Provincial Department of Education
Northern Province
Pilot Exam-2021

Combined Mathematics I **Grade:13(2021)**

10	E	I

Three Hours
Additional Reading Time : 10 minutes

Index No: | | | | | | |

Instructions

- This question paper consists of two parts; Part A (questions 1 -10) and part B (questions 11- 17).

Part - A

- Answer all questions. Answers should be written in the space provided on the questions paper. If additional space needed, you may use additional answer sheets.

Part - B

- Answer only 5 questions.
- After the allocated time hand over the paper to the supervisor with both parts attached together.
- Only part B of the paper is allowed to be taken out of the examination hall.

Combined Mathematics I		
Part	Question	Marks
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
Total		

Combined Mathematics I	
Combined Mathematics II	
Total	
Final Marks	

Part–A

1. Using the principal of mathematical induction prove that. $\sum_{r=1}^{n}\frac{1}{r(r+1)}=\frac{n}{n+1}$ For all $n \in Z^{+}$

2. Sketch the graphs of $y=|3x+2|$ and $y=|x-1|+4$ in the same diagram hence or otherwise find the real values of x satisfying the inequality $|3x-1|-|x-2| \leq 4$

3. Determine the locus of the complex number Zwhich satisfies $|\mathcal{Z} - 2 - i| = |\mathcal{Z} + 4 - 9i|$ in an Argon diagram form minimum values of |Z|.

4. If the coefficient of x^tin the binomial expansion of $\left(x^3 - \frac{1}{x^2}\right)^n$. provethat $3n - t$ is multiple of five. Where $t \leq 3n$

5. Show that $\lim_{x \to 0} \frac{1 - \cos x \sqrt{\cos 2x}}{x^2} = \frac{3}{2}$

6. The graph of $y = \sin x$ $(0 \leq x \leq \pi)$ is rotated completely around the x- axis. Show that the volume generated by the above is $\frac{\pi^2}{2}$.

7. Show that equation of tangent drawn at the point $p \equiv (a\cos\alpha, b\sin\alpha)$ on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ for $a, b \in \mathbb{R}^+$ is given by $\frac{x}{a}\cos\alpha+\frac{y}{b}\sin\alpha=1$ If this tangent goes through the point $(2a, 0)$ find the value of α $(0<\alpha<\frac{\pi}{2})$

8. Let $A=(1,-1)$ and $B=(5,-3)$. The point $C=(2t,t), t \in \mathbb{R}$ lies on the perpendicular bisector of AB. Find the value of t and find the coordinates of the point D such that $ACBD$ is a rhombus.

9. Find the equation of the circle of radius 1unit which touches both the straight lines $x - y = 0$ and $y = 0$, and lies entirely in the first quadrant.

10. Write $\tan 2\alpha$ in terms of $\tan \alpha$ .Prove that $2\tan^{-1}\left(\frac{1}{5}\right) = \tan^{-1}\left(\frac{5}{12}\right)$

Hence show thattan $\left(2\tan^{-1}\frac{5}{12}\right)$=$\frac{120}{119}$

Part–B

Answer any five questions

11. a) Let $f(x)=(x-a)(x-b)-1$ where $b>a$.

i) Show that $f(x)$ has minimum.

ii) Find the minimum value of $f(x)$

iii) Sketch the rough diagram of $f(x)$

iv) Show that the roots of the equation $f(x)=0$ are in the intervals $(-\infty,a)$ and (b,∞)

b) Let $ac \neq bc$, α and β be the roots of the equation $x^2+ax+bc=0$, and γ and δ be the roots of the equation $x^2+bx+ac=0$, where $a, b, c \in \mathbb{R}$. Show that the quadratic equation whose roots are α and γ. $x^2+cx+ab=0$.

c) A polynomial function $P(x)$ is a trinomial function. Its leading coefficient is one. The remainders when $P(x)$ are divided by $(x-1)$ and $(x-3)$ are 7 and 13 respectively. Find the remainder when $P(x)$ is divided by $(x-1)(x-3)$.

If $P(2)=6$, when $P(x)$ is divided by $(x-1)(x-3)$, find the quotient and write the polynomial function $P(x)$.

12. (a) The twelve member's mobile corona team implemented a seven days program .the team included two motorists, four doctors, and six nurses' .A motorist, two doctors, four nurses must work on a particular day.

i. Find the total number of groups that can be setup.
ii. Find the number of groups that two nurses can work together
iii. Find the number of groups that can be formed by the refused of the two doctors to work together.
iv. A particular day work load is high, in addition a nurse and a doctor will be selected find the number of groups.

(b) Show that $\sum_{r=1}^{n} r = \frac{n(n+1)}{2}$ for $r \in \mathcal{Z}^+$

Given $u_r - u_{r-1} = 2r$ for $r \in \mathcal{Z}^+$ Show that $u_n = n^2+n-1$ *where* $u_1 = 1$

Let $v_r = \frac{u_r}{(r+2)!}$ for $r \in \mathcal{Z}^+$ Show that $\sum_{r=1}^{n} v_r = \frac{1}{2} - \frac{n+1}{(n+2)!}$ for $n \in \mathcal{Z}^+$

Hence the infinite series $\sum_{r=1}^{\infty} v_r$ convergent and find its sum.

13. (a) If $M = \begin{pmatrix} 1 & 3 \\ -2 & 3 \end{pmatrix}$ then show that $M^2 - 3M + 8I = 0$ **hence** find the M^{-1} where I is the 2x2 unit Matrix

If $A = \begin{pmatrix} -1 & -5 \\ 1 & 3 \end{pmatrix}$ find the value of $A^2 - 2A + 2I$

(b) Let Z , ω are complex numbers, prove that

i. $\overline{Z + \omega} = \overline{Z} + \overline{\omega}$
ii. $|Z|^2 = Z.\overline{\overline{Z}}$
iii. $Z + \overline{Z} = 2Re(Z)$

Hence Show that $|Z - \omega|^2 = |Z|^2 + |\omega|^2 - 2Re\ (Z\overline{\omega})$

(c) Using de Movie's Theorem for a positive integral index, show that if $z = \cos\theta + i\sin\theta$, then $z^{-n} = \cos n\,\theta - i\sin n\,\theta$, where $\theta \in \mathbb{R}$ and $n \in \mathbb{Z}^+$.

Express each of the complex numbers $-1 + i\sqrt{3}$ and $\sqrt{3} + i$ in the form $r(\cos\theta + i\sin\theta)$, where $r > 0$ and $-\pi < \theta \le \pi$.

Let $m, n \in \mathbb{Z}^+$. Show that if $\frac{(-1+i\sqrt{3})^n}{(\sqrt{3}+i)^m} = 8$, then $n = m + 3$ and $n = 4k - 1$, where $k \in \mathbb{Z}$.

14. (a) Let $f(x) = \frac{1-x}{x^2}$ for $x \neq 0$ show that the derivative of $f(x)$ is given by $f'(x) = \frac{1-x}{x^3}$. Hence find the interval in which $f(x)$ is increasing and decreasing. Find the coordinates of turning points. Given that for $x \neq 3$, $f''(x) = \frac{-2(x-3)}{x^4}$, find the coordinates of infection point. Sketch the graph of $y = f(x)$ by indicating the asymptotes , turning points and infection points.

(b) It was decided to make a tank of volume 45π , which has hemispherical lid protruding outside on a cylinder. If the radius and height of the cylinder are x and y units, show that the total area of the tank is given by $A = 3\pi x^2 + 2\pi xy$, Show that $y = \frac{45}{x^2} - \frac{2x}{3}$, further find the value of y for which A is minimum.

15. a) Find the constants A, B which satisfy

$x^2 = A(x-1)(x^2+4) + B(x^2+4) - \frac{4}{35}(2x-1)(x-1)^2$ for every $x \in \mathbb{R}$.

Hence write $\frac{x^2}{(x-1)^2(x^2+4)}$ in partial fractions and find $\int \frac{x^2}{(x-1)^2(x^2+4)} dx$

b) Using Integration by parts evaluate $\int e^{3x} \cos 4x\, dx$

c) using the formula $\int_0^a f(x)dx = \int_0^a f(a-x)\, dx$ Where 'a' is a constant

Hence Show that $\int_0^\pi \frac{x^2 \sin x}{(2x-\pi)(1+\cos^2 x)} dx = \frac{\pi^2}{4}$

16. Equation of the side OA of a rhombus $OABC\ is\ 4x-3y=0$. The equations of the diagonal OB is $x-y=0$. O is the origin. Find the Equation of OC. If $B=(5,5)$ Find the equations of , AC , BC and OC.
A circle lying entirely in the first quadrant with radius 1 unit is drawn to touch the sides OC and OA of the rhombus $OABC$.Find the equation of the circle and coordinates of center. Find the Equation of the chord of contact drawn to the circle from O.

17. a) State the sine rule with usual notation

In triangle ABC in the angle of bisector of $\hat{A}$ meet the line BC at D show that $\frac{CD}{BD}=\frac{AC}{AB}$

In the same triangle $2BC=AB=a, \hat{C}=\frac{\pi}{2}$ then Show that $CD=\frac{\sqrt{3}a}{2+\sqrt{3}}$

deduce that $\tan\frac{\pi}{12}=2-\sqrt{3}$

b) Given $\alpha=\tan^{-1}\frac{5}{12}$, $\beta=\tan^{-1}\frac{3}{4}$

Show that $\cos(\alpha-\beta)=\frac{63}{65}$ **deduce** the value of $\sin(\alpha-\beta)$

c) Solve: $\cos 2x-\sin 2x+2(\cos x-\sin x)+1=0$