Provincial Department of Education
Northern Province
Pilot Exam-2021

Combined Mathematics II **Grade:13(2021)**

10 | E | II

Three Hours
Additional Reading Time : 10 minutes

Index No:

Instructions

- This question paper consists of two parts; Part A (questions 1 -10) and part B (questions 11- 17).

Part - A

- Answer all questions. Answers should be written in the space provided on the questions paper. If additional space needed, you may use additional answer sheets.

Part - B

- Answer only 5 questions.
- After the allocated time hand over the paper to the supervisor with both parts attached together.
- Only part B of the paper is allowed to be taken out of the examination hall.

Combined Mathematics II		
Part	Question	Marks
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
Total		

Combined Mathematics I	
Combined Mathematics II	
Total	
Final Marks	

Part – A

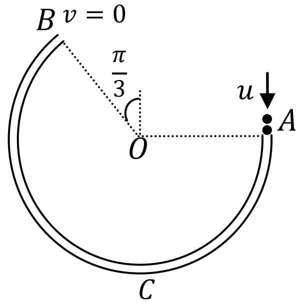
1. Impulse I is applied to a particle B of mass m which moves with the velocity u in a straight line. Show that the increase in kinetic energy is $\dfrac{I(I+2mu)}{2m}$.

2. A particle is projected with horizontal and vertical velocities u and v respectively. Show that the distance between the points which is at the same height h is $\dfrac{2u}{g}\sqrt{v^2-2gh}$.

3. Two particles of mass m and $3m$ are attached, to the ends of an inextensible string, which passes over a fixed smooth pulley. The system is held with the particles at the depths a and b below the pulley, and released from rest with string taut. Find the tension in the string and acceleration of the system.

4. A vehicle of mass $M\ kg$ travels on straight horizontal road attains a maximum speed $u\ ms^{-1}$. When the car travels upward on a straight road of inclination α which attains a maximum speed $\frac{u}{2}\ ms^{-1}$. The resistance $R = k\ w^2 N$ is same both motion; where w is the velocity of the vehicle in $ms^{-1}$and k is constant. show that $k = \frac{4Mg \sin\alpha}{7u^2}$

5. A thin smooth tube ACB in the shape of a circular arc of radius l that subtendeds an angle $\frac{7\pi}{6}$ at its centre O is fixed in a vertical plane with OA horizontal as shown in the figure. The particle P of mass m is gently released into the tube at the end A and the same time another particle of mass m collides vertically with downward velocity u with the particle P and coalesces with it. The composite particle just reaches the end B. Show that $u = 2\sqrt{gl}$.



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6. In the usual notation. Let $\underline{a}, \underline{b}$ and $(1-\lambda)\underline{a} + \lambda\underline{b}$ be the position vectors of three points A, B and C respectively with respect to a fixed origin O; where $\lambda \in \mathbb{R}$.
   (i) Show that A, B and C are collinear
   (ii) If $\underline{a} = \underline{i} + 5\underline{j}$, $\underline{b} = 4\underline{i} + 4\underline{j}$ and $OC \perp AB$ then find the value of λ.

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7. A heavy uniform rod AB of length $2a$ rests partly within and partly without a fixed smooth hemispherical bowl of radius r. The rim of the bowl is horizontal and one point of the rod is in contact with the rim. If θ is the inclination of the rod to the horizontal, Show that

(i) $2r\cos 2\theta = a\cos\theta$

(ii) $\theta < \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

8. A uniform sphere of weight w is held in equilibrium on a rough inclined plane of inclination α to the horizontal, by a horizontal string attached to the highest point of sphere and inclined plane. If the angle of friction between the sphere and the plane is λ. Show that $\alpha \leq 2\lambda$ and the tension in the string is $w\tan\frac{\alpha}{2}$.

9. Let A and B be any two events. If $P(A \cap B^1) = 0.20$, $P(A^1 \cap B) = 0.15$, $P(A \cap B) = 0.10$; where A' and B' are the complementary event of A and B respectively. Show that $P(A \cup B) = 0.45$ and $P(B/A) = \frac{1}{3}$.

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10. Suger is packed in polythene bags which the manufacturer claims, contains $50kg$ each. The following information is given for 100 such bags whose actual weights are not known. $\sum_{i=1}^{100}(x_i - 50) = 57.2$ and $\sum_{i=1}^{100}(x_i - 50)^2 = 95.1$, where $x_i\,(i = 1, 2, \ldots\ldots 100)$ denotes the actual weight of the i^{th} bag. Using an appropriate linear transformation or otherwise, find the mean and the variance of the actual weight of the 100 bags.

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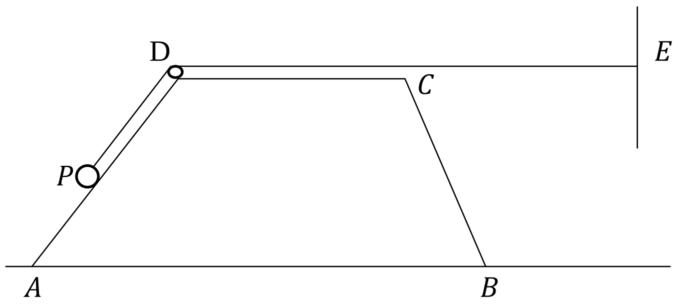
Part – B

Answer any five questions

11. (a) Two cars P and Q travel in the same direction; starting from rest from the same points. The car P acquires a maximum velocity in a time t, under a uniform acceleration f and continues its motion with that velocity. The car Q starts its motion a time $\frac{t}{2}$ after P started its motion and it moves with a uniform acceleration of $\frac{2f}{3}$. Draw the velocity time graph for each car on the same diagram. Hence, show that P has been in motion for a time $\frac{7t}{2}$ before Q over takes P. Also show that the distance travel of car P is $3ft^2$

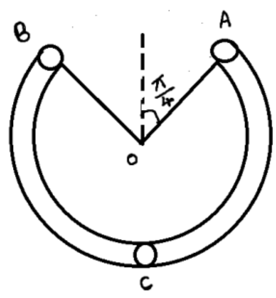
(b) A war ship sails in a straight course with a uniform speed. On a certain day a boat becomes visible at a distance $d\ km$ east of the war ship. The boat sails due north with a uniform speed of w km h^{-1}. The maximum speed attainable by the war ship is $u(< w)$ and the maximum range of its guns is $R\ km$. using the principle of relative velocity, Show that the boat is safe if $R < \frac{d}{w}\sqrt{w^2 - u^2}$; when $R > \frac{d}{w}\sqrt{w^2 - u^2}$ the war ship is sailing at maximum speed. Show that the time in which the boat can be attacked is $\frac{2\sqrt{R^2w^2 - d^2w^2 + d^2u^2}}{w\sqrt{w^2 - u^2}}$.

12. (a)



$ABCD$ is given vertical cross section through the center of mass of uniform wedge of mass m[where $AB \parallel DC,\ D\hat{A}B = \alpha$] The wedge is with face contains AB on smooth horizontal table. A particle P of mass $2m$ are placed near D at the highest inclination to the horizontal. particle P is joint to an inelastic string and the string passes over a smooth pulley at D and fixed to a point E on the wall, at the CD level. When the system is released from rest keeping string taut. The particle is always contact the surface of the wedge. Obtain equations sufficient to determine the reaction between the particle and the wedge. It is **given that** the reaction between the particle and the wedge is $\frac{2mg\,(2\cos\alpha - 1)(2 - \cos\alpha)}{5 - 4\cos\alpha}$. Show that $\alpha < \frac{\pi}{3}$.

(b) A thin small three-quarter smooth circular arc tube of radius a is fixed in a vertical plane with AB horozontal as shown in the figure; where OC is the vertical symmetric axis of the arc. A particle P of mass m is placed in the very low point on the inside of the tube, given the horizontal speed u to the particle, the particle P exists from A and falls inside the end B



(i) Show that $u^2 = 2ag\left(1+\sqrt{2}\right)$

(ii) Show that the height at which the particle reached above the absolute circle of the tube is $\frac{(3\sqrt{2}-4)a}{4}$.

13. The two points A and B are on a smooth horizontal table such that $AB = 8l$. A smooth particle P of mass m is placed in between A and B. The particle P is connected by light extensible string from the both side such as a light extensible string of natural length $\frac{12l}{5}$ and with the elastic modulus mg which tied with A and light extensible string of natural length $4l$ and with the elastic modulus $4mg$ which tied with B. Show that the particle P is in equilibrium at the point of distance $\frac{60l}{17}$ from A. If the particle P is placed on the point C such that $AP = \frac{12l}{5}$ and released from rest. Let x be the distance from A to the particle P after time t, for $\frac{12l}{5} \le x \le 4l$. Show that the equation for the motion of P can be given as $\ddot{x} + \frac{17g}{12l}\left(x - \frac{60l}{17}\right) = 0$. **Hence,** find the center of motion and amplitude. Find the velocity of P at $x = 4l$. Further, **using the conservation of energy** if the velocity of P is zero at $x = x_o$ $(> 4l)$ then show that $x_0 = \frac{12l}{5} + \frac{16\sqrt{3}l}{5\sqrt{5}}$.

Show that the equation of motion of P is given by $\ddot{x} + \frac{5g}{12l}\left(x - \frac{12l}{5}\right) = 0$ for $4l \le x \le x_0$. Find the center and amplitude of the motion.

14. (a) P and Q are two points such that $\overrightarrow{OP} = \underline{p}$ and $\overrightarrow{OQ} = \underline{q}$. R is the mid point of OP. X is a point on OQ such that $OX: XQ = 3: 1$ and Y is a point on PX such that $PY: YX = 4: 1$

(i) Find $\overrightarrow{OR}$, $\overrightarrow{OX}$ and $\overrightarrow{OY}$ in terms of $\underline{p}$ and $\underline{q}$.

(ii) Show that $\overrightarrow{QY} = \frac{1}{5}\left(\underline{p} - 2\underline{q}\right)$.

(iii) Show that Q, Y and R are collinear and calculate $QY: YR$.

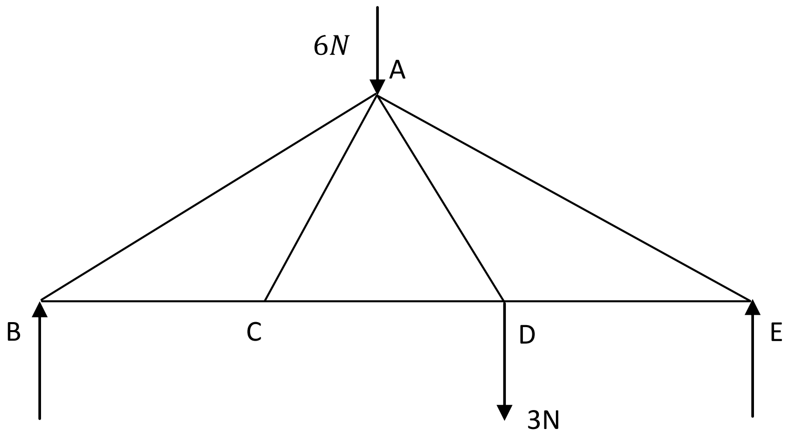
(b) Let $ABCDEF$ be a regular hexagon of side $2a$ metres. Forces of magnitudes $1, 2, 3, 4, 5$ and 6 newtons act along $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CD}$, $\overrightarrow{DE}$, $\overrightarrow{EF}$ and $\overrightarrow{FA}$ taken in order. Reduce the system to

(i) a force through the center of the hexagon and a couple.

(ii) a single force. Find the magnitude, direction and line of action of the single force.

15. (a) Three uniform rods AB,BC and CD each of length $2a$ and weight W are jointed freely at B and C. The rod BC is horizontal and the end A and D are kept on a smooth horizontal plane. Two light, inextensible strings of same length are connected to the midpoints of rods AB and CD and the other ends are connected to the midpoint of rod BC. The strings are kept taut and ABCD is in equilibrium in a vertical plane and $A\hat{B}C = 120^0$. Show that the tensions in the strings is 2W. Find the magnitude of the reaction at the joint B and show that the reaction makes an angle $\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$ with the horizontal.

(b)

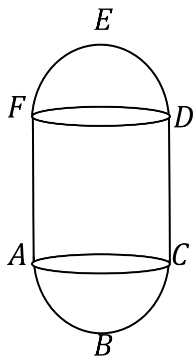


Seven light rods are freely jointed to form a vertical framework as shown in the figure. BC, CD, DE are in a horizontal position. The system is kept in equilibrium by vertical forces at B, E and carries loads $6N$, $3N$ at A, D. $BC = CD = DE = AC = AD$. Find the reaction at B and E. By using Bow's notation, Draw the stress diagram and find the stresses in all rods, distinguishing between tensions and thrusts.

16. (a) (i) Find by integration the position of center of mass of a uniform wire bent to the form of a semi – circle of radius r from its center

(ii) Find by integration the position of center of mass of a uniform solid hemisphere from its center

(b) As shown in the figure a decorative lantern is made from a uniform solid hemisphere ABC of radius $2a$ surmounted so as the flat surfaces coincide by a uniform hollow cylinder ACDF of radius $2a$ and height h, and a semi-circular wire attached across the diameter FD of the cylinder. If the ratio of density between the materials of sphere, cylinder and wire are $1 : a : a^2$. Find the distance of center of mass of this lantern from the circular edge of hemisphere



If this lantern can rest equilibrium with any part of the curved surface of hemisphere in contact with the horizontal plane find h in terms of a.

17. (a) Meat of broilers which were affected by bird flu were distributed by mistake to everyone who attended a banquet of foreign company. Because of that they have the symptoms of a serious illness. But they may not have the disease even if they have symptoms. The symptoms are severe headache or severe fever or diarrhea. A person could suffer only from one of those symptoms. Probabilty for these are $0.3,\ 0.2,\ 0.5$ respectively. Probabilty for one to get the severe disease with the symptoms are $0.1,\ 0.4,\ 0.5$ respectively. When a person is randomly selected, find the

(i) Probabilty of him affected by the disease.

(ii) Probabilty of a person who is affected by headache, given that he has disease.

(b) The set of n numbers $\{ x_1, x_2, \ldots\ldots\ldots x_n \}$ with mean $\bar{x}$ and standard deviation s_x is transformed to the set of n numbers $\{ y_1, y_2, \ldots\ldots\ldots y_n \}$ by means of the formula $y_i = a x_i + b$ for $i = 1, 2, \ldots\ldots n$ where a and b are constants. Let the mean and standard deviation of the set of n numbers $\{ y_1, y_2, \ldots\ldots\ldots y_n \}$ be $\bar{y}$, s_y respectively. Show that $\bar{y} = a\bar{x} + b,\ s_y = |a| s_x$.

The table below shows the means and standard deviation of the maks in Combined mathematics and Physics obtained by the candidates who sat a certain examination.

	Mean	standard deviation
Combined mathematics	m	15
Physics	45	p

Suppose that the maks in each subject were scaled linearly to have a mean of 50 and a standard deviation of 20. The original and the scaled marks if particular candidates are shown below.

	Original Marks	Scaled Marks
Combined mathematics	40	40
Physics	61	65

Find the values of m and the value of p. The candidates were allowed to apply for rescrutiny of their answer scripts. After rescrutiny, Combined Mathematics marks of 1% of the total number of candidates who sat for Combined Mathematics were changed. The mean of Combined Mathematics marks of the candidates whose marks were changed increase from 60 to 64. Find the mean of marks, after rescrutiny, of all candidates who sat for Combined Mathematics.