30

Tessellation

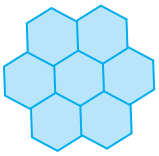
By studying this lesson, you will be able to,

- identify what regular tessellations and semi-regular tessellations are,
- select suitable polygons to create regular and semi-regular tessellations, and
- create regular and semi-regular tessellations.

30.1 Tessellation

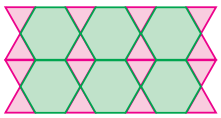
Let us recall what was learnt in Grade 7 about tessellation.

Covering a certain space using one or more shapes, in a repeated pattern, without gaps and without overlaps is called **tessellation**. An arrangement of shapes of this form is also called a tesselation.



If a tessellation consists of one shape only, it is called a **pure tessellation.**

If a tessellation consists of two or more shapes, it is called a **semi-pure tessellation**.



In tessellations where rectilinear plane figures are used, the sum of the angles around each vertex point is 360^{o}.

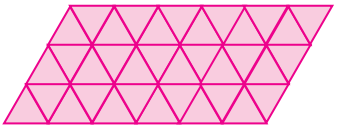
Therefore, the shapes that are selected for such tessellations should be such that the 360^{o} around a point on a plane can be covered without gaps and without overlaps with the selected shapes.

Do the following review exercise to revise the facts you have learnt previously on tessellation.

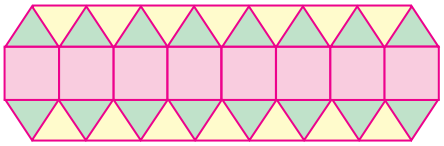
Review Exercise

(1) In your exercise book, draw a tessellation consisting of only equilateral triangular shapes.

(2) For each of the following tessellations, write with reasons whether it is a pure tessellation or a semi-pure tessellation.



(a)



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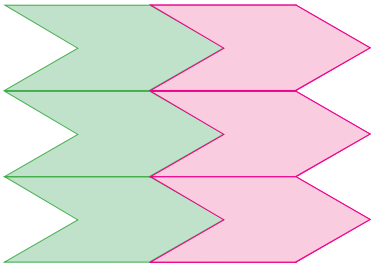

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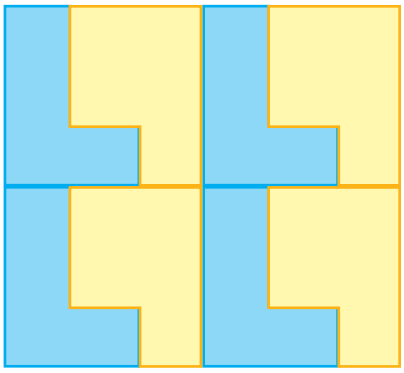

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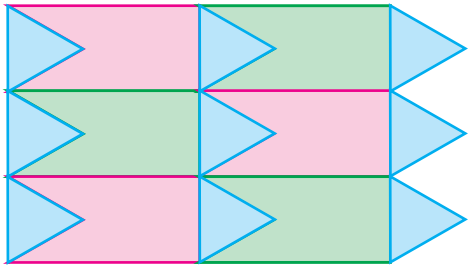
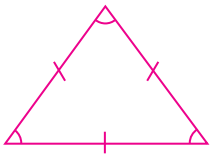
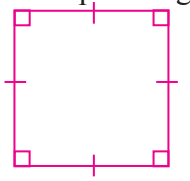


(e)

(3) Select and write the numbers of the plane figures which are regular polygons.



(i)



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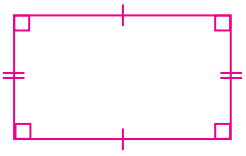

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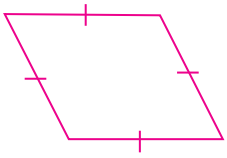

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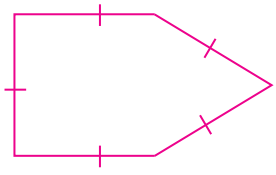

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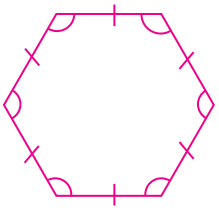


(vi)

30.2 Regular tessellation

We know that a polygon with sides of equal length and interior angles of equal magnitude is a regular polygon. Equilateral triangles, squares, regular pentagons and regular hexagons are examples of regular polygons.

A tessellation created using only one regular polygonal shape is known as a **regular tessellation**.

When creating a regular tessellation,

- a vertex of one geometrical shape should not be on a side of another geometrical shape.

In the tessellation in Figure 1 created with equilateral triangles, all the shapes are identical regular polygons. A vertex of any triangle is not located on a side of another triangle. Therefore this is a regular tessellation.

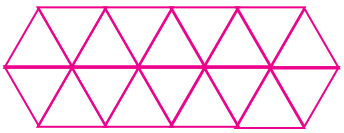


Figure 1

In the creation in Figure 2, although identical regular polygons have been used, the vertices of some polygons lie on the sides of other polygons. Therefore, this is not a regular tessellation.

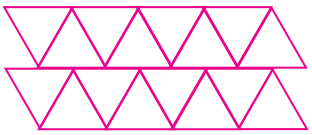
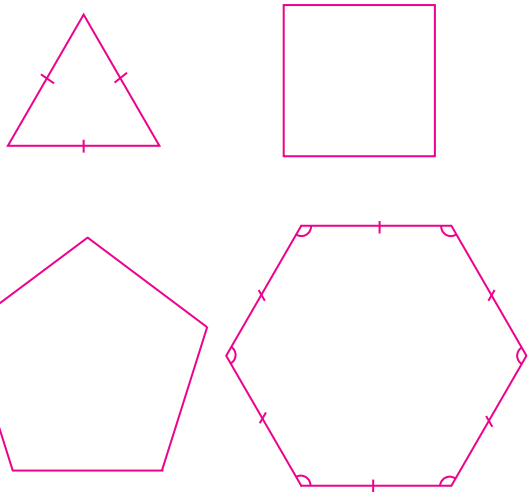


Figure 2

Activity 1

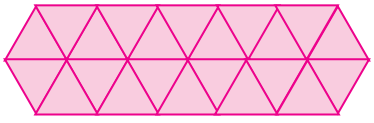
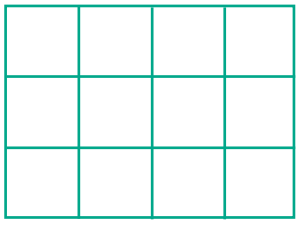
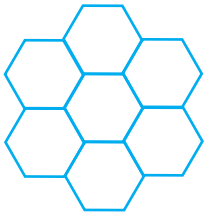
Step 1 - Trace these regular polygonal shapes onto coloured paper using a tissue paper and cut out 10 shapes of each kind.

Step 2 - Create a regular tessellation using only the triangular shapes and paste it in your exercise book.



Step 3 -	Examine each of the other shapes carefully and check whether a regular tessellation can be created.
Step 4 -	Using the shapes that were identified above as those with which regular tessellations can be created, create regular tessellations and paste them in your exercise book.
Step 5 -	Find out how many types of regular polygons can be used to create regular tessellations.
Step 6 -	Investigate the condition that needs to be satisfied by an interior angle of a regular polygon, to be able to create a regular tessellation with that polygon.

According to the above activity, we can create regular tessellations by using either equilateral triangles or squares or regular hexagons only.

In the creation of regular tessellations, the vertices of the regular shapes used should meet at particular points. These are called the vertices of the tessellations.

The sum of the angles around each vertex point of a tessellation is 360^{o}.

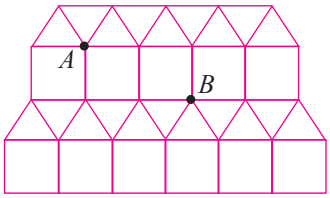
It must be clear to you through the above activity that a regular tessellation can be created by using a particular regular polygon, only if 360^{o} is a multiple of the magnitude of an interior angle of that polygon.

An interior angle of a regular pentagon is 108^{o}. Since 360^{0} is not divisible by 108^{o}, we cannot create a regular tessellation by using a regular pentagon.

30.3 Semi-regular tessellation

Tessellations created using two or more regular polygons, and such that the same polygons in the same order (when considered clockwise or anticlockwise) surround each vertex point are called **semi-regular tessellations.**

Given here is a semi-regular tessellation created using squares and equilateral triangles.

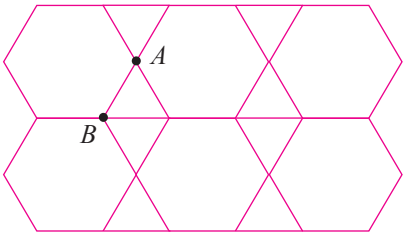


Observe how the polygons are positioned at the vertex points A and B. You can see that three triangular shapes and two square shapes meet at each of these two points. At both points, the three triangles and the two squares are positioned in the same order, with the three triangles together followed by the two squares next to each other.

This feature can be observed in the whole tessellation.

This is a feature of a semi-regular tessellation. That is, in a semi-regular tessellation, the same polygonal shapes should surround each vertex point and they should be positioned in the same order around these points.

This tessellation is made up of equilateral triangles and regular hexagons. Observe the vertex points A and B carefully. We can clearly see that the orders in which the polygons are positioned around these two points are different to each other.



Since the orders in which the shapes are positioned at different vertex points are not identical, this tessellation is **not a semi-regular tessellation**.

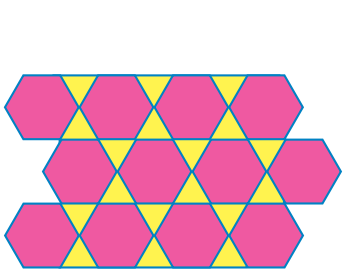
Activity 2

Step 1 - Cut out the shapes used in Activity 1 again using coloured paper.

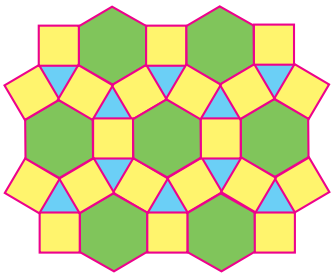
Step 2 - Create semi-regular tessellations using two types of shapes and paste them in your exercise book.

Step 3 - Create semi-regular tessellations using three types of shapes and paste them in your exercise book.

There are 8 types of semi-regular tessellations that can be created on a plane. They are given below.



(i)



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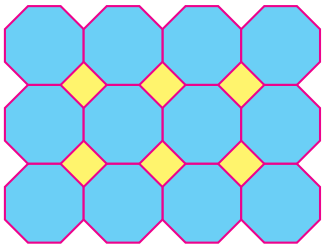
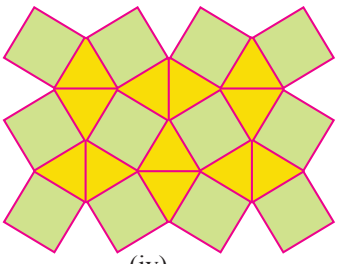
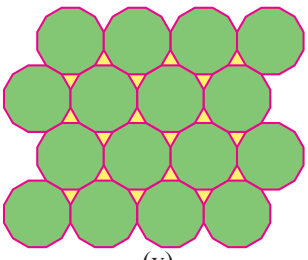


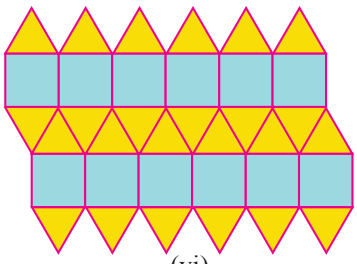
(iii)



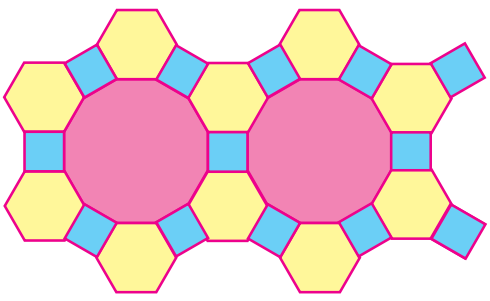
(iv)



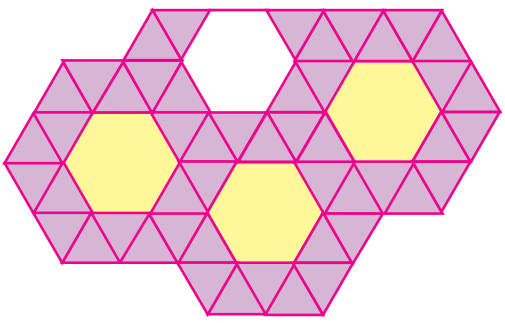
(v)



(vi)



(vii)



(viii)

Exercise 30.1

(1) (i) What are the regular polygons that can be used to create regular tessellations?

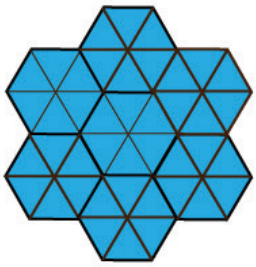
(ii) How many types of regular tessellations are there?

(iii) Each interior angle of a certain regular polygon is 98°. Explain whether a regular tessellation can be created using this polygon.

(2) Some figures are given below.

(i) Select and write the letters corresponding to the figures which are regular tessellations.

(ii) Select and write the letters corresponding to the figures which are semi-regular tessellations.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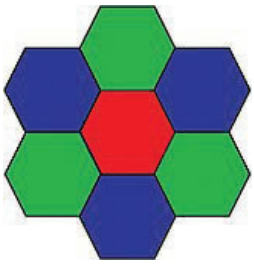
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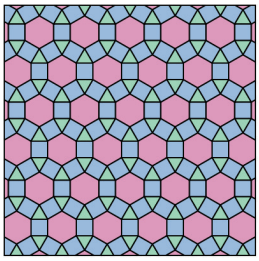
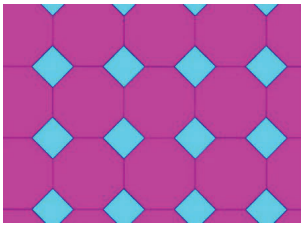

(c)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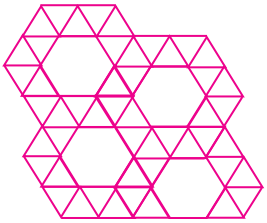
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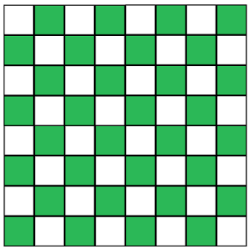
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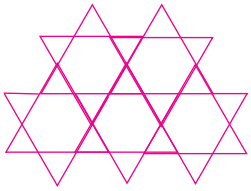
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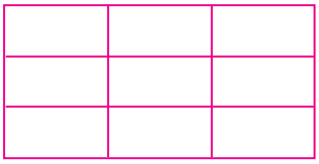
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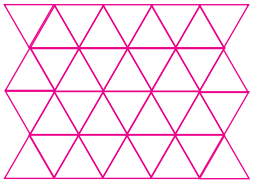
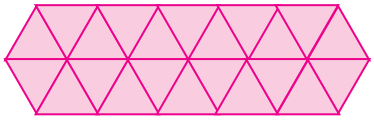
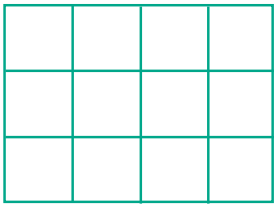
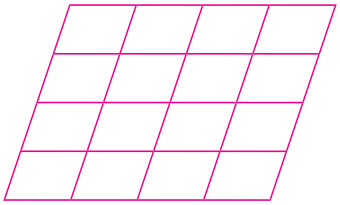

(i)


(j)


(k)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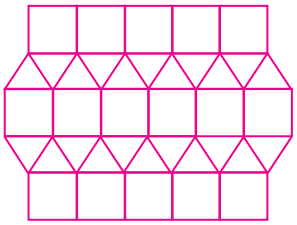
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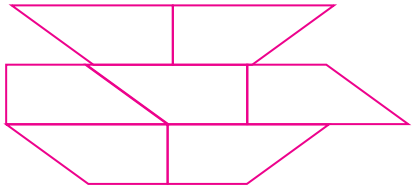
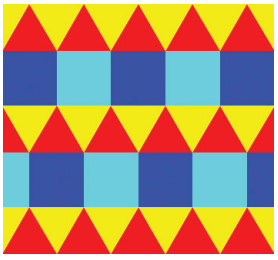
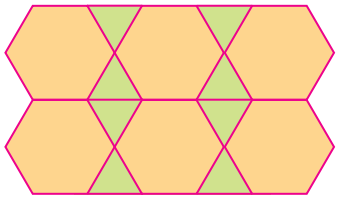

(n)


(o)



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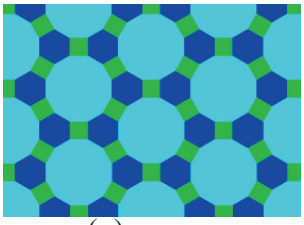

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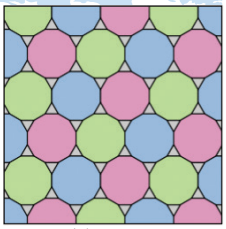
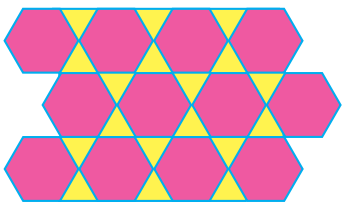
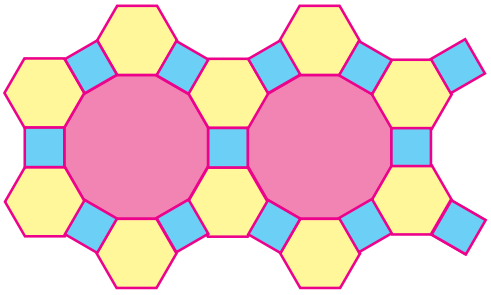


(r)

(3) Explain with reasons whether each of the following tessellations which have been created using regular polygons is a semi-regular tessellation or not.





Miscellaneous Exercise

Prepare several regular/semi-regular tessellations that are suitable for wall hangings.

Summary

- A tessellation created using only one regular polygonal shape is known as a regular tessellation.
- Tessellations created using two or more regular polygonal shapes, and such that the same polygons in the same order (when considered clockwise or anticlockwise) surround each vertex point are called semi-regular tessellations.

Revision Exercise 3

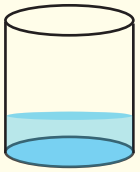
(1) A wax cube of side length 6 cm is given.

(i) Find the volume of the wax cube.

(ii) Write the above answer as a product of prime factors.

(iii) The given wax cube is melted and eight equal size cubes are made without wastage. If the side lengths of the two cubes are integral values, write the side length of each cube separately.

(2) The shaded portion of the cylindrical container in the figure contains 550 ml of water. Estimate the capacity of the container.



(3) The length, breadth and height of a cuboidal shaped container are 8 cm, 6 cm and 10 cm respectively. Find the following.

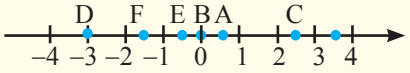
(i) The capacity of the container.

(ii) The volume of water in the container if water is filled up to a height of 6 cm.

(4) With the aid of figures, explain the terms given below which are related to circles.

- Chord
- Arc
- Sector
- Segment

(5) For each part given below, select the correct answer from within the brackets by considering the given number line.



(i) The number indicated by A is
$(1\frac{1}{2}\ ,\ -0.5,\ \frac{1}{2})$

(ii) The number indicated by F is
$(-2.5,\ -1.5,\ -3\frac{1}{2}\ )$

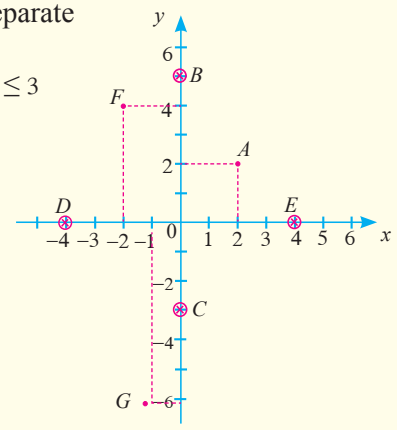
(iii) According to the numbers indicated by B and D,
(B > D, D > B)

(iv) According to the numbers indicated by C, D and E,
(C > E and D > E, D > E > C, D < E < C)

(6) Represent each of the following inequalities on a separate number line.

(i) $x > 2$ (ii) $x < -1$ (iii) $x \leq 3$ (iv) $-2 < x \leq 3$
(v) $0 \leq x < 5$

(7) Write the coordinates of the points A, B, C, D, E, F and G that are marked on the Cartesian plane.



(8) Draw a Cartesian plane with the x and y axes marked from -5 to 5.

(i) Draw the graphs of the straight lines given by $x = -2$, $y = 3$, $x = 5$ and $y = -4$ on the above Cartesian plane.

(ii) Write the coordinates of the points of intersection of the above graphs.

(9) From the sets of length measurements given below, write the sets that could be the lengths of the sides of a triangle.

(i) 4.2 cm , 5.3 cm, 6 cm

(ii) 12.3 cm , 5.7 cm, 6.6 cm

(iii) 8.5 cm , 3.7 cm, 4.3 cm

(iv) 15 cm , 9 cm, 12 cm

(10) Construct triangles with the following measurements as side lengths.

(i) 8 cm , 6 cm, 10 cm

(ii) 6.3 cm , 3.5 cm, 8.2 cm

(11) (i) Construct the triangle ABC such that $AB = 7.2$ cm, $BC = 5$ cm and $AC = 6.7$ cm.

(ii) Measure and write the magnitude of $A\hat{B}C$ in the above triangle.

(12) The lengths of the calls received on a certain day by a person who uses a mobile phone are given below to the nearest minute.

3, 2, 5, 10, 1, 3, 7, 3, 4, 6, 2, 4, 3, 8, 11, 4, 3, 2

(i) Write the range of the given set of data.

(ii) What is the mode?

(iii) Write the median.

(iv) Using the mean, estimate the time in hours and minutes that could be expected to be spent on 100 calls that are received by this person.

(13) Write the scales given below using a different method.

(i) Representing 100 m by 1 cm.

(ii) Representing 0.25 km by 1 cm.

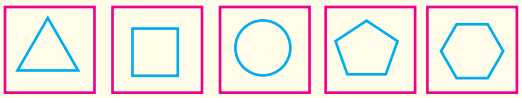
(iii) 1 : 50000

(v) Representing $\frac{3}{4}$ km by 1 cm.

(14) (i) In a scale diagram drawn to the scale 1: 50000, what is the actual distance in kilometres represented by 3.5 cm?

(ii) The scale selected to draw a scale diagram is 1: 0.5. Find the length of the straight line segment that needs to be drawn to represent 3.5 km.

(15) Three points A, B and C are located on a flat ground. B is situated 800 m away from A is 60^o east of north and C is situated 600 m away from B is 30^o east of south. Illustrate this information with a sketch.

(16) 

The figure shows five types of plane figures printed on 5 identical cards. The cards are mixed well and one card is picked randomly. The plane figure on the picked card is recorded and the card is replaced. Another card is picked randomly as before, and again the plane figure on it is recorded. The results obtained by conducting this experiment repeatedly are given in the following table.

Figure	△	□	○	⬠	⬡
Tally marks	𝍸 ///	𝍸 //	.....	𝍸 𝍸	𝍸 ////
Number of outcomes	.....	.....	9	.....	.....

(i) Copy the table and complete it.

(ii) How many times was this experiment repeated?

(iii) Write the fraction of success of obtaining the shape □ .

(iv) Draw the shape of the plane figure with the highest fraction of success.

(v) Draw the shapes of the plane figures with equal fractions of success and write this fraction.

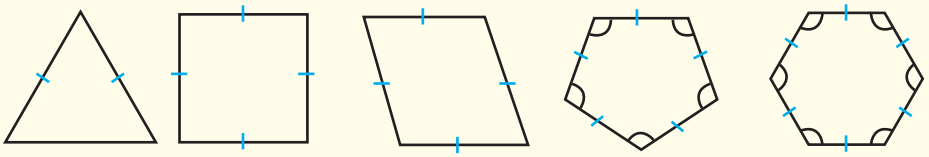
(17) A bag contains 2 red pens, 3 blue pens and 1 black pen of identical shape and size. A pen is taken out randomly. Find the probability of it being,

(i) a black pen.

(ii) a blue pen or a black pen

(iii) a green pen.

(18) From the given figures, select the shapes that can be used to create regular tessellations and write their corresponding letters.



(*a*) (*b*) (*c*) (*d*) (*e*)

(19) Copy each of the statements given below and place a "✓" before the statement if it is correct and a "×" if it is incorrect.

(i) A circle has no rotational symmetry.

(ii) Only rectilinear plane figures have rotational symmetry.

Glossary

Arc of a circle	වෘත්ත චාපය	வட்டவில்
Area	වර්ගඵලය	பரப்பளவு
Base	ආධාරකය	அடி
Capacity	ධාරිතාව	கொளள் ளவு
Cartesian co-ordinate plane	කාටිසීය ඛණ්ඩාංකතලය	தெகக் ாடடின்ஆளகூற்றுத்தத்ளம்
Centre	කේන්ද්‍රය	மையம்
Chord	ජ්‍යාය	நாண்
Circle	වෘත්තය	வட்டம்
Closed figures	සංවෘත රූප	மூடிய உரு
Commiunication	සන්නිවේදනය	தொடர்பாடல்
Continued ratios	සංයුක්ත අනුපාත	கூட்டுவிகிதம்
Compound solids	සංයුක්ත ඝනවස්තු	கூட்டுத்திண்மங்கள்
Construction	නිර්මාණය	அமைப்பு
Conversion	පරිවර්තනය	வகுப்பு எல்லை
Cube	ඝනකය	சதுரமுகி
Cuboid	ඝනකාභය	கனவுரு
Data	දත්ත	தரவு
Decimal numbers	දශම සංඛ්‍යා	தசம எண்கள்
Denomínator	හරය	பகுதி
Direction	දිශාව	திசை
Distance	දුර	தூரம்
Elements	අවයව	மூலகம
Events that do not occur	සිදු නොවන සිද්ධි	நடகக்ும்நிகழச்சிகள்
Events that definitely occur	ස්ථිර වශයෙන් සිදු වන සිද්ධි	
Events	සිද්ධි	நிகழ்ச்சிகள்
Experiment	පරීක්ෂණය	பரிசோதனை
Experimental probability	පරීක්ෂණාත්මක සම්භාවිතාව	பரிசோதனை முறை நிகழ்சசிகள்
Flow chart	ගැලීම් සටහන	பாய்ச்சற் கோட்டுப்படம்
Formula	සූත්‍රය	சூத்திரம்
Fraction of success	සාර්ථක භාගය	வெற்றிப்பின்னம்
Fraction	භාගය	பின்னம்
Fractions	භාග	பின்னம்
Greenwich meridian line	ග්‍රිනිච් මධ්‍යාහ්න රේඛාව	கிறின்வீச்கிடைக்கோடு
Greater than	වඩා විශාල	இலும் பெரிய
Perpendicular height (or altitude)	උච්චය	உயரம்
Infinite	අපරිමිත	முடிவிலி
International date line	ජාත්‍යන්තර දින රේඛාව	சர்வதேச திகதிக்கோடு

Latitude	අක්ෂාංශ	அகலக்கோடு
Location	පිහිටීම	அமைவு
Longitude	දේශාංශ	நெடுங்கோடு
Maximum value	උපරිම අගය	கூடிய பெறுமானம்
Minimum value	අවම අගය	குறைந்த பெறுமானம்
Null set	අභිශූන්‍ය කුලකය	வெறுந்தொடை
Number of elements of a set	කුලකයක අවයව සංඛ්‍යාව	மூலகங்களின் எண்ணிக்கை
Numerator	ලවය	தொகுதி
Ordered pairs	පටිපාටිගත යුගල	வரிசைப்பட்ட சோடி
Percentages	ප්‍රතිශත	சதவீதம்
Polygon	බහු අස්‍රය	பல்கோணி
Likelihood	විය හැකියාව	இயல்தகவு
Probability	සම්භාවිතාව	நிகழ்தகவு
Protractor	කෝණ මානය	பாகைமாணி
Quadrant	වෘත්ත පාදක	காற்பகுதி
Random events	සමහර විට සිදු වන සිද්ධි (අහඹු සිද්ධි)	சிலவேளை நடக்கும்நிகழ்ச்சிகள் (எழுமாறான நிகழ்ச்சிகள)
Range	පරාසය	எண் தொடரி
Ratio	අනුපාතය	விகிதம்
Rectangle	සෘජුකෝණාස්‍රය	செவ்வகம்
Right angled triangle	සෘජුකෝණි ත්‍රිකෝණය	செங்கோண முக்கோணி
Regular tessellation	සවිධි ටෙසලාකරණ	ஒழுங்கான தெசலாக்கம்
Rough sketch	දළ සටහන	பரும்படி படம்
Scale	පරිමාණය	அளவிடை
Sector of a circle	කේන්ද්‍රික ඛණ්ඩය	ஆரைச்சிறை
Segment of a circle	වෘත්ත ඛණ්ඩය	வட்டத்துண்டம்
Semi-regular tessellation	අර්ධ සවිධි සෙලාකරණ	அரைத் தூய தெசலாக்கமி
Set	කුලකය	தொடை
Sides of a triangle	ත්‍රිකෝණයක පාද	முக்கோணியின் பக்கங்கள்
Simple equation	සරල සමීකරණ	எளிய சமன்பாடுகள்
Solution	විසඳුම	தீர்வு
Square	සමචතුරස්‍රය	சதுரம்
Stem and leaf diagram	වෘන්ත පත්‍ර සටහන	தண்டு - இலை வரைபு
Symmetry	සමමිතිය	சமச்சீர்
Tesselation	ටෙසලාකරණ	தெசலாக்கம்
Theoretical probability	සෛද්ධාන්තික සම්භාවිතාව	அறிமுறை நிகழ்தகவு
Time zones	කාල කලාප	காலவலயம்
Triangle	ත්‍රිකෝණය	முக்கோணி
True length	සැබෑ දිග	உண்மை நீளம்
Unknown	අඥාතය	தெரியாக்கணியம்
Volume	පරිමාව	கனவளவு