Provincial Department of Education - Northern Province

Combined Mathematics - II

Grade:13 (2023) | **10 | T | II** | **Three hours** **Additional Reading Time: 10 minutes**

Index No. | | | | | | |

Instructions

- This question paper consists of two parts;
  Part A (questions 1-10) and **part B**(questions 11- 17).

Part - A

- Answer **all** questions. Answers should be written in the space provided on the questions paper. If additional space needed, you may use additional answer sheets.

Part - B

- Answer only 5 questions.
- After the allocated time hand over the paper to the supervisor with both parts attached together.
- **Only part B** of the paper is allowed to be taken out of the Examination Hall..

(10) Combined Mathematics - II		
Part	**Question No.**	**Marks**
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
Total		

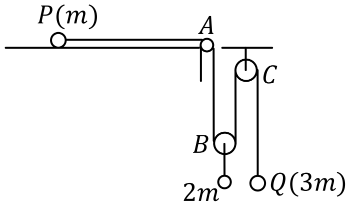
Combined Mathematics-I	
Combined Mathematics-II	
Final Marks	

Part A

1. Two small smooth spheres A and B of masses m and em are placed on a smooth table and given velocities u and eu respectively, in the direction $\overrightarrow{AB}$, where $e(0 < e < 1)$ is the coefficient of restitution of the spheres. Find the velocities of A and B after the collision and show that the velocity of B after the collision does not depend on e. If e varies, find the value of e such that velocity of A after the collision is minimum.

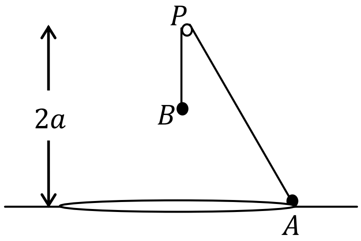
2. A particle is projected horizontally with velocity u from a point A at a height $\frac{3a}{2}$ vertically above a point O on the horizontal ground. The particle hits the horizontal ground at 60^0 to the horizontal at the point B. Show that $u = \sqrt{ag}$ and find OB in terms of a.

3. A light inextensible string attached to a particle P of mass m on a smooth horizontal table passes over a fixed smooth small pulley A at the edge of the table and passes under a light smooth movable pulley B hung with a mass of $2m$ and passes over a fixed smooth pulley C and is attached with a particle Q of mass $3m$. The system is released from rest with the string taut. Write down equations sufficient to determine the tension of the string.



4. The total mass of a train is $150 \times 10^3\, kg$. The train engine is working at a rate $90\, kW$. If the train moves on a horizontal straight track with maximum speed of $54\, kmh^{-1}$, find the resistant force of the train. Find the maximum speed in kmh^{-1} when the train moves up along a straight track inclined to the horizontal at an angle α with the same power and the same resistance force, where $\sin\alpha = \frac{1}{300}$ and $g = 10ms^{-2}$.

5. Particles A and B of mass m each are attached to the ends of a light inextensible string of length $5a$, passes over a fixed small smooth pulley P at a height $2a$ from the smooth horizontal table. The particle A moves with a constant angular velocity $\sqrt{\frac{g}{4a}}$ in a horizontal circle on the table and the particle B hang in equilibrium. Show that inclination of PA to the vertical is $\frac{\pi}{3}$. Find the reaction on A exerted by the table.



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6. Let $\boldsymbol{a}$ and $\boldsymbol{b}$ be two non zero vectors. Find the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$ in each of the following cases
(i) $\boldsymbol{a}.(\boldsymbol{a}+2\boldsymbol{b}) = 0$ and $|\boldsymbol{b}| = |\boldsymbol{a}|$
(ii) $|\boldsymbol{a}+\boldsymbol{b}| = |\boldsymbol{a}-\boldsymbol{b}|$.

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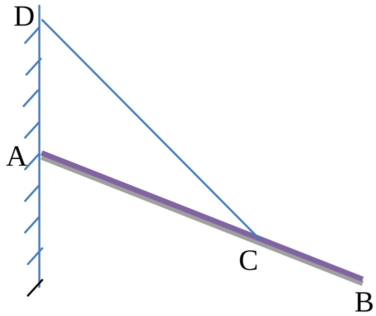
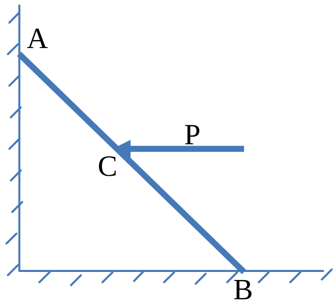
7. A uniform rod AB of weight w and length $2a$ has the end A in contact with a smooth vertical wall, and one end of a string is fastened to the rod at a point C, such that $AC:CB = 3:1$, and the other end of the string is fastened to the point D on the wall vertically above at a height $\frac{3a}{2}$ from A. The rod AB is in equilibrium. Find the angle the rod makes to the vertical and the tension in the string.



8. A uniform rod AB of weight W is kept in equilibrium against a smooth vertical wall with its lower end B on a rough horizontal ground, by a horizontal force of magnitude P applied at the mid-point C of the rod as shown in the figure. The rod makes an angle 45^0 to the horizontal. The coefficient of friction between the rod and the ground is $\frac{1}{3}$. Show that $\frac{W}{3} \leq P \leq \frac{5W}{3}$.



9. Let A and B be two events of a sample space Ω. It is given that $P(A/B) = \frac{1}{3}$, $P(B) = \frac{1}{2}$ and $P(A \cup B) = \frac{2}{3}$. Find $P(A \cap B)$, $P(A)$ and show that A and B are independent events.

10. The mean and standard deviation of marks obtained by 100 students in an examination are 50 and 10 respectively. If the z-score of a student in an examination is 2.2, what is the marks obtained by him in the examination? It was later found that this mark has been entered erroneously and it should have been 10 more than this mark instead. Find the correct value of the mean of the marks obtained for this examination.

Part B

11. (a) A and B are two points on a straight road with a meter apart. A car P starting from rest at A travels towards B with constant acceleration $2f\ ms^{-2}$ and then travels with a constant retardation $f\ ms^{-2}$ for the remaining distance and comes to rest at B. A car Q starts from rest at B at the same time as P starts and travels towards A with a constant acceleration for time of $\frac{1}{2}\sqrt{\frac{3a}{f}}$ second and then travels with constant velocity and reaches A at the same instant when the car P comes to rest at B.

Sketch the velocity-time graphs for the motion of P and Q in the **same diagram. Hence,**

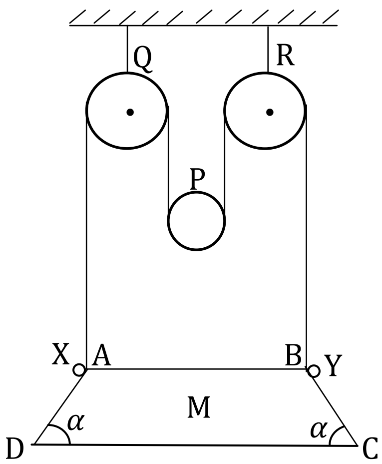
(i) Show that the maximum speed of the car P is $2\sqrt{\frac{af}{3}}ms^{-1}$.

(ii) Find the magnitude of the acceleration of the car Q.

(b) A ship P is sailing due north with a uniform speed $\sqrt{3}u\ kmh^{-1}$ and at the same time a ship Q is sailing due east with a uniform speed $\frac{3}{2}u\ kmh^{-1}$. A third ship R appears to move in the direction 60^0east of north when it is observed from P and the ship R appears to move due north when it is observed from Q. Draw the velocity triangles for the motion of the ship R in the **same diagram**. Show that the ship R moves with speed $3u\ kmh^{-1}$ and find the direction of the ship R.

Also, it is given that initially the ship R is in $d\ km$ west of P and $\sqrt{3}d\ km$ south of Q. Find the shortest distance between P and R and the time taken to cover the distance and show that the distance between Q and R in this time is $\frac{\sqrt{3}d}{4}\ km$.

12. (a) $ABCD$ is a cross-section with the shape of trapezium of a symmetrical wedge of mass M. A light inextensible string passes over fixed smooth pulleys Q, R and passes under a smooth movable pulley P of mass m has its ends attached to the points A and B on the wedge as shown in the figure. The particles X and Y each of mass m are placed on the faces AD and BC respectively. The string, pulleys and particles are in the vertical plane through the centre of mass of the wedge. The system is released from rest with the string taut. Write down equations sufficient to determine the tension of the string and the acceleration of the wedge.



(b) A solid sphere with center O and radius a is fixed. A particle P of mass m is placed at a point A on the smooth surface of the sphere and released, where OA makes an angle $\cos^{-1}\left(\frac{3}{4}\right)$ to the vertical. Let OP makes an angle θ to the upward vertical and assume that the particle is on the surface of the sphere and its velocity is v.

(i) Show that $v^2 = \frac{ag}{2}(3 - 4\cos\theta)$.

(ii) Find the reaction of the particle at that moment.

Hence, show that the particle leaves the surface of the sphere at an angle $\frac{\pi}{3}$ to the upward vertical of OP.

Find the horizontal and vertical components of the velocity of the particle at that moment.

13. One end of a light elastic string of natural length a is attached to a point O on the smooth long table and the other end to a particle P of mass m. When the particle P is held at a point A on the table at a distance $2a$ from O and given a velocity $2\sqrt{6ag}$ in the direction $\overrightarrow{OA}$, if the particle comes to instantaneous rest at a distance $6a$ from O in the direction $\overrightarrow{OA}$, show that the modulus of elasticity is mg.

Now particle P is removed and a particle Q of mass $2m$ is attached to the end of the string and the particle Q is held at O and given a velocity $5\sqrt{2ag}$ in the direction $\overrightarrow{OA}$. Using the **conservation of mechanical energy,** show that $\dot{x}^2 = -\frac{g}{2a}(x^2 - 2ax - 99a^2)$ for $x \geq a$, where x is the length of the string.

Hence, show that the equation of the motion of Q is given by $\ddot{x} = -\frac{g}{2a}(x - a)$ for $x \geq a$.

Re-write this equation in the form $\ddot{X} = -\frac{g}{2a}X$, where $X = x - a$.

Find the centre of the oscillation of the simple harmonic motion.

It is given that $\dot{X}^2 = \omega^2(b^2 - X^2)$ is a solution of $\ddot{X} = -\frac{g}{2a}X$.

Find ω and amplitude b.

Show that the particle Q takes the minimum time of $\sqrt{\frac{2a}{g}}\left(\frac{1}{10} + \sin^{-1}\frac{4}{5}\right)$ to reach point B at a distance $9a$ from O in the direction $\overrightarrow{OA}$.

Find the speed of the particle Q at B.

When the particle Q first reaches B, Q collides with another particle of mass m at rest in its path and coalesces with it.

Find the speed of the composite particle just after the collision.

Find the distance from O on which the composite particle comes to instantaneous rest.

Write down the centre of oscillation and amplitude of the simple harmonic motion for the composite particle with reasons.

Show that the equation of the motion of the composite particle is given by $\ddot{x} = -\frac{g}{3a}(x-a)$ for $x \geq a$, where x is the length of the string.

Show that the time which the composite particle performs from B until the string becomes slack for the first time is $\sqrt{\frac{3a}{g}}\left\{\frac{\pi}{2} + \cos^{-1}\frac{2\sqrt{22}}{11}\right\}$.

14. (a) $ABCD$ is a rectangle. The side BC is produced to F such that $BC = CF$ and the side AB is produced to P such that $AB:BP = 1:m$. AF and DP intersect at E.

Let $\overrightarrow{AB} = \underline{a}$, $\overrightarrow{AD} = \underline{b}$, $|\underline{a}| = 1$ and $|\underline{b}| = 2$.

(i) Find $\overrightarrow{AF}$ in terms of $\underline{a}, \underline{b}$ and $\overrightarrow{DP}$ in terms of $\underline{a}, \underline{b}, m$.

Hence, if $A\hat{E}D = 90^0$, show that $m = 7$.

(ii) Let $m = 7$, $AE : EF = 1 : r$ and $DE : EP = 1 : k$.

Find $\overrightarrow{AE}$ in terms of $\underline{a}, \underline{b}, r$ and also in terms of $\underline{a}, \underline{b}, k$.

Hence, find the values of r and k.

(b) The coordinates of points A, C, D about $Ox.Oy$ axes are respectively $(4,0)$, $(4, 4\sqrt{3})$, $(0, 4\sqrt{3})$. B is a point on the Oxy plane such that $O\hat{A}B = \frac{5\pi}{6}$, $D\hat{C}B = \frac{2\pi}{3}$ and $OABCD$ forms a pentagon. Forces $\alpha\underline{i} + 2\sqrt{3}\underline{j}$, $8\underline{i} + 4\sqrt{3}\underline{j}$, $6\underline{i} - 5\sqrt{3}\underline{j}$ and $-4\underline{i} + 3\sqrt{3}\underline{j}$ are acting at the points A, B, C and D respectively.

(i) Show that $B \equiv (7, \sqrt{3})$.

(ii) Show that for any value of α, the system is equivalent to a single resultant force passes through the origin O.

(iii) It is given that $\alpha = -6$. Find the resultant force of the system and also, show that this resultant passes through the point C. Also, find the equation of the line of action of the resultant.

(iv) If the resultant of the forces passes through the point A, find the couple that should be added.

(v) If a force is added at O and a couple is added to the system such that new system of forces is equivalent to a single force of magnitude $8\sqrt{3}$ acting along $\overrightarrow{AB}$, find the force and the couple.

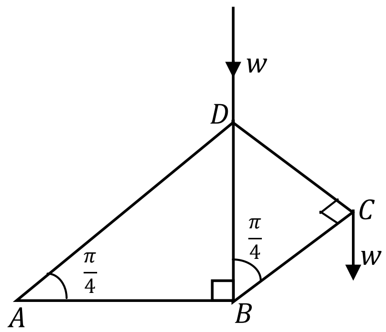
15. (a) Three uniform rods OA, AB, BC each of length $2a$ and weight w are smoothly jointed at A and B and hang independently from a fixed point O. A horizontal force P is applied to the rod BC at C. system is in equilibrium and BC makes an angle 45^0 to the horizontal.

(i) Find the force P in terms of W.

(ii) Show that the reaction at the joint O is $\frac{W}{2}\sqrt{37}$ and find the direction of this reaction.

(iii) Show that the angle of OA to the horizontal is $\tan^{-1} 5$.

(b) The framework shown in the figure is made of five light rods freely jointed at their ends. It is given that $A\hat{B}D = \frac{\pi}{2}$, $B\hat{C}D = \frac{\pi}{2}$ and other angles are $\frac{\pi}{4}$. A load of weight w each is hang from C and D and the framework is in equilibrium in a vertical plane with AB horizontal and BD vertical, supported by two vertical forces acting at A and B.

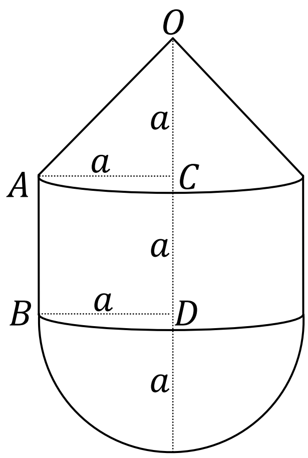


Draw a stress diagram using Bow's notation and **hence**, find the stresses in all the rods and state whether these stresses are tensions or thrusts. Also, find the vertical forces acting at A and B.

16. Show that the centre of mass of

(i) a uniform solid hemisphere of radius r is at a distance $\frac{3r}{8}$ from its centre.

(ii) a uniform solid right circular cone of base radius r and height h is at a distance $\frac{3h}{4}$ from its vertex.

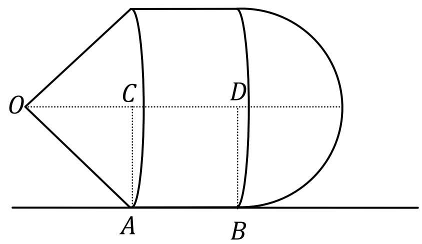


A composite body is made by rigidly joining a solid right circular cone of radius a and height a, a solid right circular cylinder of radius a and height a and a uniform solid hemisphere of radius a, as shown in the figure. Density of the cone and the cylinder is σ and the hemisphere is $k\sigma$.

Show that the centre of mass of the composite body is at a distance $\frac{(19k+21)a}{8(k+2)}$ from the vertex O.

If the composite body is freely suspended from a point A and hangs in equilibrium, find the angle of OD to the vertical.

Now if the composite body is removed from the point of suspension and kept in equilibrium, with the cylindrical surface AB placed on a horizontal plane as shown in the figure, show that $0 < k < \frac{11}{3}$.



17. (a) Three identical boxes A, B and C each contain 6 balls identical in all respects except their colours. Box A contains 4 red balls and 2 white balls. Box B contains 3 red balls and 3 white balls. Box C contains 2 red balls and 4 white balls. Two boxes are chosen at random and a ball is drawn from each of them.

Find the probability that

(i) the two balls drawn are red,

(ii) the two balls drawn are of the different colours,

(iii) the boxes A and B were chosen, given that the two balls drawn are different colours.

(b) Let the mean and the standard deviation of the set of data $\{x_1, x_2, x_3, \ldots.., x_n\}$ be $\bar{x}$ and σ_x respectively, and $y_i = \frac{x_i - a}{b}$ for $i = 1, 2, 3, \ldots, n,$ where a and b are real constants. Show that $\bar{y} = \frac{\bar{x} - a}{b}$ and $\sigma_y = \frac{\sigma_x}{|b|}$, where $\bar{y}$ and σ_y are respectively the mean and the standard deviation of the set of data $\{y_1, y_2, y_3, \ldots.., y_n\}$.

Marks obtained by 100 students in an exam are given in the following frequency table.

Marks	No. of students
30 - 39	8
40 - 49	15
50 - 59	28
60 - 69	20
70 - 79	19
80 - 89	10

By means of the transformation $y = \frac{x - 54.5}{10}$, estimate the mean and the standard deviation of y, and also the coefficient of skewness of y defined by $\frac{3(\text{mean - median})}{\text{standard deviation}}$.

Hence, estimate the mean, the standard deviation and the coefficient of skewness of x.