \sqrt{256\sqrt{64\sqrt[4]{4}}}}{\left(\log_2 \sqrt[4]{32} + \log_e e^5\right)}$ අගය සොයන්න.

$$\frac{\log_4 \left[256\left(64 \times 4^{\frac{1}{4}}\right)^{1/2}\right]^{1/2}}{\left(\log_2 32^{\frac{1}{4}} + \log_e e^5\right)}$$

$$\frac{\log_4 \left[4^4\left(4^3 \times 4^{1/4}\right)^{1/2}\right]^{1/2}}{\log_2 2^5 \times + \ 5\log_e e}$$

$$\frac{\log_4 \left[4^{4\times 1/2} \times 4^{3\times 1/4} \times 4^{\frac{1}{4}\times\frac{1}{4}}\right]}{\log_2 2^{5/4} + 5 \times 1}$$

$$\frac{\log_4 \left[4^2 \times 4^{3/4} \times 4^{1/16}\right]}{\frac{5}{4}\log_2 2 + 5 \times 1}$$

$$\frac{\log_4 4^{\left(2+\frac{3}{4}+\frac{1}{16}\right)}}{\frac{5}{4} \times 1 + 5}$$

$$\frac{\log_4 4^{45/16}}{\frac{25}{4}}$$

$$\frac{\frac{45}{16} \times \log_4 4}{\frac{25}{4}}$$

$$\frac{\cancel{45}^{\ 9}}{\cancel{16}^{\ 4}} \times \frac{\cancel{4}^{\ 1}}{\cancel{25}^{\ 5}}$$

$$\frac{9}{20}$$

03. $\log_a\left(\frac{65}{19}\right) + \log_a\left(\frac{133}{143}\right) + \log_a\left(\frac{11}{35}\right)$ අගයන්න.

$$\log_a\left(\frac{\cancel{65}^{\cancel{13}^{1}}}{\cancel{19}^{1}} \times \frac{\cancel{133}^{\cancel{7}^{1}}}{\cancel{143}^{\cancel{11}^{1}}} \times \frac{\cancel{11}^{1}}{\cancel{35}^{\cancel{5}^{1}}}\right)$$

$$\log_a(1)$$

$$0$$

04. $\log_a\left(\frac{162}{108}\right) + 2\log_a\left(\frac{36}{27}\right) - 3\log_a\left(\frac{6}{3}\right)$ අගය සොයන්න.

$$\log_a\left(\frac{162}{108}\right) + 2\log_a\left(\frac{36}{27}\right) - 3\log_a\left(\frac{6}{3}\right)$$

$$\log_a\left(\frac{162}{108}\right) + \log_a\left(\frac{36^2}{27^2}\right) - \log_a\left(\frac{6^3}{3^3}\right)$$

$$\log_a\left(\frac{162}{108} \times \frac{36^2}{27^2} \times \frac{3^3}{6^3}\right)$$

$$\log_a\left(\frac{\cancel{162}^{18}}{108} \times \frac{\cancel{36}^{6} \times \cancel{36}}{\cancel{27} \times \cancel{27}^{9}} \times \frac{\cancel{27}}{\cancel{216}^{6}}\right)$$

$$\log_a\left(\frac{108}{108}\right)$$

$$\log_a(1)$$

$$0$$

05. $2\log_a\left(\frac{15}{8}\right) - \log_a\left(\frac{25}{162}\right) + 3\log_a\left(\frac{4}{9}\right) = \log_a(2)$ බව පෙන්වන්න.

ව.පැ. $2\log_a\left(\frac{15}{8}\right) - \log_a\left(\frac{25}{162}\right) + 3\log_a\left(\frac{4}{9}\right)$

$\log_a\left(\frac{15^2}{8^2}\right) - \log_a\left(\frac{25}{162}\right) + \log_a\left(\frac{4^3}{9^3}\right)$

$\log_a\left(\frac{15^2}{8^2} \times \frac{162}{25} \times \frac{43}{93}\right)$

$\log_a\left(\frac{\cancel{225}^{\,9\,1}}{\cancel{64}} \times \frac{\cancel{162}^{\,2}}{\cancel{25}} \times \frac{\cancel{64}}{\cancel{729}_{81\,1}}\right)$

$\log_a(2)$ ද.පැ.

06. $7^{2x} - 4\,(7^x) + 3 = 0$ විසඳන්න

$7^{2x} - 4\,(7^x) + 3 = 0$

$(7^x)^2 - 4\,(7^x) + 3 = 0$

$y = 7^x$ නම්

$y^2 - 4y + 3 = 0$

මෙය y වල වර්ගසමීකරණයකි. සාධක වලට වෙන්කිරීමෙන්

$(y-3)(y-1) = 0$

$y = 3$ හෝ $y = 1$

එනම් $7^x = 3$ හෝ $7^x = 1$

$x\ln|3| = \ln|3|$ හෝ $7^x = 7^0$

$x = \frac{\ln|3|}{\ln|7|}$ $\qquad$ $x = 0$

07. $x > 0$ නම් $\log_a(x+1) + \log_a(x-1) = 4\log_a 2 + \log_a 5$ විසඳන්න.

$$\log_a(x+1) + \log_a(x-1) = 4\log_a 2 + \log_a 5$$

$\log_a[(x+1)(x-1)] = \log_a 2^4 + \log_a 5$

$\log_a[x^2-1] = \log_a[16\times 5]$

$x^2 - 1 = 80$

$x^2 = 81$

$x = 9$

x ධනඅගයක්නිසා $x = 9$

08. $3\log_2 x - \log_x 2 = 2$ විසඳන්න.

$3\log_2 x - \log_x 2 = 2$

$3\log_2 x - \dfrac{1}{\log_2 x} = 2$

$3(\log_2 x)^2 - 1 = 2\log_2 x$

$3(\log_2 x)^2 - 2\log_2 x - 1 = 0$

සාධක වලට වෙන්කිරීමෙන්

$(3\log_2 x + 1)(\log_2 x - 1) = 0$

$3\log_2 x + 1 = 0$ හෝ $\log_2 x - 1 = 0$

$\log_2 x = -\dfrac{1}{3}$ හෝ $\log_2 x = 1$

$x = 2^{-\frac{1}{3}}$ හෝ $x = 2^1$

$x = \dfrac{1}{\sqrt[3]{2}}$ හෝ $x = 2$

9. $\log_b x = (2-a) + \log_b\left[\frac{a^2b^a}{b^2}\right]$ විසඳන්න.

$$\log_b x - \log_b\left[\frac{a^2b^2}{b^2}\right] = (2-a)$$

$$\log_b\left[\frac{xb^2}{a^2b^2}\right] = (2-a)$$

$$\frac{xb^2}{a^2b^2} = b^{(2-a)}$$

$$\frac{xb^2}{a^2b^a} = \frac{b^2}{b^a}$$

$$x = a^2$$

10. $x > 0$ නම්

$2\log_b x = 2\log_b(1-a) + 2\log_b(1+a) - \log_b\left(\frac{1}{a} - a\right)^2$ විසඳන්න.

$$2\log_b x = 2\log_b(1-a) + 2\log_b(1+a) - \log_b\left(\frac{1}{a} - a\right)^2$$

$$\log_b x^2 = \log_b(1-a)^2 + \log_b(1+a)^2 - \log_b\left(\frac{1-a^2}{a}\right)^2$$

$$\log_b x^2 = \log_b\left[\frac{(1-a)^2(1+a`)^2}{\frac{(1-a^2)^2}{a^2}}\right]$$

$$\log_b x^2 = \log_b\left[\frac{(1-a^2)^2 \times a^2}{(1-a^2)^2}\right]$$

$$x^2 = a^2$$

$$x = a$$

11. $\log_{\sqrt{3}} x + \log_{\sqrt[3]{3}} x + \log_{\sqrt[4]{3}} x = 9$ **නම් x හි අගය සොයන්න.**

$$\log_{\sqrt{3}} x + \log_{\sqrt[3]{3}} x + \log_{\sqrt[4]{3}} x = 9$$

$$\frac{\log_a x}{\log_a \sqrt{3}} + \frac{\log_a x}{\log_a \sqrt[3]{3}} + \frac{\log_a x}{\log_a \sqrt[4]{3}} = 9$$

$$\frac{\log_a x}{\log_a 3^{1/2}} + \frac{\log_a x}{\log_a 3^{1/3}} + \frac{\log_a x}{\log_a 3^{1/4}} = 9$$

$$\frac{\log_a x}{\frac{1}{2}\log_a 3} + \frac{\log_a x}{\frac{1}{3}\log_a 3} + \frac{\log_a x}{\frac{1}{4}\log_a 3} = 9$$

$$\frac{\log_a x}{\log_a 3}(2 + 3 + 4) = 9$$

$$\frac{\log_a x}{\log_a 3} \times 9 = 9$$

$$\log_a x = \log_a 3$$

$$x = 3$$

12. $\log_b a^2 \times \log_c b^3 \times \log_a c^4 = 24$ **බව පෙන්වන්න.**

ව.පැ. $\log_b a^2 \times \log_c b^3 \times \log_a c^4$

$$2\log_b a \times 3\log_c b \times 4\log_a c$$

$$24 \times \frac{\cancel{\log_x a}}{\cancel{\log_x b}} \times \frac{\cancel{\log_x b}}{\cancel{\log_x c}} \times \frac{\cancel{\log_x c}}{\cancel{\log_x a}}$$

24 ද.පැ.

13. $\mathbf{a}, \mathbf{b}, \mathbf{c}$ යනු අසමාන තාත්වික ධන සංඛ්‍යා නම් සහ $\mathbf{a} > b > c$ නම්

$\mathbf{2\log_m(a-c)}, \mathbf{\log_m(a^2-c^2)}, \mathbf{\log_m(a^2+2b^2+c^2)}$

පද සමාන්තර ශ්‍රේඪියක අනුයාත පදනම්,

$\boldsymbol{a, b, c}$ ගුණෝත්තරශ්‍රේඪියක අනුයාත පද බව සාධනයකරන්න.

$\mathbf{2\log_m(a-c)}, \mathbf{\log_m(a^2-c^2)}, \mathbf{\log_m(a^2+2b^2+c^2)}$

පදසමාන්තරශ්‍රේඪියකඅනුයාතපදනිසා,

$$\log_m(a^2-c^2) - 2\log_m(a-c) = \log_m(a^2+2b^2+c^2) - \log_m(a^2-c^2)$$

$$2\log_m(a^2-c^2) = 2\log_m(a-c) + \log_m(a^2+2b^2+c^2)$$

$$2\log_m(a^2-c^2) - 2\log_m(a-c) = \log_m(a^2+2b^2+c^2)$$

$$2\log_m\left[\frac{a^2-c^2}{(a-c)}\right] = \log_m(a^2+2b^2+c^2)$$

$$2\log_m\left[\frac{(a-c)(a+c)}{(a-c)}\right] = \log_m(a^2+2b^2+c^2)$$

$$\log_m[(a+c)^2] = \log_m(a^2+2b^2+c^2)$$

$$(a+c)^2 = a^2+2b^2+c^2$$

$$a^2+c^2+2ac = a^2+2b^2+c^2$$

$$ac = b^2$$

$$\frac{a}{b} = \frac{b}{c}$$

$\therefore \boldsymbol{a, b, c}$ ගුණෝත්තර ශ්‍රේඪියක අනුයාතපදවේ.

14. $\boldsymbol{x = \log_{2a} a}, \boldsymbol{y = \log_{3a} 2a}$ සහ $\boldsymbol{z = \log_{4a} 3a}$ නම්

$\boldsymbol{xyz + 1 = 2yz}$ බවපෙන්වන්න.

$x = \log_{2a} a$

$$x = \frac{\log_b a}{\log_b 2a}$$

මෙලෙසම $y = \frac{\log_b 2a}{\log_b 3a}$

$$z = \frac{\log_b 3a}{\log_b 4a}$$

$$xyz + 1 = \frac{\log_b a}{\cancel{\log_b 2a}} \times \frac{\cancel{\log_b 2a}}{\cancel{\log_b 3a}} \times \frac{\cancel{\log_b 3a}}{\log_b 4a} + 1$$

$$xyz + 1 = \frac{\log_b a}{\log_b 4a} + 1$$

$$xyz + 1 = \frac{\log_b a + \log_b 4a}{\log_b 4a}$$

$$xyz + 1 = \frac{\log_b 4a^2}{\log_b 4a}$$

$$xyz + 1 = \frac{\log_b (2a)^2}{\log_b (4a)}$$

$$xyz + 1 = \frac{2\log_b 2a}{\log_b 4a}$$

$$xyz + 1 = 2 \times \frac{\log_b 2a}{\log_b 3a} \times \frac{\log_b 3a}{\log_b 4a}$$

$$xyz + 1 = zyz$$

$$\therefore xyz + 1 = zyz$$