



G.C.E. A/L Examination March - 2017

Conducted by Field Work Centre, Thondaimanaru

In Collaboration with

Provincial Department of Education North District.

Grade :- 12 (2018)

Combined Mathematics

Time :- Three hours

Instructions

- This question paper consists of two parts; Part A (questions 1 -10) and part B (questions 11- 17).

Part - A

- Answer all questions. Answers should be written in the space provided on the question paper. If additional space is needed, you may use additional answer sheets.

Part - B

- Answer only 5 questions.
- After the allocated time hand over the paper to the supervisor with both parts attached together.
- Only part B of the paper is allowed to be taken out of the examination hall.

Combined Maths		
Part	Question	Marks
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
	Total	

Combined Maths

Final Marks

Part – A

- (1) Let $a, b, c \in R$ and a, b and c are distinct.

Show that the quadratic function $y = x^2 - \frac{2}{3}(a + b + c)x + \frac{1}{3}(a^2 + b^2 + c^2)$ is above the x - axis for all values of x .

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- (2) Find the value of $\frac{1}{\log_{xy} xyz} + \frac{1}{\log_{yz} xyz} + \frac{1}{\log_{zx} xyz}$, where x , y and z are positive real numbers.

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The National Clean

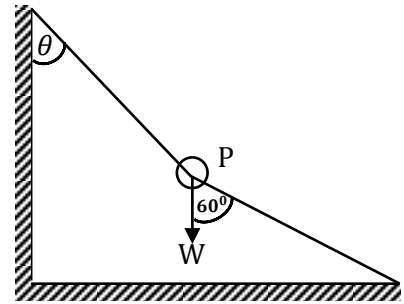
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- The General Education

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- (9) As shown in the diagram a particle P of weight W is attached to the ends of two strings one of which is at angle 60° with the vertical and the other is at angle $\theta (< 60^\circ)$ with the vertical, and it is in equilibrium. If the tension in the string attached to the ground is $2W$, find the tension in the string and θ .



- (10) The weights w_1 and w_2 are tied to the ends of a light rod AB . When suspended about a point C on the rod, the rod attains equilibrium horizontally. Find $AC : CB$.



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Part - B

(11) (a) Let the roots of the equation $x^2 - px + q = 0$ be α and β .

(i) Find the quadratic equation, in terms of p and q , whose roots are $\alpha + 3$ and $\beta + 3$.

(ii) Find the roots of the quadratic equation $x^2 - p(p^2 - 3q)x + q^3 = 0$ in terms of α and β .

(iii) If $x^2 - px + q = 0$ has distinct real roots, show that the equation $x^2 + (2k - p)x + q - kp = 0$ has distinct real roots for all real values of k .

(b) If a factor of $f(x) \equiv (k^2 - 3)x^4 + kx^3 - 2x^2 + k^2 + 4$ is $(x - 2)^2$, find the value of k .

For this value of k , show that $f(x) \geq 0$ for all real values of x .

(12) (a) Considering the equations $(2x)^{\ell n 2} = (3y)^{\ell n 3}$ and $3^{\ell n x} = 2^{\ell n y}$, show that $x = \frac{1}{2}$.

(b) Express $\frac{2x^3}{(x+1)(x-2)}$ in partial fractions.

(c) Show that the function $\frac{x+2}{x^2+3x+6}$ takes the values from $-\frac{1}{5}$ to $\frac{1}{3}$ for all real values of x .

(d) Show that $ab \leq \frac{1}{4}(a+b)^2$, where a and b are real.

Hence or otherwise, show that $\left(1 + \frac{1}{a}\right)\left(1 + \frac{1}{b}\right) \geq 9$, where $a, b > 0$ and $a + b = 1$.

(13) (a) From first principles, find the derivative of $\sqrt{1+x}$.

(b) Find $\frac{dy}{dx}$ in the following.

(i) $y = \frac{x^2-x+1}{x^2+x+1}$

(ii) $y = \tan^{-1} \left\{ \frac{1+\sin x}{1-\sin x} \right\}^{\frac{1}{2}}$, where $0 < x < \frac{\pi}{2}$

(c) If $y = (\sin^{-1}x)^2 + a \sin^{-1}x + b$, show that $(1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} = 2$, where a and b are constants.

(d) If $x^2y = a \cos nx$, show that

$$x^2 \frac{d^2y}{dx^2} + 4x \frac{dy}{dx} + (n^2x^2 + 2)y = 0 \text{ , where } a \text{ and } n \text{ are constants.}$$

(14) (a) Solve : $\sin x - 3 \sin 2x + \sin 3x = \cos x - 3 \cos 2x + \cos 3x$

(b) Solve : $\tan^{-1} \left(\frac{x+1}{x+2} \right) + \tan^{-1} \left(\frac{x-1}{x-2} \right) = \frac{\pi}{4}$

(c) With the usual notation in a triangle ABC , state and prove the **Sine Rule**.

With the usual notation, show that

$$(a^2-b^2) \cot C + (b^2-c^2) \cot A + (c^2-a^2) \cot B = 0 \text{ .}$$

(15) A particle is projected under gravity in a vertical plane with velocity u at angle θ to the horizontal from the origin O . If at $t = t$ the coordinates of the point on its path with respect to O are (x, y) , show that $y = x \tan \theta - \frac{gx^2}{2u^2} \sec^2 \theta$.

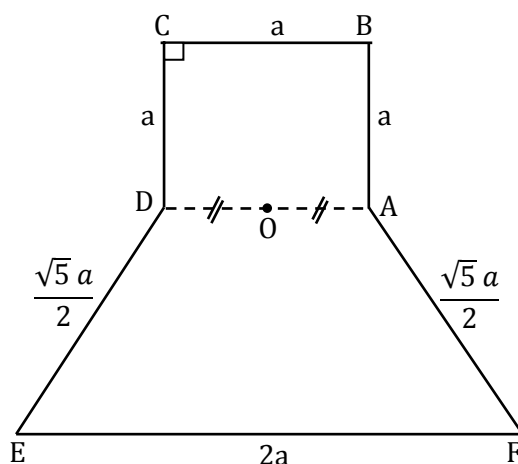
Two particles P and Q are projected upwards in the same vertical plane horizontally and at angle 60° to the horizontal respectively with velocities u and v respectively from a point O at a height h from the horizontal ground. If the both particles hit the horizontal ground at distance R at the same point,

(i) show that $R^2 = \frac{2u^2h}{g}$.

(ii) show that $h = \frac{2gR^2}{v^2} - \sqrt{3} R$.

(iii) show that $\frac{4u}{v^2} - \frac{1}{u} = \sqrt{\frac{6}{gh}}$.

(16)



On the plane shown in the diagram the forces of magnitude $1, 2\sqrt{5}, 3\sqrt{5}, 4, 5\sqrt{2}, 6\sqrt{2}$ N act along OA, OB, OC, OD, OE, OF respectively. Find the magnitude of the resultant and the angle it makes with OA .

Now, on the plane the forces of magnitude P, Q N act along OG, OH respectively; where G and H are the mid – points of AB and AF respectively. If the system is in equilibrium, find the values of P and Q .

- (17) (a) The position vectors of the points A and B are \mathbf{a} and \mathbf{b} respectively. Show that the position vector of any point C on AB can be written in the form of $(1 - \alpha)\mathbf{a} + \alpha\mathbf{b}$.

$ABCD$ is a trapezium. $AB \parallel DC$, $AB = 2DC$ and $AD = BC$. The mid – point of BC is M . The extended AM meets the extended DC at N . Taking that $AN = \lambda AM$, $DN = \mu DC$, $\overrightarrow{DC} = \mathbf{a}$ and $\overrightarrow{AD} = \mathbf{b}$,

- (i) find \overrightarrow{AN} in \mathbf{a} and \mathbf{b} .
- (ii) find \overrightarrow{DN} in \mathbf{a} and \mathbf{b} .
- (iii) using the vector addition, find λ and μ .
- (iv) hence, show that M is the mid – point of AN .
- (v) what is the ratio where C divides DN ?

- (b) A uniform rod AB of length 12 m and weight 270 kg is supported in a horizontal position by two vertical strings attached to the two points at distances 5 m, and 4 m from its mid – point. Find the tensions in the strings.

If the maximum weight which each string can sustain is 225 kg, find the maximum weight which can be suspended from the rod.

Find the distance between the point where this weight is suspended and the mid – point of the rod.