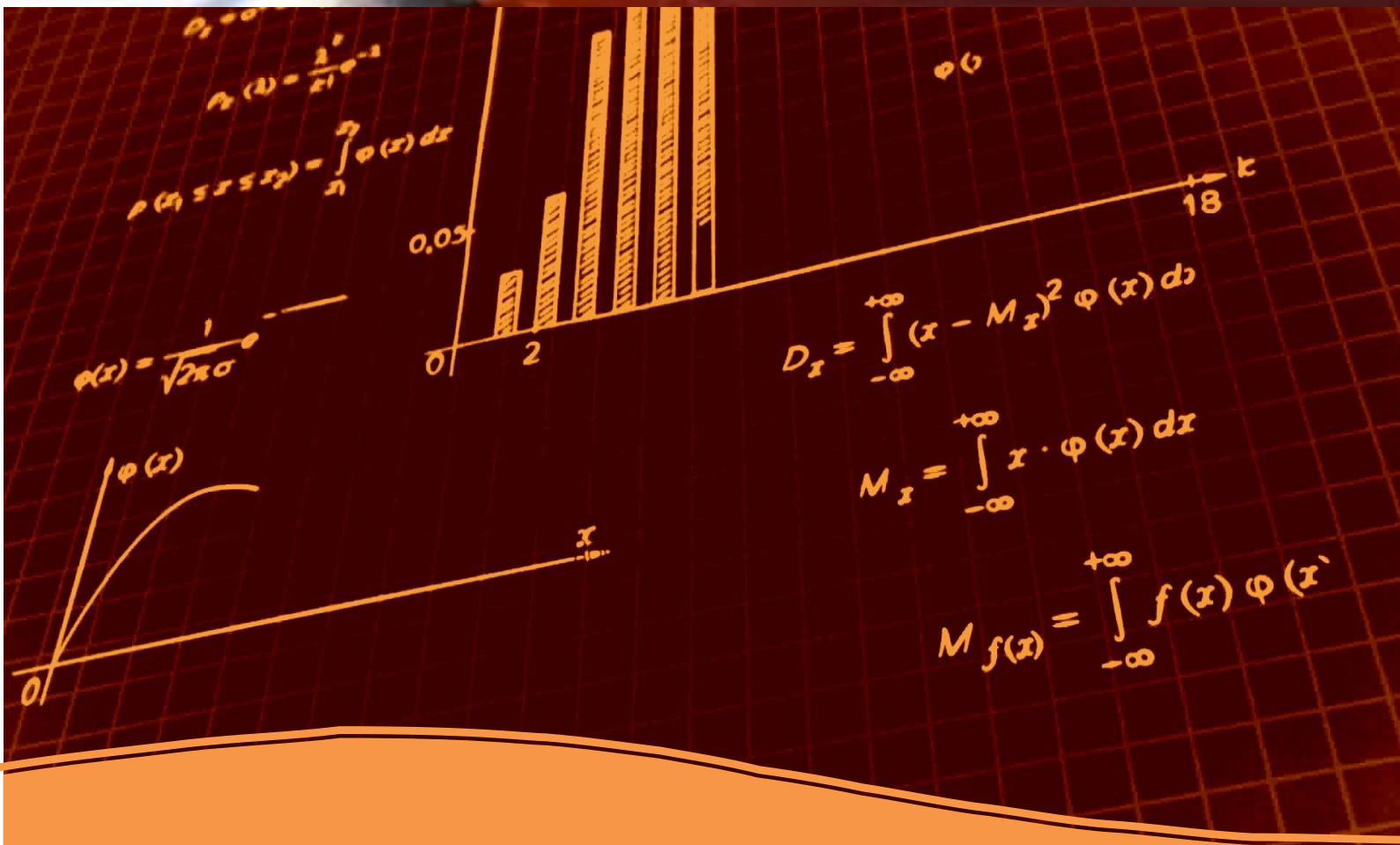


# අ.පො.ස. උසස්පෙළ සංයුක්ත ගණිතය



## ඒකකය 8

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01. (I).  $\sin(105^\circ) + \cos(105^\circ) = \frac{1}{\sqrt{2}}$  බව පෙන්වන්න.

ච.පැ.  $\sin(105^\circ) + \cos(105^\circ)$

$$\sin(105^\circ) + \cos(90^\circ + 15^\circ)$$

$$\sin(105^\circ) - \sin(15^\circ)$$

$$2 \cos\left(\frac{105^\circ + 15^\circ}{2}\right) \sin\left(\frac{105^\circ - 15^\circ}{2}\right)$$

$$2\cos(60^\circ) \sin(45^\circ)$$

$$2 \times \frac{1}{2} \times \frac{1}{\sqrt{2}}$$

$$\frac{1}{\sqrt{2}} //$$

(II).  $\cos(10^\circ) + \cos(130^\circ) + \cos(110^\circ) = 0$  බව පෙන්වන්න.

ච.පැ.  $\cos(10^\circ) + \cos(130^\circ) + \cos(110^\circ)$

$$\cos(10^\circ) + 2 \cos\left(\frac{130^\circ + 110^\circ}{2}\right) \cos\left(\frac{130^\circ - 110^\circ}{2}\right)$$

$$\cos(10^\circ) + 2\cos(120^\circ) \cos(10^\circ)$$

$$\cos(10^\circ) + 2\cos(90^\circ + 30^\circ) \cos(10^\circ)$$

$$\cos(10^\circ) + 2(-\sin 30^\circ) \cos(10^\circ)$$

$$\cos(10^\circ) - 2 \times \frac{1}{2} \times \cos(10^\circ)$$

$$\cos(10^\circ) - \cos(10^\circ) = 0 //$$
 ද.පැ.

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$$02. \frac{\sin(7\theta) + \sin(8\theta) + \sin(10\theta) + \sin(11\theta)}{\cos(7\theta) + \cos(8\theta) + \cos(10\theta) + \cos(11\theta)} = \text{Tan}(9\theta)$$

බව සාධනය කර පෙන්වන්න.

$$\begin{aligned} \text{ච.පැ.} \quad & \frac{(\sin(7\theta)+\sin(11\theta))+(\sin(8\theta)+\sin(10\theta))}{(\cos(7\theta)+\cos(11\theta))+\cos(8\theta)+\cos(10\theta))} \\ & \frac{(\sin(7\theta) + \sin(11\theta)) + (\sin(8\theta) + \sin(10\theta))}{(\cos(7\theta) + \cos(11\theta) + \cos(8\theta) + \cos(10\theta))} \\ & \frac{\cancel{\sin\left(\frac{7\theta+11\theta}{2}\right)} \cos\left(\frac{7\theta-11\theta}{2}\right) + \cancel{\sin\left(\frac{8\theta+10\theta}{2}\right)} \cos\left(\frac{8\theta-10\theta}{2}\right)}{\cancel{\cos\left(\frac{7\theta+11\theta}{2}\right)} \cos\left(\frac{7\theta-10\theta}{2}\right) + \cancel{\cos\left(\frac{8\theta+10\theta}{2}\right)} \cos\left(\frac{8\theta-10\theta}{2}\right)} \\ & \frac{\sin(9\theta) \cos(-2\theta) + \sin(9\theta) \cos(-\theta)}{\cos(9\theta) \cos(-2\theta) + \cos(9\theta) \cos(\theta)} \\ & \frac{\sin(9\theta)[\cancel{\cos 2\theta} + \cos \theta]}{\cos(9\theta)[\cancel{\cos 2\theta} + \cos \theta]} \\ & \tan(9\theta) = \text{ඳ.පැ.} // \end{aligned}$$

$$03. \frac{\sin(11\theta) \sin(\theta) + \sin(7\theta) \sin(3\theta)}{\cos(11\theta) \sin(\theta) + \cos(7\theta) \sin(3\theta)} = \tan(8\theta)$$

බව සාධනය කරන්න.

$$\begin{aligned} \text{ච.පැ.} \quad & \frac{\sin(11\theta) \sin(\theta) + \sin(7\theta) \sin(3\theta)}{\cos(11\theta) \sin(\theta) + \cos(7\theta) \sin(3\theta)} \\ & \frac{2\sin(11\theta) \sin(\theta) + 2\sin(7\theta) \sin(3\theta)}{2 \cos(11\theta) \sin(\theta) + 2\cos(7\theta) \sin(3\theta)} \\ & \frac{\{\cos(11\theta - \theta) - \cos(11\theta + \theta)\} + \{\cos(7\theta - 3\theta) - \cos(7\theta + 3\theta)\}}{\{\sin(11\theta + \theta) - \sin(11\theta - \theta)\} + \{\sin 7\theta + 3\theta - \sin(7\theta - 3\theta)\}} \\ & \frac{\cancel{\cos(10\theta)} - \cancel{\cos(12\theta)} + \cancel{\cos(4\theta)} - \cancel{\cos(10\theta)}}{\cancel{\sin(12\theta)} - \cancel{\sin(10\theta)} + \cancel{\sin(10\theta)} - \cancel{\sin(4\theta)}} \\ & \frac{\cos(4\theta) - \cos(12\theta)}{\sin(12\theta) - \sin(4\theta)} \end{aligned}$$

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$$\frac{\cancel{2} \sin\left(\frac{4\theta+12\theta}{2}\right) \sin\left(\frac{12\theta-4\theta}{2}\right)}{\cancel{2} \cos\left(\frac{12\theta+4\theta}{2}\right) \sin\left(\frac{12\theta-4\theta}{2}\right)}$$

$$\frac{\sin(8\theta) \cancel{\sin(4\theta)}}{\cos(8\theta) \cancel{\sin(4\theta)}}$$

$$\tan(8\theta) = \text{ද. පැ.} //$$

04.  $\cos A + \cos B + \cos C + \cos(A + B + C) = 4 \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{B+C}{2}\right) \cos\left(\frac{C+A}{2}\right)$

බව පෙන්වන්න.

ච.පැ.  $\cos A + \cos B + \cos C + \cos(A + B + C)$

$$(\cos A + \cos B) + (\cos A + B + C + \cos C)$$

$$2 \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right) + 2 \cos\left(\frac{A+B+2C}{2}\right) \cos\left(\frac{A+B+C-C}{2}\right)$$

$$2 \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right) + 2 \cos\left(\frac{A+B+2C}{2}\right) \cos\left(\frac{A+B}{2}\right)$$

$$2 \cos\left(\frac{A+B}{2}\right) \left[ \cos\left(\frac{A-B}{2}\right) + \cos\left(\frac{A+B+2C}{2}\right) \right]$$

$$2 \cos\left(\frac{A+B}{2}\right) \left[ 2 \cos\left(\frac{A+B+2C+A-B}{4}\right) \right] \cos\left(\frac{A+B+2C+A+B}{2}\right)$$

$$4 \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{2A+2C}{4}\right) \cos\left(\frac{2B+2C}{4}\right)$$

$$4 \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{B+C}{2}\right) \cos\left(\frac{A+C}{2}\right) = \text{ද. පැ.} //$$

$$05. \sin(110^\circ) \sin(130^\circ) \sin(150^\circ) \sin(170^\circ) = \frac{1}{16} \text{ බව සාධනය කරන්න.}$$

$$\text{ච.පෑ.} \sin(110^\circ) \sin(130^\circ) \sin(150^\circ) \sin(170^\circ) = \frac{1}{16}$$

$$\sin(180^\circ - 70^\circ) \sin(180^\circ - 50^\circ) \sin(180^\circ - 30^\circ) \sin(180^\circ - 10^\circ)$$

$$\sin(70^\circ) \sin(50^\circ) \sin(30^\circ) \sin(10^\circ)$$

$$\sin(30^\circ) \frac{1}{2} [2 \sin(50^\circ) \sin(10^\circ)] \sin(70^\circ)$$

$$\frac{1}{2} \cdot \frac{1}{2} [\cos(50^\circ - 10^\circ) - \cos(50^\circ + 10^\circ)] \sin(70^\circ)$$

$$\frac{1}{4} [\cos(40^\circ) - \cos(60^\circ)] \sin(70^\circ)$$

$$\frac{1}{4} \cos(40^\circ) \sin(70^\circ) - \frac{1}{4} \cos(60^\circ) \sin(70^\circ)$$

$$\frac{1}{8} [2 \cos(40^\circ) \sin(70^\circ)] - \frac{1}{4} \cdot \frac{1}{2} \sin(70^\circ) = \frac{1}{16}$$

$$\frac{1}{8} [\sin(40^\circ + 70^\circ) - \sin(40^\circ - 70^\circ)] - \frac{1}{8} \sin(70^\circ) = \frac{1}{16}$$

$$\frac{1}{8} \sin(110^\circ) - \frac{1}{8} \sin(-30^\circ) - \frac{1}{8} \sin(70^\circ) = \frac{1}{16}$$

$$\frac{1}{8} \sin(180^\circ - 70^\circ) + \frac{1}{8} \times \frac{1}{2} - \frac{1}{8} \sin(70^\circ) = \frac{1}{16}$$

$$\frac{1}{8} \sin(70^\circ) + \frac{1}{16} - \frac{1}{8} \sin(70^\circ) = \frac{1}{16}$$

$$\frac{1}{16} //$$

06.  $\cos(200^\circ) \cos(220^\circ) \cos(240^\circ) \cos(260^\circ) = \frac{1}{16}$  බව සාධනය කරන්න.

ච.පැ.  $\cos(200^\circ) \cos(220^\circ) \cos(240^\circ) \cos(260^\circ)$

$$\cos(180^\circ + 20^\circ) \cos(180^\circ + 40^\circ) \cos(180^\circ + 60^\circ) \cos(180^\circ + 80^\circ)$$

$$(-\cos(20^\circ)) (-\cos(40^\circ)) (-\cos(60^\circ)) (-\cos(80^\circ))$$

$$\cos(20^\circ) \cos(40^\circ) \cos(60^\circ) \cos(80^\circ)$$

$$\cos(60^\circ) \frac{1}{2} [2\cos(20^\circ) \cos(60^\circ) \cos(40^\circ)] \cos(80^\circ)$$

$$\frac{1}{2} \cdot \frac{1}{2} [\cos(20^\circ + 40^\circ) + \cos(20^\circ - 40^\circ)] \cos(80^\circ)$$

$$\frac{1}{4} [\cos(60^\circ) + \cos(-20^\circ)] \cos(80^\circ)$$

$$\frac{1}{4} \cdot \frac{1}{2} \cos(80^\circ) + \frac{1}{4} \cos(20^\circ) \cos(80^\circ)$$

$$\frac{1}{8} \cos(80^\circ) + \frac{1}{8} [2\cos(20^\circ) \cos(80^\circ)]$$

$$\frac{1}{8} \cos(80^\circ) + \frac{1}{8} [\cos(20^\circ + 80^\circ) + \cos(20^\circ - 80^\circ)]$$

$$\frac{1}{8} \cos(80^\circ) + \frac{1}{8} [\cos(100^\circ) + \cos(-60^\circ)]$$

$$\frac{1}{8} \cos(80^\circ) + \frac{1}{8} \cos(180^\circ - 80^\circ) + \frac{1}{8} \cdot \frac{1}{2}$$

$$\frac{1}{8} \cancel{\cos(80^\circ)} - \frac{1}{8} \cancel{\cos(80^\circ)} + \frac{1}{16}$$

$$\frac{1}{16} = \text{ච. පැ.} //$$

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07.  $\tan(20^\circ) \tan(40^\circ) \tan(80^\circ) = \sqrt{3}$  බව සාධනය කරන්න.

ච.පැ.  $\tan(20^\circ) \tan(40^\circ) \tan(80^\circ)$

$$\frac{\sin(20^\circ)}{\cos(20^\circ)} \cdot \frac{\sin(40^\circ)}{\cos(40^\circ)} \cdot \frac{\sin(80^\circ)}{\cos(80^\circ)}$$

$$\frac{(2\sin(20^\circ) \sin(40^\circ)) \sin(80^\circ)}{(2\cos(20^\circ) \cos(40^\circ)) \cos(80^\circ)}$$

$$\frac{[\cos(20^\circ - 40^\circ) - \cos(20^\circ + 40^\circ)] \sin(80^\circ)}{[\cos(20^\circ + 40^\circ) + \cos(20^\circ - 40^\circ)] \cos(80^\circ)}$$

$$\frac{[\cos(-20^\circ) - \cos(60^\circ)] \sin(80^\circ)}{[\cos(60^\circ) + \cos(-20^\circ)] \cos(80^\circ)}$$

$$\frac{\cos(20^\circ) \sin(80^\circ) - \frac{1}{2} \sin(80^\circ)}{\left[\frac{1}{2} \cos(80^\circ) + \cos(20^\circ)\right] - \frac{1}{2} \cos(80^\circ)}$$

$$\frac{\frac{1}{2}[2 \cos(20^\circ) \sin(80^\circ)] - \frac{1}{2} \sin(80^\circ)}{\frac{1}{2} \cos(80^\circ) + \frac{1}{2}[\cos(20^\circ) \cos(80^\circ)]}$$

$$\frac{\sin(20^\circ + 80^\circ) - \sin(20^\circ - 80^\circ) - \sin(80^\circ)}{\cos(80^\circ) + \cos(20 + 80^\circ) + \cos(20^\circ - 80^\circ)}$$

$$\frac{\sin(100^\circ) - \sin(-60^\circ) - \sin(80^\circ)}{\cos(80^\circ) + \cos(100^\circ) + \cos(-60^\circ)}$$

$$\frac{\sin(180^\circ - 80^\circ) + \sin(-60^\circ) - \sin(80^\circ)}{\cos(80^\circ) + \cos(180 - 80^\circ) + \cos(60^\circ)}$$

$$\frac{\cancel{\sin(80^\circ)} + \sin(60^\circ) - \cancel{\sin(80^\circ)}}{\cancel{\cos(80^\circ)} - \cancel{\cos(80^\circ)} + \cos(60^\circ)}$$

$$\frac{\sin(60^\circ)}{\cos(60^\circ)}$$

$$\tan(60^\circ)$$

$$\sqrt{3} = \text{ච. පැ.} //$$



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08.  $b \sin(2A) = a \sin(2B)$  නම්,  $\frac{\tan(A+B)}{\tan(A-B)} = \frac{a+b}{a-b}$  බව සාධනය කරන්න.

ච.පැ.  $b \sin(2A) = a \sin(2B)$

$$\frac{\sin(2A)}{\sin(2B)} = \frac{a}{b}$$

$$\frac{\sin(2A) + \sin(2B)}{\sin(2A) - \sin(2B)} = \frac{a+b}{a-b}$$

$$\frac{2 \sin\left(\frac{2A+2B}{2}\right) \cos\left(\frac{2A-2B}{2}\right)}{2 \cos\left(\frac{2A+2B}{2}\right) \sin\left(\frac{2A-2B}{2}\right)} = \frac{a+b}{a-b}$$

$$\frac{\sin(A+B) \cos(A-B)}{\cos(A+B) \sin(A-B)} = \frac{a+b}{a-b}$$

$$\tan(A+B) \cot(A-B) = \frac{a+b}{a-b}$$

$$\frac{\tan(A+B)}{\tan(A-B)} = \frac{a+b}{a-b} //$$