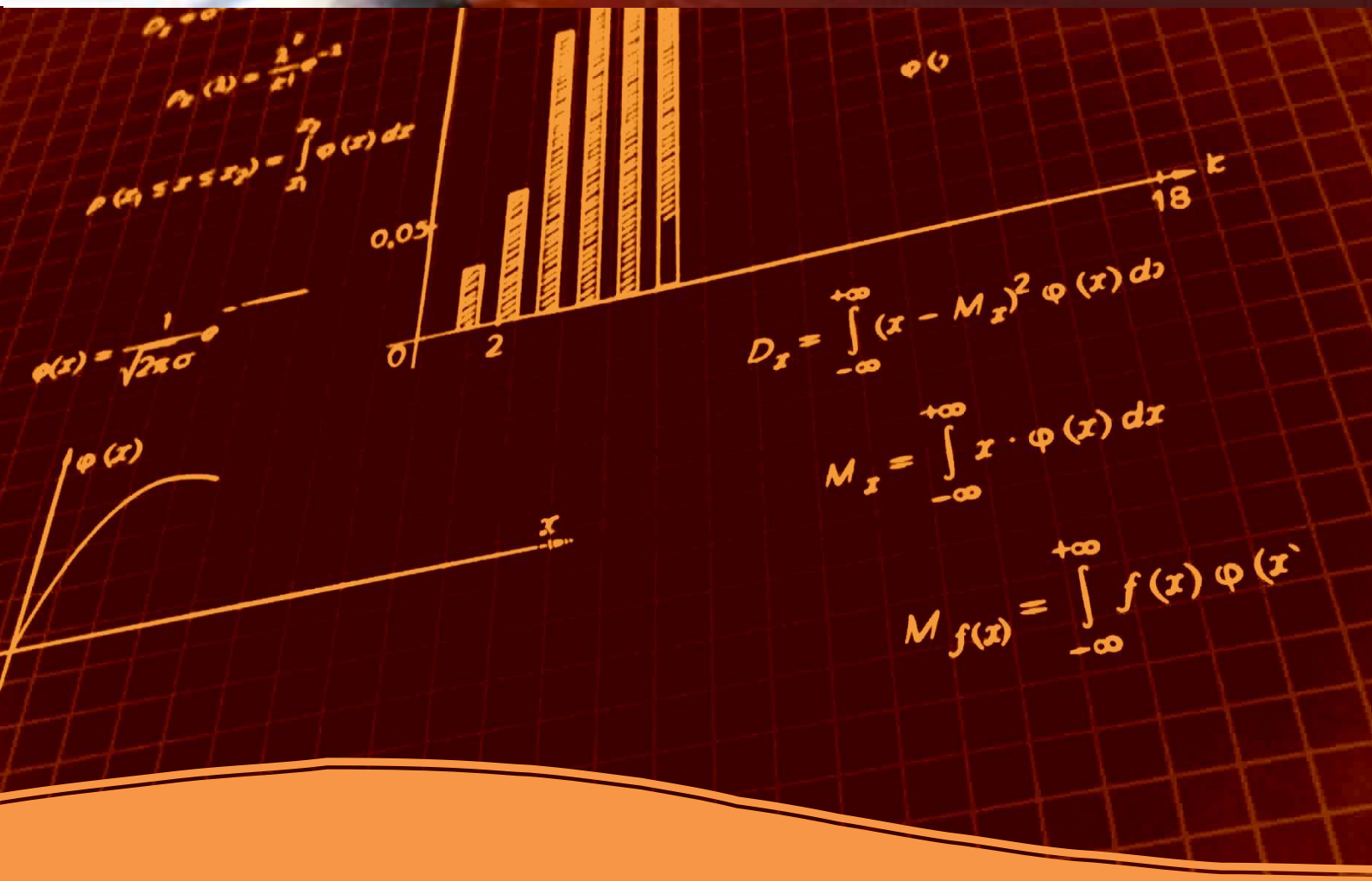
අ.පො.ස. උසස්පෙළ

සංයුක්ත ගණිතය



ඒකකය 8

1. ආකලන ව්‍යකලන සූත්‍ර භාවිතයෙන් අගයන්න.

(I) Sin(75°) (II) Cos(105°) (III) Sin(15°)

(I) Sin(75°)

Sin (A+B) = Sin A . Cos B + Cos A . Sin B

A= 30° B = 45°නම්,

Sin (30° + 45°) = Sin(30°) . Cos(45°) + Cos(30°) . Sin(45°)

$$= \frac{1}{2} x \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} x \frac{1}{\sqrt{2}}$$

$$= \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}$$

$$\text{Sin}(75°) = \frac{1+\sqrt{3}}{2\sqrt{2}}$$

$$\text{Sin}(75°) = \frac{\sqrt{2}+\sqrt{6}}{4}$$

(II) Cos(105°)

Cos(A+B) = CosA .Cos B - SinA . Sin B

A= 60° B = 45°නම්,

Cos (60° + 45°) = Cos(60°) . Cos(45°) - Sin(60°) . Sin(45°)

$$= \frac{1}{2} x \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} x \frac{1}{\sqrt{2}}$$

$$= \frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}}$$

$$\text{Cos}(105°) = \frac{1-\sqrt{3}}{2\sqrt{2}}$$

$$\text{Sin}(75°) = \frac{\sqrt{2}-\sqrt{6}}{4}$$

(III) Sin(15°)

Sin(A-B) = Sin A . Cos B – Cos A . Sin B

A = 45° B = 30°

Sin(45° - 30°) = Sin(45°) . Cos(30°) – Cos(45°) . Sin(30°)

$$= \frac{1}{\sqrt{2}} x \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} x \frac{1}{2}$$

$$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$$

$$\text{Sin}(15°) = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

$$\text{Sin}(15°) = \frac{\sqrt{6}-\sqrt{2}}{4}$$

2. අගය සොයන්න.

I. Sin(12°) . Cos(48°) + Cos(12°) . Sin(48°)
II. Sin(88°) . Cos(58°) - Cos(88°) . Sin(58°)
III. Cos(54°) . Cos(90°) + Sin(54°) . Sin(90°)
IV. Cos(50°) . Cos(40°) – Sin(50°) . Sin(40°)

පිළිතුරු

I. $\text{Sin}(12°) \cdot \text{Cos}(48) + \text{Cos}(12°) \cdot \text{Sin}(48°)$

$\text{Sin A} \cdot \text{Cos B} + \text{Cos A} \cdot \text{Sin B} = \text{Sin}(A+B)$

$$\begin{aligned}\text{Sin}(12°) \cdot \text{Cos}(48°) + \text{Cos}(12°) \cdot \text{Sin}(48°) &= \text{Sin}(12°+ 48°)\\ &= \text{Sin}(60°)\\ &= \frac{\sqrt{3}}{2}\end{aligned}$$

II. $\text{Sin}(A) \cdot \text{Cos}(B) - \text{Cos}(A) \cdot \text{Sin}(B) = \text{Sin}(A+B)$

$$\begin{aligned}\text{Sin}(88°) \cdot \text{Cos}(58°) - \text{Cos}(88°) \cdot \text{Sin}(58°) &= \text{Sin}(88°-58°)\\ &= \text{Sin}(30°)\\ &= \frac{1}{2}\end{aligned}$$

III. $\text{Cos}(A) \cdot \text{Cos}(B) + \text{Sin}(A) \cdot \text{Sin}(B) = \text{Cos}(A-B)$

$$\begin{aligned}\text{Cos}(54°) \cdot \text{Cos}(90°) + \text{Sin}(54°) \cdot \text{Sin}(90°) &= \text{Cos}(54°-90°)\\ &= \text{Cos}(45°)\\ &= \frac{1}{\sqrt{2}}\end{aligned}$$

IV. $\text{Cos}(A) \cdot \text{Cos}(B) - \text{Sin}(A) \cdot \text{Sin}(B) = \text{Cos}(A+B)$

$$\begin{aligned}\text{Cos}(50°) \cdot \text{Cos}(40°) - \text{Sin}(50°) \cdot \text{Sin}(40°) &= \text{Cos}(50° + 40°)\\ &= \text{Cos}(90°)\\ &= 0\end{aligned}$$

3. $\frac{Cos(22°)+Sin(22°)}{Cos(22°)-Sin(22°)} = Tan(67°)$බව පෙන්වන්න.

බ.පැ. $\frac{Cos(22°)+Sin(22°)}{Cos(22°)-Sin(22°)}$

හරයේ සහ ලවයේ සියලුම පද $Cos(22°)$න් බෙදමු.

$\frac{1 + Tan(22°)}{1 - Tan(22°)}$

$\frac{Tan(45°)+Tan(22°)}{1 - Tan(45°).Tan(22°)}$

$Tan(45° + 22°)$

$Tan(67°)$ =ද. පැ.

4. $Tan A = \frac{m}{m+1}$සහ $Tan B = \frac{1}{2m+1}$නම් $A+B = \frac{\pi}{4}$බව පෙන්වන්න.

$$Tan(A + B) = \frac{Tan A+Tan B}{1-Tan A .Tan B}$$

$$= \frac{\frac{(m)}{(m+1)} + \frac{1}{(2m+1)}}{1-\frac{m}{(m+1)} \times \frac{1}{(2m+1)}}$$

$$\text{Tan}(A+B) = \frac{\frac{m(2m+1)+(m+1)}{(m+1)(2m+1)}}{\frac{(m+1)(2m+1)-m}{(m+1)(2m+1)}}$$

$$= \frac{2m^2+m+m+1}{2m^2+m+2m+1-m}$$

$$= \frac{(2m^2+2m+1)}{(2m^2+2m+1)}$$

$$\text{Tan}(A+B) = 1$$

$$\therefore A+B = \frac{\pi}{4} \text{වේ.}$$

5. Tan(A + B) = a ද, Tan(A-B) = b ද නම්,

I. Tan(2A)

II. Tan(2B)අගය සොයන්න.

$$\text{Tan}(2A) = \text{Tan}[(A+B)+(A-B)]$$

$$= \frac{\text{Tan}(A+B)-\text{Tan}(A-B)}{1+\text{Tan}(A+B).\text{Tan}(A-B)}$$

$$= \frac{a+b}{1-ab}$$

$$\text{Tan}(2B) = \text{Tan}[(A+B) - (A-B)]$$

$$= \frac{\text{Tan}(A+B) - \text{Tan}(A-B)}{1 + \text{Tan}(A+B).\text{Tan}(A-B)}$$

$$= \frac{a-b}{1+ab}$$

6. $\text{Tan}(11\theta) - \text{Tan}(9\theta) - \text{Tan}(2\theta) = \text{Tan}(11\theta).\text{Tan}(9\theta).\text{Tan}(2\theta)$ බව සාධනය කරන්න.

$$\text{Tan}(11\theta) = \text{Tan}(9\theta + 2\theta)$$

$$\text{Tan}(11\theta) = \frac{\text{Tan}(9\theta) + \text{Tan}(2\theta)}{1-\text{Tan}(9\theta).\ \text{Tan}(2\theta)}$$

$$\text{Tan}(11\theta)\,[1 - \text{Tan}(9\theta).\text{Tan}(2\theta)] = \text{Tan}(9\theta) + \text{Tan}(2\theta)$$

$$\text{Tan}(11\theta) - \text{Tan}(11\theta).\text{Tan}(9\theta).\text{Tan}(2\theta) = \text{Tan}(9\theta) + \text{Tan}(2\theta)$$

$$\text{Tan}(11\theta) - \text{Tan}(9\theta) - \text{Tan}(2\theta) = \text{Tan}(11\theta).\text{Tan}(9\theta).\text{Tan}(2\theta)$$

7. A+B = 45°නම්,

(Cot A – 1) (Cot B – 1) = 2 බව සාධනය කරන්න. එනයින් $\text{Tan}\left(\frac{\pi}{8}\right)$හි අගය සොයන්න.

$$A + B = 45°$$

$$\text{Tan}(A+B) = \text{Tan}(45°)$$

$$\frac{\text{Tan}(A) + \text{Tan}(B)}{1 - \text{Tan A . Tan B}} = 1$$

$$\text{Tan}(A) + \text{Tan}(B) = 1 - \text{Tan}(A) \text{ . } \text{Tan}(B)$$

දෙපස සියළුම පද Tan(A) . Tan(B) වලින් බෙදීමෙන්,

$$\text{Cot B} + \text{Cot A} = \text{Cot A . Cot B} - 1$$

$$\text{Cot A . Cot B} - \text{Cot A} - \text{Cot B} = 1$$

$$\text{Cot A}(\text{Cot B} - 1) - (\text{Cot B} - 1) = 2$$

$$(\text{Cot B} - 1)(\text{Cot A} - 1) = 2$$

A = B නම්,

$$A + A = \frac{\pi}{4}$$

$$A = \frac{\pi}{8}$$

$$(\text{Cot A} - 1)^2 = 2$$

$$\text{Cot A} - 1 = \pm\sqrt{2}$$

$$\text{Cot A} = 1 + \sqrt{2}$$

$$\text{Tan}\left(\frac{\pi}{8}\right) = \frac{1 \times (\sqrt{2}-1)}{(1+\sqrt{2}) \times (\sqrt{2}-1)}$$

$$\text{Tan}\left(\frac{\pi}{8}\right) = \sqrt{2} - 1$$

8. $\frac{\text{Sin}(A+B))+\text{Sin}(A-B)}{\text{Cos}(A+B)+\text{Cos}(A-B)} = \text{Tan}(A)$ බව සාධනය කරන්න.

බ.පැ. $\frac{\text{Sin}(A+B))+\text{Sin}(A-B)}{\text{Cos}(A+B)+\text{Cos}(A-B)}$

$$\frac{\text{Sin A .Cos B}+\cancel{\text{Cos A .Sin B}}+\text{Sin A .Cos B}-\cancel{\text{Cos A . Sin B}}}{\text{Cos A .Cos B}-\cancel{\text{Sin A .Sin B}}+\text{Cos A .Cos B}+\cancel{\text{Sin A .Sin B}}}$$

$$\frac{2\,\cancel{\text{Sin}}\text{ A .}\cancel{\text{Cos B}}}{2\,\cancel{\text{Cos}}\text{ A .}\cancel{\text{Cos B}}}$$

$\text{Tan A} =$ ද. පැ.

9. $\frac{\text{Sin}(A-B)}{\text{SinA . Sin B}} + \frac{\text{Sin}(B-C)}{\text{SinB . Sin C}} + \frac{\text{Sin}(C-A)}{\text{SinC . Sin A}} = 0$ බව පෙන්වන්න.

$$\frac{\text{Sin}(A-B)}{\text{SinA . Sin B}} = \frac{\text{Sin A .Cos B}-\text{Cos A .Sin B}}{\text{SinA . Sin B}}$$

$$= \frac{\cancel{\text{Sin A .Cos B}}}{\cancel{\text{SinA}}\text{ . Sin B}} - \frac{\text{Cos A .}\cancel{\text{Sin B}}}{\text{SinA . }\cancel{\text{Sin B}}}$$

$$= \text{Cot B} - \text{Cot A} \quad \text{——— (1)}$$

මෙලෙසම $\frac{\text{Sin}(B-C)}{\text{SinB . Sin C}} = \text{Cot C} - \text{Cot B}$ ——— (2)

$$\frac{\text{Sin}\,(C-A)}{\text{Sin}C\,.\,\text{Sin}\,A} = \text{Cot}\,A - \text{Cot}\,C \quad \text{——} \quad (3)$$

(1) + (2) + (3)

$$\frac{\text{Sin}\,(A-B)}{\text{Sin}A\,.\,\text{Sin}\,B} + \frac{\text{Sin}\,(B-C)}{\text{Sin}B\,.\,\text{Sin}\,C} + \frac{\text{Sin}\,(C-A)}{\text{Sin}C\,.\,\text{Sin}\,A} =$$

$$(\cancel{\text{Cot}}\,B - \cancel{\text{Cot}}\,A) - (\cancel{\text{Cot}}\,C - \cancel{\text{Cot}}\,B) - (\cancel{\text{Cot}}\,A - \cancel{\text{Cot}}\,C)$$

$$= 0$$

10. a Tan A = b Tan B නම්, $\frac{\text{Sin}(A-B)}{\text{Sin}(A+B)} = \frac{b-a}{b+a}$ බව සාධනය කරන්න.

a Tan A = b Tan B නම්,

$$\frac{\text{Tan}\,A}{\text{Tan}\,B} = \frac{b}{a}$$

$$\frac{\text{Sin}\,A\,.\text{Cos}\,B}{\text{Cos}\,A\,.\,\text{Sin}\,B} = \frac{b}{a}$$

$$\frac{\text{Sin}\,A\,.\text{Cos}\,B - \text{Cos}\,A\,.\text{Sin}\,B}{\text{Sin}\,A\,.\,\text{Cos}\,B + \text{Cos}\,A\,.\text{Sin}\,B} = \frac{b-a}{b+a}$$

$$\frac{\text{Sin}(A-B)}{\text{Sin}(A+B)} = \frac{b-a}{b+a}$$

11. Cos A + Sin B = a ද, Sin A + Cos B = b ද නම්,

$\text{Sin(A+B)} = \frac{1}{2}(a^2 + b^2 - 2)$ බව පෙන්වන්න.

Cos A + Sin B = a ——— (1)

Sin A + Cos B = b ——— (2)

$(1)^2 + (2)^2$

$$(\text{Cos A + Sin B})^2 + (\text{Sin A + Cos B})^2 = a^2 + b^2$$

$$\text{Cos}^2\text{A} + \text{Sin}^2\text{B} + 2\text{CosA .SinB} + \text{Sin}^2\text{A} + \text{Cos}^2\text{B} + 2\text{SinA .CosB} = a^2 + b^2$$

$$(\text{Cos}^2\text{A} + \text{Sin}^2\text{A}) + (\text{Cos}^2\text{B} + \text{Sin}^2\text{B}) + 2(\text{Sin A.CosB} + \text{CosA.SinB}) = a^2 + b^2$$

$$1 + 1 + 2\,.\text{Sin(A + B)} = a^2 + b^2$$

$$2\,\text{Sin(A + B)} = a^2 + b^2 - 2$$

$$\text{Sin(A + B)} = \frac{1}{2}\{a^2 + b^2 - 2\}$$

12. $90° < A < 180°$ ද, $180° < B < 270°$ ද නම් සහ $\text{Cos } A = -\frac{12}{13}$ ද, $\text{Tan } B = \frac{3}{4}$ ද නම්,

I. Sin (A + B)
II. Cos (A + B)
III. Tan (A + B)

A කෝණය දෙවන වෘත්ත පාදයේ ඇති නිසා Sin A ධන වේ.

$$\text{Sin A} = \sqrt{1 - Cos^2 A} = \sqrt{1 - (-\frac{12}{13})} = \sqrt{\frac{13^2 - 12^2}{13^2}} = \sqrt{\frac{25}{13^2}}$$

$$\text{Sin A} = \frac{5}{13}$$

A කෝණය දෙවන වෘත්ත පාදයේ ඇති නිසා Sin B , Cos B , Cosec B අනුපාත සෘණ වේ.

$$\text{Cosec B} = \sqrt{1 + Cot^2 B} = -\sqrt{1 - (\frac{4}{3})^2}$$

$$= -\sqrt{\frac{3^2 - 4^2}{3^2}}$$

$$\text{Cosec B} = -\frac{5}{3}$$

$$\text{Sin B} = -\frac{3}{5}$$

$$\text{Cos B} = \text{Cot B . Sin B}$$

$$= \frac{4}{3} \times (-\frac{3}{5})$$

$$= (-\frac{4}{5})$$

I. $\text{Sin}(A + B) = \text{Sin}\,A \,.\, \text{Cos}\,B + \text{Cos}\,A \,.\, \text{Sin}\,B$

$$= \left(\frac{5}{13}\right)\times\left(-\frac{4}{5}\right) + \left(-\frac{12}{13}\right)\times\left(-\frac{3}{5}\right)$$

$$= -\frac{20}{65} + \frac{36}{65}$$

$$= \underline{\underline{\frac{16}{65}}}$$

II. $\text{Cos}(A + B) = \text{Cos}\,A \,.\, \text{Cos}\,B + \text{Sin}A \,.\, \text{Sin}\,B$

$$= \left(\frac{5}{13}\right)\times\left(-\frac{4}{5}\right) + \left(-\frac{12}{13}\right)\times\left(-\frac{3}{5}\right)$$

$$= -\frac{20}{65} + \frac{36}{65}$$

$$= \underline{\underline{\frac{16}{65}}}$$

III. $\text{Tan}(A + B) = \dfrac{\text{Sin}(A+B)}{\text{Cos}(A+B)}$

$$= \frac{16}{\cancel{65}} + \frac{\cancel{65}}{63}$$

$$= \underline{\underline{\frac{16}{63}}}$$

13. $\pi < A < \frac{3\pi}{2}$, $\frac{\pi}{2} < B < \pi$ ද නම්, $\text{Cot } A = \frac{1}{2}$ ද $\text{Sec } B = -\frac{5}{3}$ ද නම්, $\text{Tan}(A + B)$ හි අගය සොයා තවද කෝණය පිහිටන වෘත්ත පාදය සොයන්න.

$$\text{Cot } A = \frac{1}{2} \longrightarrow \text{Tan } A = 2$$

B කෝණය දෙවන වෘත්ත පාදයේ ඇති නිසා Tan B අනුපාතය සෘණ වේ.

$$\text{Tan } B = -\sqrt{\text{Sec}^2 B - 1} = -\sqrt{(\frac{5}{3})^2 - 1} = -\sqrt{\frac{25}{9} - 1}$$

$$= -\sqrt{\frac{16}{9}}$$

$$\text{Tan } B = -\frac{4}{3}$$

$$\text{Tan}(A + B) = \frac{\text{Tan } A + \text{Tan } B}{1 - \text{Tan } A \,.\, \text{Tan } B}$$

$$= \frac{2 + (-\frac{4}{3})}{1 - 2 \times (-\frac{4}{3})}$$

$$= \frac{2 + (-\frac{4}{3})}{1 - 2 \times (-\frac{4}{3})}$$

$$= \frac{\frac{6 - 4}{3}}{\frac{3 + 8}{3}}$$

$$\text{Tan } (A + B) = \frac{2}{11}$$

Tan (A + B) අනුපාතය ධන වේ. තව ද Tanඅනුපාතය ධන වන්නේ කෝණය පළමු හෝ තෙවන වෘත්ත පාදයේ පිහිටන විට ය.

තව ද $\frac{3\pi}{2} < \alpha + \beta < \frac{5\pi}{2}$

$\therefore \alpha + \beta$ කෝණය පළමු වෘත්ත පාදයේ පිහිටයි.

14. $0< C < B < A <\frac{\pi}{2}$ ද, $\text{Tan } B = \frac{\text{Cos } C-\text{Cos } A}{\text{Sin } A-\text{Sin } C}$ ද නම් A,B,C කෝණ සමාන්තර ශ්‍රේඩියක පිහිටන බව පෙන්වන්න.

$$\text{Tan } B = \frac{\text{Cos } C-\text{Cos } A}{\text{Sin } A-\text{Sin } C}$$

$$\frac{\text{Sin } B}{\text{Cos } B} = \frac{\text{Cos } C-\text{Cos } A}{\text{Sin } A-\text{Sin } C}$$

$$\text{Sin } B(\text{Sin } A - \text{Sin } C) = \text{Cos } B(\text{Cos } C - \text{Cos } A)$$

$$\text{Sin } B \,.\, \text{Sin } A - \text{Sin } B \,.\, \text{Sin } C = \text{Cos } B \,.\, \text{Cos } C - \text{Cos } B \,.\, \text{Cos } A$$

$$\text{Cos } A \,.\, \text{Cos } B + \text{Sin } A \,.\, \text{Sin } B = \text{Cos } B \,.\, \text{Cos } C + \text{Sin } B \,.\, \text{Sin } C$$

$$\text{Cos}(A - B) = \text{Cos}(B - C)$$

$$A - B = B - C$$

$$2A = A + C$$

$\therefore A , B , C$ සමාන්තර ශ්‍රේඩියක පිහිටයි.