FWC

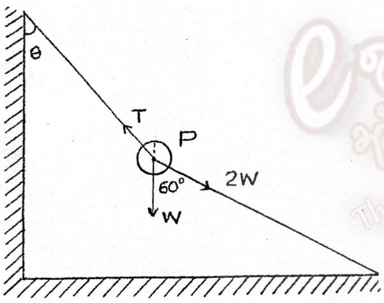
Field Work Centre - Thondaimanaru

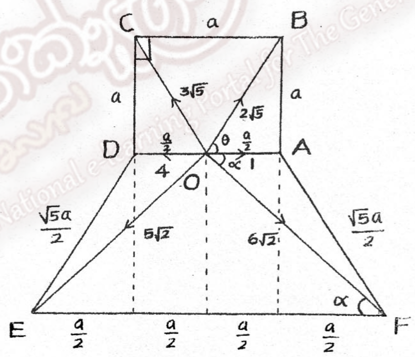
Term Examination, March – 2018

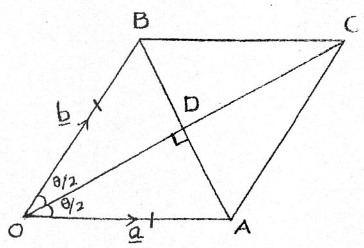
Grade : 12 (2018)

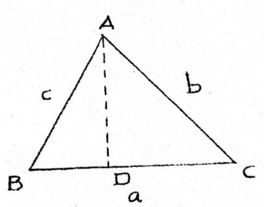
Combined Mathematics

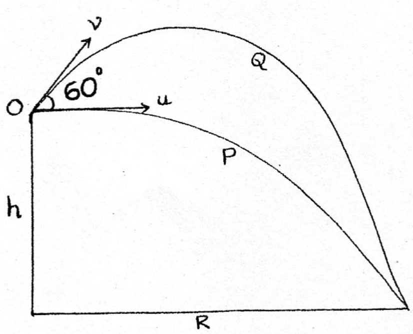
Marking Scheme

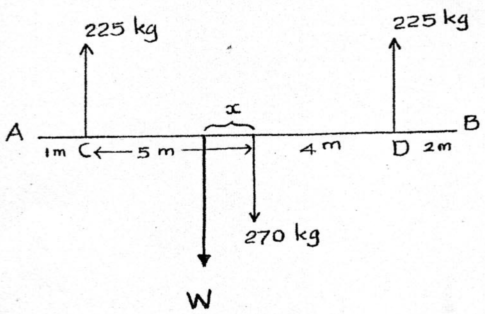












FWC

01. $y = x^2 - \frac{2}{3}(a+b+c)x + \frac{1}{3}(a^2+b^2+c^2)$

$\Delta = \frac{4}{9}(a+b+c)^2 - 4\left(\frac{1}{3}\right)(a^2+b^2+c^2)$ 10

$= \frac{4}{9}\left\{a^2+b^2+c^2+2ab+2bc+2ca-3a^2-3b^2-3c^2\right\}$ 5

$= -\frac{4}{9}\left\{2a^2+2b^2+2c^2-2ab-2bc-2ca\right\}$

$= -\frac{4}{9}\left\{(a-b)^2+(b-c)^2+(c-a)^2\right\}$

< 0 5

$\therefore\ y > 0$ for all x 5

$\therefore$ The quadratic function is above the x- axis.

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02. $\frac{1}{\log_{xy} xyz} + \frac{1}{\log_{yz} xyz} + \frac{1}{\log_{zx} xyz}$

$= \log_{xyz} xy + \log_{xyz} yz + \log_{xyz} zx$ 10

$= \log_{xyz}(xy \cdot yz \cdot zx)$ 5

$= \log_{xyz}(xyz)^2$

$= 2 \times \log_{xyz} xyz$ 5

$= 2 \times 1 = 2$ 5

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03. $\frac{x-1}{x-2} \geqslant \frac{x-2}{x-3}$

$\frac{x-1}{x-2} - \frac{x-2}{x-3} \geqslant 0\ ;\ x \neq 2, 3$ 5

$\frac{(x-1)(x-3)-(x-2)^2}{(x-2)(x-3)} \geqslant 0$

$\frac{x^2-4x+3-x^2+4x-4}{(x-2)(x-3)} \geqslant 0$ 5

$\frac{-1}{(x-2)(x-3)} \geqslant 0$ 5

$(x-2)(x-3) < 0 \quad [(x-2)^2 > 0,\ (x-3)^2 > 0]$

$(x-2)(x-3)$	+++	----	+++
	+++	----	+++

5

$\therefore$ The set of solutions is $2 < x < 3$. 5

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04. $\lim_{x\to 0} \frac{1-\cos 4x}{\sqrt{x^2+9}-3}$

$= \lim_{x\to 0} \frac{2\sin^2 2x\left\{\sqrt{x^2+9}+3\right\}}{(x^2+9-9)}$ 10

$= 2 \times 4\left\{\lim_{2x\to 0} \frac{\sin 2x}{2x}\right\}^2 \lim_{x\to 0}\left\{\sqrt{x^2+9}+3\right\}$ 5

$= 8\ (1)^2\left\{\sqrt{9}+3\right\}$ 5 5

$= 48$

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05. $\sin\theta = \dfrac{12}{13}$

$$\cos^2\theta = 1 - \sin^2\theta$$
$$= 1 - \frac{144}{169}$$
$$= \frac{25}{169}$$

$$\cos\theta = \frac{5}{13} \quad \left(0 < \theta < \frac{\pi}{2}\right) \quad 5$$

$$\sin^2\phi = 1 - \cos^2\phi$$
$$= 1 - \frac{9}{25}$$
$$= \frac{16}{25}$$

$$\sin\phi = -\frac{4}{5} \quad \left(\pi < \phi < \frac{3\pi}{2}\right) \quad 5$$

$$\sin(\theta+\phi) = \sin\theta\cos\phi + \cos\theta\sin\phi \quad 5$$
$$= \frac{12}{13}\cdot\left(-\frac{3}{5}\right) + \frac{5}{13}\cdot\left(-\frac{4}{5}\right) \quad 5$$
$$= -\frac{56}{65} \quad 5$$

$\boxed{25}$

06. $$\frac{\cos\theta}{(2\cos\theta-1)(3\cos\theta-1)} = \frac{A}{2\cos\theta-1} + \frac{B}{3\cos\theta-1} \quad 5$$

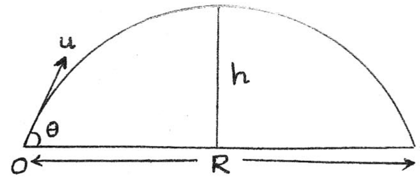
$$\cos\theta = A(3\cos\theta-1) + B(2\cos\theta-1) \quad 5$$

$$\left.\begin{array}{l} 3A+2B = 1 \\ -A-B = 0 \end{array}\right\} \Rightarrow \begin{array}{l} A = 1 \quad 5 \\ B = -1 \quad 5 \end{array}$$

$$\therefore \frac{\cos\theta}{(2\cos\theta-1)(3\cos\theta-1)} = \frac{1}{2\cos\theta-1} - \frac{1}{3\cos\theta-1}$$

5 $\boxed{25}$

07.



$$R = \frac{u^2}{g}\sin 2\theta \quad 10$$

$$\Rightarrow \sin 2\theta = \frac{Rg}{u^2} \quad \ldots\ldots (1)$$

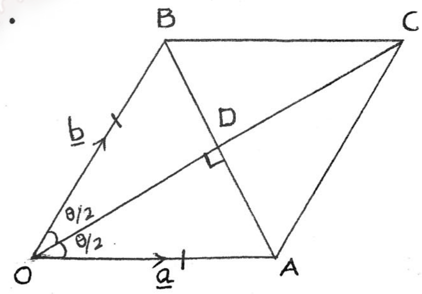
$$h = \frac{u^2}{2g}\sin^2\theta \quad 5$$

$$2gh = u^2\sin^2\theta$$
$$4gh = u^2(1-\cos 2\theta)$$
$$\Rightarrow \cos 2\theta = \frac{u^2-4gh}{u^2} \quad \ldots\ldots (2) \quad 5$$

$$\frac{(1)}{(2)} \Rightarrow \tan 2\theta = \frac{Rg}{u^2-4gh} \quad 5$$

$\boxed{25}$

08.



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$$AB = |\underline{a} - \underline{b}| \quad 5$$
$$OC = |\underline{a} + \underline{b}| \quad 5$$

$$\tan\frac{\theta}{2} = \frac{AD}{OD} = \frac{\frac{1}{2}AB}{\frac{1}{2}OC} \quad 10$$
$$= \frac{|\underline{a}-\underline{b}|}{|\underline{a}+\underline{b}|}$$

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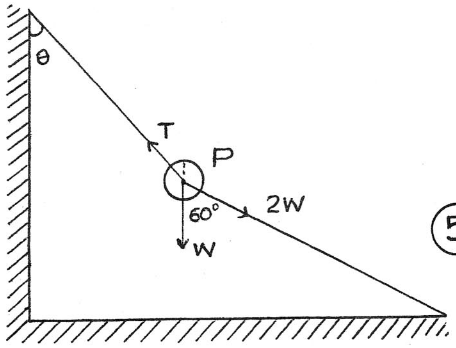
09.



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By Lami's theorem,

$$\frac{T}{\sin 60} = \frac{W}{\sin(120+\theta)} = \frac{W}{\sin\theta} \quad 5$$

$$\sin\theta = 2\sin(120+\theta)$$

$$\sin\theta = 2\sin 120\cos\theta + 2\cos 120\cdot\sin\theta \quad 5$$

$$\tan\theta = \sqrt{3} - \tan\theta$$

$$\tan\theta = \frac{\sqrt{3}}{2}$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{3}}{2}\right) \quad 5$$

$$T = \frac{W\sqrt{3}}{\sin\theta}$$

$$= \sqrt{7}\,W \quad 5$$

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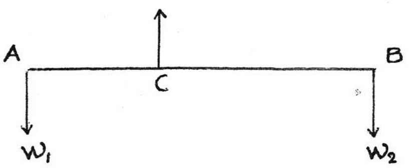
10.



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$\overset{\curvearrowleft}{C}$ 5 $\quad W_1\cdot AC - W_2\cdot BC = 0$ 10

$$\frac{AC}{BC} = \frac{W_2}{W_1}$$

$AC : CB = W_2 : W_1$ 5

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11. (a)

(i) $x^2 - px + q = 0$ has the roots α and β.

$$\left.\begin{aligned}\alpha+\beta &= p\\ \alpha\beta &= q\end{aligned}\right\} \quad 5$$

$$(\alpha+3)+(\beta+3)$$
$$= (\alpha+\beta)+6$$
$$= p+6 \quad 5$$

$$(\alpha+3)(\beta+3)$$
$$= \alpha\beta + 3(\alpha+\beta) + 9 \quad 5$$
$$= q + 3p + 9 \quad 5$$

$\therefore$ The quadratic equation whose roots are $\alpha+\beta$ and $\beta+3$ is

$$x^2 - (p+6)x + 3p + q + 9 = 0.$$

5

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(ii) $x^2 - p(p^2 - 3q)x + q^3 = 0$

$$x^2 - (\alpha+\beta)\left((\alpha+\beta)^2 - 3\alpha\beta\right)x + (\alpha\beta)^3 = 0 \quad 5$$

$$x^2 - (\alpha+\beta)(\alpha^2 - \alpha\beta + \beta^2)x + (\alpha\beta)^3 = 0 \quad 5$$

$$x^2 - (\alpha^3+\beta^3)x + \alpha^3\beta^3 = 0 \quad 5$$

$$(x-\alpha^3)(x-\beta^3) = 0 \quad 5$$

$$x = \alpha^3 \text{ or } x = \beta^3$$

$\therefore$ The roots are α^3 and β^3. 5

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(iii) If the roots of $x^2 - px + q = 0$ are real and distinct,

$p^2 - 4q > 0$ ---(1) 10

$x^2 + (2k-p)x + q - kp = 0$

$\Delta = (2k-p)^2 - 4(q-kp)$ 10

$= 4k^2 - 4kp + p^2 - 4q + 4kp$

$= 4k^2 + p^2 - 4q$

> 0 (by (1)) 10

∴ For all real values of k, the roots of $x^2 + (2k-p)x + q - kp = 0$ are real and distinct.

$\boxed{30}$

(b) $f(x) \equiv (k^2-3)x^4 + kx^3 - 2x^2 + k^2 + 4$

$\equiv (x-2)^2 \phi(x)$ 10

If $x = 2$, $16(k^2-3) + 8k - 8 + k^2 + 4 = 0$ 5

$17k^2 + 8k - 52 = 0$

$(k+2)(17k-26) = 0$ 5

$k = -2$ or $k = \frac{26}{17}$ ---(1)

5

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$f'(x) = (k^2-3)4x^3 + 3kx^2 - 4x$

$= (x-2)^2\phi'(x) + \phi(x)\,2(x-2)$ 10

When $x = 2$, $(k^2-3)32 + 12k - 8 = 0$ 5

$8k^2 + 3k - 26 = 0$

$(k+2)(8k-13) = 0$ 5

$k = -2$ or $k = \frac{13}{8}$ ---(2)

5

(1), (2) $\Rightarrow$ $k = -2$ 5

$x^4 - 2x^3 - 2x^2 + 8 \equiv (x^2 - 4x + 4)\cdot(x^2 + 2x + 2)$ 10

$\equiv (x-2)^2\{(x+1)^2 + 1\}$

≥ 0 5

∴ $f(x) \geq 0$, for all x.

$\boxed{45}$

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12. (a) $(2x)^{\ln 2} = (3y)^{\ln 3}$ --- (1)

$3^{\ln x} = 2^{\ln y}$ --- (2)

(1) $\Rightarrow$ $\ln 2 \cdot \ln 2x = \ln 3 \cdot \ln 3y$ 5

$\Rightarrow$ $\ln 2\,(\ln 2 + \ln x) = \ln 3\,(\ln 3 + \ln y)$ --- (3)

(2) $\Rightarrow$ $\ln x \cdot \ln 3 = \ln y \cdot \ln 2$ 5 5

$\Rightarrow$ $\ln y = \frac{\ln x \cdot \ln 3}{\ln 2}$ --- (4) 5

(3), (4) $\Rightarrow$ $\ln 2\,(\ln 2 + \ln x) = \ln 3\left\{\ln 3 + \frac{\ln x \cdot \ln 3}{\ln 2}\right\}$ 5

$\Rightarrow (\ln 2)^2 + \ln 2 \cdot \ln x = (\ln 3)^2 + \ln x \cdot \frac{(\ln 3)^2}{\ln 2}$

$\Rightarrow (\ln 3)^2 - (\ln 2)^2 = \frac{\ln x}{\ln 2}\{(\ln 2)^2 - (\ln 3)^2\}$ 5

$\Rightarrow -1 = \frac{\ln x}{\ln 2} \Rightarrow -1 = \log_2 x$ 5

$\Rightarrow 2^{-1} = x \Rightarrow x = \frac{1}{2}$ 5

$\boxed{40}$

(b) $\dfrac{2x^3}{(x+1)(x-2)} = Ax+B+\dfrac{C}{x+1}+\dfrac{D}{x-2}$ 10

$2x^3 = (Ax+B)(x+1)(x-2) +$

$C(x-2) + D(x+1)$

$\Rightarrow$ $A = 2$ 5, $B = 2$ 5, $C = \dfrac{2}{3}$ 5, $D = \dfrac{16}{3}$ 5

$\therefore \dfrac{2x^3}{(x+1)(x-2)} = 2x+2+\dfrac{2}{3(x+1)}+\dfrac{16}{3(x-2)}$

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(c) Let $E = \dfrac{x+2}{x^2+3x+6}$.

$\Rightarrow Ex^2+(3E-1)x+6E-2=0$ 5

For the real values of x,

$\Delta \geqslant 0$ 5

$(3E-1)^2 - 4E(6E-2) \geqslant 0$ 5

$15E^2 - 2E - 1 \leqslant 0$

$(5E+1)(3E-1) \leqslant 0$

	$E < -\frac{1}{5}$	$-\frac{1}{5} < E < \frac{1}{3}$	$E > \frac{1}{3}$
$(5E+1)(3E-1)$	$++$ $++$	$---$ $---$	$++$ $++$

10

$-\dfrac{1}{5} \leqslant E \leqslant \dfrac{1}{3}$ 5

$\therefore$ E takes the values from $-\frac{1}{5}$ to $\frac{1}{3}$.

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(d) $a, b \in \mathbb{R}$, $a, b > 0$

$(a-b)^2 \geqslant 0$ 5

$a^2 - 2ab + b^2 \geqslant 0$

$a^2 - 2ab + b^2 + 4ab \geqslant 4ab$ 5

$(a+b)^2 \geqslant 4ab$ 5

$ab \leqslant \frac{1}{4}(a+b)^2$ --- (R)

15

$\dfrac{ab}{a+b} \leqslant \dfrac{1}{4}(a+b)$ [$\because a+b>0$] 5

when $a+b=1$

(R) $\Rightarrow$ $ab \leqslant \frac{1}{4} \Rightarrow \frac{1}{ab} \geqslant 4$ 10

Consider $\left(1+\frac{1}{a}\right)\left(1+\frac{1}{b}\right)$

$= 1+\dfrac{1}{a}+\dfrac{1}{b}+\dfrac{1}{ab}$ 5

$= 1+\dfrac{a+b}{ab}+\dfrac{1}{ab}$

$= 1+\dfrac{2}{ab}$ 5

$\geqslant 1+2(4) = 9$ 5

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13. (a) $f(x) = \sqrt{1+x}$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \quad 5$$

$$= \lim_{h\to 0} \frac{\sqrt{1+x+h} - \sqrt{1+x}}{h} \quad 5$$

$$= \lim_{h\to 0} \frac{(1+x+h) - (1+x)}{h\left\{\sqrt{1+x+h} + \sqrt{1+x}\right\}} \quad 10$$

$$= \lim_{h\to 0} \frac{1}{\left\{\sqrt{1+x+h} + \sqrt{1+x}\right\}} \quad 5$$

$$= \frac{1}{\sqrt{1+x} + \sqrt{1+x}} \quad 5$$

$$= \frac{1}{2\sqrt{1+x}} \quad 5$$

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(b) (i) $y = \dfrac{x^2 - x + 1}{x^2 + x + 1}$

$$\frac{dy}{dx} = \frac{(x^2+x+1)(2x-1) - (x^2-x+1)(2x+1)}{(x^2+x+1)^2} \quad 15$$

$$= \frac{2(x^2-1)}{(x^2+x+1)^2} \quad 5$$

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(ii) $y = \tan^{-1}\left\{\dfrac{1+\sin x}{1-\sin x}\right\}^{\frac{1}{2}}$

$$= \tan^{-1}\left\{\frac{(1+\sin x)^2}{\cos^2 x}\right\}^{\frac{1}{2}} \quad 5$$

$$= \tan^{-1}\{\sec x + \tan x\} \quad 5$$

$$\frac{dy}{dx} = \frac{1}{1 + (\sec x + \tan x)^2}\left\{\sec x \tan x + \sec^2 x\right\} \quad 10$$

$$= \frac{\sec x(\sec x + \tan x)}{2\sec x(\sec x + \tan x)} \quad 5$$

$$= \frac{1}{2} \quad 5$$

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(c) $y = (\sin^{-1}x)^2 + a\sin^{-1}x + b$

$$\frac{dy}{dx} = 2\sin^{-1}x \cdot \frac{1}{\sqrt{1-x^2}} + a \cdot \frac{1}{\sqrt{1-x^2}} \quad 15$$

$$\sqrt{1-x^2}\,\frac{dy}{dx} = 2\sin^{-1}x + a \quad 5$$

$$\sqrt{1-x^2}\,\frac{d^2y}{dx^2} + \frac{dy}{dx} \cdot \frac{1}{2\sqrt{1-x^2}}(-2x) = 2 \cdot \frac{1}{\sqrt{1-x^2}} \quad 15$$

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 2$$

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(d) $x^2 y = a\cos nx$ ---(*)

$$x^2\frac{dy}{dx} + y \cdot 2x = a(-\sin nx) \cdot n \quad 10$$

$$x^2\frac{d^2y}{dx^2} + \frac{dy}{dx} \cdot 2x + 2\left\{x\frac{dy}{dx} + y \cdot 1\right\} = -an\cos nx \cdot n \quad 15$$

$$x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = -n^2x^2y \quad 5 \text{ (by(*))}$$

$$x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + (n^2x^2 + 2)y = 0$$

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14. (a) $\sin x - 3\sin 2x + \sin 3x = \cos x - 3\cos 2x + \cos 3x$

5 $2\sin 2x \cos x - 3\sin 2x = 2\cos 2x \cos x - 3\cos 2x$ 5

$\sin 2x\,(2\cos x - 3) = \cos 2x\,(2\cos x - 3)$ 5

$(2\cos x - 3)(\sin 2x - \cos 2x) = 0$ 5

$2\cos x - 3 = 0$ or $\sin 2x - \cos 2x = 0$

$\cos x = \frac{3}{2}$ $\qquad$ $\tan 2x = 1 = \tan\frac{\pi}{4}$ 5

impossible 5 $\qquad$ $2x = n\pi + \frac{\pi}{4}$

$x = \frac{n\pi}{2} + \frac{\pi}{8}$; 5 $n \in \mathbb{Z}$

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(b) $\tan^{-1}\left(\frac{x+1}{x+2}\right) + \tan^{-1}\left(\frac{x-1}{x-2}\right) = \frac{\pi}{4}$

Let $\alpha = \tan^{-1}\left(\frac{x+1}{x+2}\right)$

$\beta = \tan^{-1}\left(\frac{x-1}{x-2}\right)$ 5

$\alpha + \beta = \frac{\pi}{4}$

$\tan(\alpha+\beta) = 1$ 5

$$\frac{\left(\frac{x+1}{x+2}\right) + \left(\frac{x-1}{x-2}\right)}{1 - \left(\frac{x+1}{x+2}\right)\left(\frac{x-1}{x-2}\right)} = 1 \quad 10$$

$$\frac{(x+1)(x-2) + (x-1)(x+2)}{(x^2-4) - (x^2-1)} = 1 \quad 10$$

$$\frac{x^2 - x - 2 + x^2 + x - 2}{-3} = 1$$

$$\frac{2x^2 - 4}{-3} = 1 \quad 10$$

$$2x^2 = 1$$

$$x = \pm\frac{1}{\sqrt{2}} \quad 5$$

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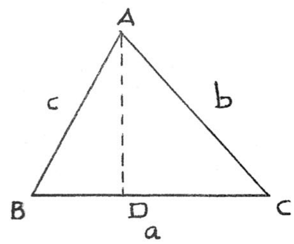
(c) sine rule with the usual notation :

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ 5

Situation I $\quad$ ABC is an acute-angled triangle

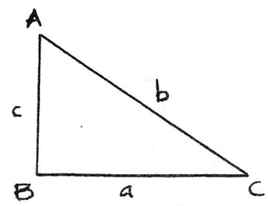


$AD = AB \sin B = AC \sin C$

$\therefore \frac{\sin B}{b} = \frac{\sin C}{c}$ 5

similarly, $\frac{\sin A}{a} = \frac{\sin C}{c}$

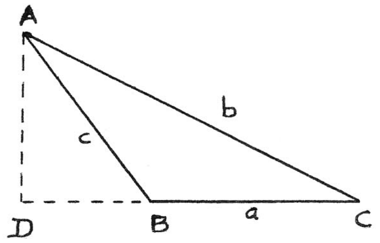
Situation II $\quad$ ABC is a right-angled triangle



$AB = AC \sin C = AB \sin B$

$\therefore \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$ 5

<u>Situation III</u> ABC is an obtuse-angled triangle



$AD = AB \sin(\pi - B) = AC \sin C$

$\therefore \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$ 5

$\boxed{20}$

Let $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C} = k.$

Consider $(a^2 - b^2)\cot C$

$= (k^2\sin^2 A - k^2\sin^2 B)\dfrac{\cos C}{\sin C}$ 5

$= \dfrac{k^2}{2}\{(1-\cos 2A)-(1-\cos 2B)\}\dfrac{\cos C}{\sin C}$ 10

$= \dfrac{k^2}{2}\{\cos 2B - \cos 2A\}\dfrac{\cos C}{\sin C}$

$= \dfrac{k^2}{2}\, 2\sin(A+B)\sin(A-B)\dfrac{\cos C}{\sin C}$ 10

$= -\dfrac{k^2}{2}\, 2\sin(A-B)\cos(A+B)$ 5

$= -\dfrac{k^2}{2}(\sin 2A - \sin 2B)$

$= \dfrac{k^2}{2}(\sin 2B - \sin 2A)$ 5

Similarly

$(b^2 - c^2)\cot A = \dfrac{k^2}{2}(\sin 2C - \sin 2B)$ 5

$(c^2 - a^2)\cot B = \dfrac{k^2}{2}(\sin 2A - \sin 2C)$ 5

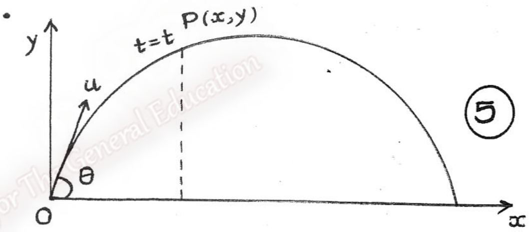
$\therefore (a^2-b^2)\cot C + (b^2-c^2)\cot A + (c^2-a^2)\cot B$

$= \dfrac{k^2}{2}\{\sin 2B - \sin 2A + \sin 2C - \sin 2B + \sin 2A - \sin 2C\}$ 5

$= 0$

$\boxed{50}$

15.



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$O \rightarrow P : \rightarrow \quad s = ut + \frac{1}{2}at^2$

$x = u\cos\theta \cdot t$ 10

$O \rightarrow P : \uparrow \quad s = ut + \frac{1}{2}at^2$

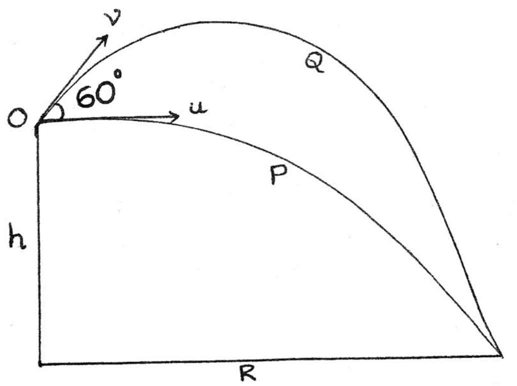
$y = u\sin\theta \cdot t - \frac{1}{2}gt^2$ 10

$= u\sin\theta \cdot \dfrac{x}{u\cos\theta} - \dfrac{1}{2}g \cdot \dfrac{x^2}{u^2\cos^2\theta}$ 10

$y = x\tan\theta - \dfrac{gx^2}{2u^2}\sec^2\theta$ 5

$\boxed{40}$

(i)



(10)

$y = x\tan\theta - \dfrac{gx^2}{2u^2}\sec^2\theta$

When $y = -h$, $x = R$, $\theta = 0$ for the particle P ; (15)

$-h = -\dfrac{1}{2}\dfrac{gR^2}{u^2}$ (15)

$R^2 = \dfrac{2u^2h}{g}$ (5)

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(ii) When $y = -h$, $x = R$, $\theta = 60°$ for the particle Q ; (15)

$-h = R\sqrt{3} - \dfrac{1}{2}\dfrac{gR^2}{v^2}\cdot 4$ (15)

$-h = \sqrt{3}R - \dfrac{2gR^2}{v^2}$

$h = \dfrac{2gR^2}{v^2} - \sqrt{3}R$ (5)

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(iii) $\sqrt{3}R = \dfrac{2gR^2}{v^2} - h$

$3R^2 = \left(\dfrac{2gR^2}{v^2} - h\right)^2$ (10)

$\dfrac{6u^2h}{g} = \left(\dfrac{4u^2h}{v^2} - h\right)^2$ (5)

$\dfrac{6u^2}{g} = \dfrac{h}{v^4}(4u^2 - v^2)^2$

$\sqrt{\dfrac{6}{gh}}\cdot u = \dfrac{4u^2}{v^2} - 1$ (10)

$\dfrac{4u}{v^2} - \dfrac{1}{u} = \sqrt{\dfrac{6}{gh}}$ (5)

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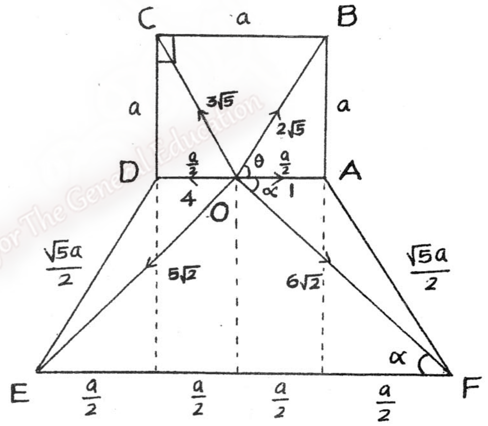
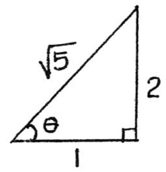
16.



(10)

$\tan\theta = 2$ (5)



$\alpha = 45°$ (5)

$\overrightarrow{OA}$ $\quad X = 1 + 2\sqrt{5}\cos\theta - 3\sqrt{5}\cos\theta$
$\quad -4 - 5\sqrt{2}\cos\alpha + 6\sqrt{2}\cos\alpha$ (15)

$= -3 - \sqrt{5}\cdot\dfrac{1}{\sqrt{5}} + \sqrt{2}\cdot\dfrac{1}{\sqrt{2}}$

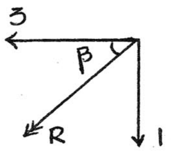
$= -3$ (5)

$OA \uparrow \quad Y = 2\sqrt{5}\sin\theta + 3\sqrt{5}\sin\theta$

$- 5\sqrt{2}\sin\alpha - 6\sqrt{2}\sin\alpha$ (15)

$= 5\sqrt{5}\cdot\dfrac{2}{\sqrt{5}} - 11\sqrt{2}\cdot\dfrac{1}{\sqrt{2}}$

$= -1$ (5)



$R = \sqrt{3^2+1^2}$

$= \sqrt{10}\ N$ (5)

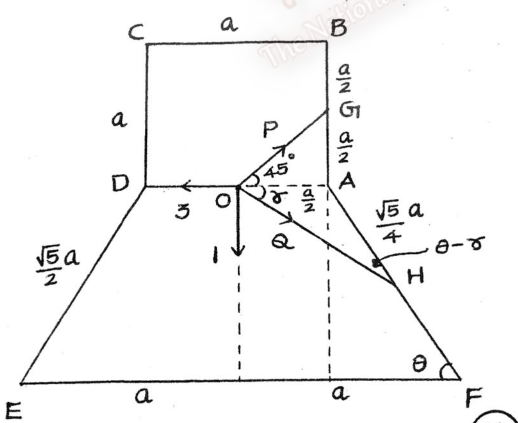
$\tan\beta = \dfrac{1}{3}$ (5)

Angle making with OA is

$\pi + \tan^{-1}\left(\dfrac{1}{3}\right)$.

(5) (5)

80



(5)

$A\hat{F}E = \theta$ (5)

$A\hat{H}O = \theta - \gamma$ (5)

By sin rule in $\triangle AOH$,

$$\frac{\frac{\sqrt{5}a}{4}}{\sin\gamma} = \frac{\frac{a}{2}}{\sin(\theta-\gamma)} \quad (5)$$

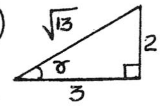
$$\frac{\sqrt{5}}{2\sin\gamma} = \frac{1}{\sin(\theta-\gamma)}$$

$$\sqrt{5}\left(\frac{2}{\sqrt{5}}\cos\gamma - \frac{1}{\sqrt{5}}\sin\gamma\right) = 2\sin\gamma \quad (5)$$

$$2\cos\gamma - \sin\gamma = 2\sin\gamma$$

$$2\cos\gamma = 3\sin\gamma$$

$$\tan\gamma = \frac{2}{3} \quad (5)$$



For the equilibrium,

$\rightarrow\ X = 0$ & $\uparrow\ Y = 0$.

(5) (5)

$\rightarrow \quad P\cos 45 + Q\cos\gamma - 3 = 0$ (10)

$$\frac{P}{\sqrt{2}} + Q\cdot\frac{3}{\sqrt{13}} = 3 \quad ---(1)$$

$\uparrow \quad P\sin 45 - Q\sin\gamma - 1 = 0$ (10)

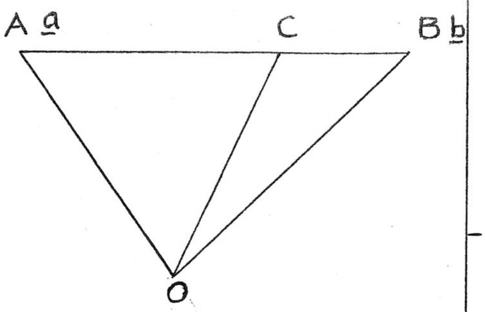
$$\frac{P}{\sqrt{2}} - Q\cdot\frac{2}{\sqrt{13}} = 1 \quad ---(2)$$

$(1), (2) \Rightarrow \quad P = \dfrac{9\sqrt{2}}{5}, \quad Q = \dfrac{2\sqrt{13}}{5}$

(5) (5)

70

17. (a)



$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC}$ 5

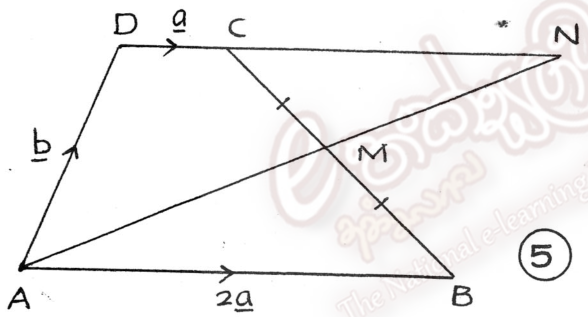
$= \underline{a} + \alpha \overrightarrow{AB}$ 5

$= \underline{a} + \alpha(\underline{b} - \underline{a})$ 5

$= (1-\alpha)\underline{a} + \alpha \underline{b}$

15

(i)



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$\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC}$

$= \overrightarrow{BA} + \overrightarrow{AD} + \overrightarrow{DC}$ 5

$= -2\underline{a} + \underline{b} + \underline{a}$

$= \underline{b} - \underline{a}$ 5

$\overrightarrow{AM} = \overrightarrow{AB} + \overrightarrow{BM}$

$= 2\underline{a} + \frac{1}{2}\overrightarrow{BC}$ 5

$= 2\underline{a} + \frac{1}{2}(\underline{b} - \underline{a})$

$= \frac{1}{2}(\underline{b} + 3\underline{a})$ 5

$\overrightarrow{AN} = \lambda \overrightarrow{AM}$

$= \lambda \cdot \frac{1}{2}(\underline{b} + 3\underline{a})$ 5

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(ii) $\overrightarrow{DN} = \mu \overrightarrow{DC}$

$= \mu \underline{a}$ 5

05

(iii) $\overrightarrow{AD} = \overrightarrow{AN} + \overrightarrow{ND}$ 5

$\underline{b} = \frac{\lambda}{2}(\underline{b} + 3\underline{a}) - \mu \underline{a}$

$\left(\mu - \frac{3\lambda}{2}\right)\underline{a} + \left(1 - \frac{\lambda}{2}\right)\underline{b} = \underline{0}$ 5

$\mu - \frac{3\lambda}{2} = 0$ & $1 - \frac{\lambda}{2} = 0$ 5

$\mu = 3$ & $\lambda = 2$ 5

20

(iv) $AN = 2AM$ 5

$\frac{AN}{AM} = \frac{2}{1}$

$\Rightarrow$ M is the mid-point of AN. 5

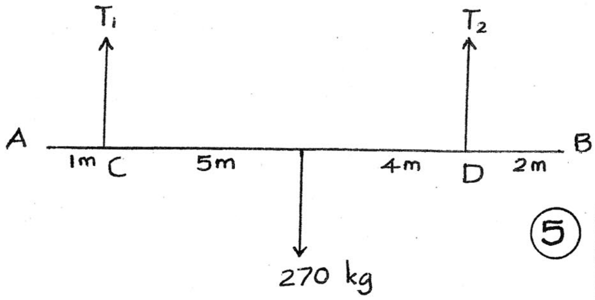
10

(v) $DN = 3DC$ 5

$\frac{DN}{CN} = \frac{1}{2}$ 5

10

(b)



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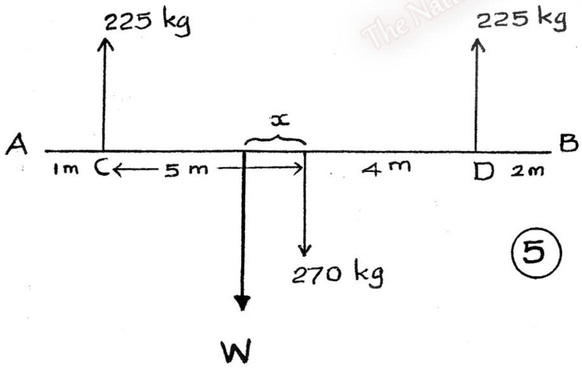
$\overset{\curvearrowleft}{C}$ $T_2 \cdot 9 - 270 \cdot 5 = 0$ 5

$T_2 = 150$ kg 5

$\uparrow$ $T_1 + T_2 = 270$ 5

$\therefore T_1 = 120$ kg 5

$\boxed{25}$



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$\downarrow$ $W + 270 = 450$ 5

$W = 180$ kg 5

$\boxed{15}$

$\overset{\curvearrowright}{C}$ $270 \cdot 5 + W(5 - x)$

$- 225 \cdot 9 = 0$ 10

$W(5 - x) = 675$

$180(5 - x) = 675$ 5

$x = 5 - 3.75$

$= 1.25$ m 5

$\boxed{20}$