## Scale Diagrams

By studying this lesson you will be able to,

- identify what a scale diagram is,
- calculate the actual lengths of a rectilinear plane figure which has been drawn according to a given scale, and
- draw a scale diagram according to a given scale, when the actual measurements of a rectilinear plane figure are given.


### 28.1 Scale diagrams

It is often difficult to draw the various objects in the environment according to the actual measurements.

In such situations, for every rectilinear plane figure,
(i) a figure drawn to represent the shape of the original figure is called a sketch, and
(ii) when a rectilinear plane figure is drawn such that every measurement of length is increased or decreased by the same ratio, the drawn figure is called a scale diagram of the given figure.

The shape of a figure in a scale diagram is exactly the same as the shape of the original figure, and only the size is different.


The floor plan of a house is drawn by decreasing the measurements

Let us recall the facts that were learnt on scale diagrams of rectangular shapes by considering the sketch given below.

The length and breadth of the floor of a rectangular room is 5 m and 4 m respectively. A sketch of it is given here.


A scale diagram of the floor of the room can be drawn in your exercise book by representing an actual measurement of 1 m by 1 cm . Since 1 m is $100 \mathrm{~cm}, 1 \mathrm{~cm}$ in the scale diagram represents 100 cm of the floor. This is represented as a ratio by 1 : 100. This ratio is known as the scale of the scale diagram.


The actual length of 5 m is represented by 5 cm in the scale diagram, and the actual length of 4 m is represented by 4 cm in the scale diagram.

Scale 1: 100
Do the following review exercise to recall these facts which you have learnt previously.

## Review Exercise

(1) Write the scale given in each of the following parts as a ratio.
(i) Representing 50 cm of the actual length by 1 cm in the scale drawing.
(ii) Representing 2 m of the actual length by 1 cm in the scale drawing.
(iii) Representing 100 m of the actual length by 2 cm in the scale drawing.
(iv) Representing 1 mm of the actual length by 5 cm in the scale drawing.
(2) A scale diagram drawn to the scale 1:200 is given here.
(i) Find the actual length represented by 1 cm according to the given scale.
(ii) Find the length and breadth of the actual figure that is represented by this scale diagram.

(ii) Draw a scale diagram of the floor plan using this scale.

### 28.2 Calculating the lengths corresponding to actual lengths when the scale of a scale diagram is given

Suppose we want to draw a scale diagram of the rectangular floor of a hall of length 6 m and breadth 4 m , using the scale $1: 200$. Let us find the length of each side in the scale diagram.
In the scale diagram,
an actual length of $200 \mathrm{~cm}=1 \mathrm{~cm}$
an actual length of $600 \mathrm{~cm}=\frac{1}{200} \times 600 \mathrm{~cm}=3 \mathrm{~cm}$
an actual length of $400 \mathrm{~cm}=\frac{1}{200} \times 400 \mathrm{~cm}=2 \mathrm{~cm}$
Since, $400 \mathrm{~cm}=4 \mathrm{~m}$ and $6 \mathrm{~m}=600 \mathrm{~cm}$, the length of the floor in the scale diagram is 3 cm and the breadth is 2 cm .
i.e., the lengths 6 m and 4 m are represented by 3 cm and 2 cm respectively in the scale diagram.

The figure shows a sketch of a right triangular vegetable plot.

Let us find the lengths of the boundaries of this plot in the scale diagrams which are drawn according to the different scales given below.


|  | Lengths in the scale diagram according to the given scale |  |  |
| :---: | :---: | :---: | :---: |
| boundaries of the vegetable plot | 1 m is represented <br> by $1 \mathrm{~cm}(1: 100)$ | 2 m is represented by $1 \mathrm{~cm}(1: 200)$ | $\frac{1}{2} \mathbf{m}$ is represented by 1 cm (1:50) |
| $\begin{aligned} & 10 \mathrm{~m} \\ & 8 \mathrm{~m} \\ & 6 \mathrm{~m} \end{aligned}$ | $\begin{aligned} & \frac{1000}{100} \mathrm{~cm}=10 \mathrm{~cm} \\ & \frac{800}{100} \mathrm{~cm}=8 \mathrm{~cm} \\ & \frac{600}{100} \mathrm{~cm}=6 \mathrm{~cm} \end{aligned}$ | $\begin{aligned} & \frac{1000}{200} \mathrm{~cm}=5 \mathrm{~cm} \\ & \frac{800}{200} \mathrm{~cm}=4 \mathrm{~cm} \\ & \frac{600}{200} \mathrm{~cm}=3 \mathrm{~cm} \end{aligned}$ | $\begin{aligned} & \frac{1000}{50} \mathrm{~cm}=20 \mathrm{~cm} \\ & \frac{800}{50} \mathrm{~cm}=16 \mathrm{~cm} \\ & \frac{600}{50} \mathrm{~cm}=12 \mathrm{~cm} \end{aligned}$ |

The scale $1: 50$ can be used if a larger scale diagram is required, and for a smaller scale diagram, the scale 1:100 can be used.


A sketch of a triangular plot of land is shown in the figure. If a scale diagram of this plot is to be drawn to the scale $1: 2500$, find the length of each boundary of this plot in the scale diagram.

The scale is $1: 2500$.
Since $2500 \mathrm{~cm}=25 \mathrm{~m}$, an actual length of 25 m is represented by 1 cm in the scale diagram.
$\therefore$ In the scale diagram,
a length of 200 m is represented by $\frac{200}{25} \mathrm{~cm}$, that is by 8 cm ,
a length of 150 m is represented by $\frac{150}{25} \mathrm{~cm}$, that is by 6 cm ,
a length of 125 m is represented by $\frac{125}{25} \mathrm{~cm}$, that is by 5 cm .

## Example 2

By what length is an actual length of 250 m represented in a scale diagram which has been drawn to the scale $1: 10000$ ?

1 cm represents 10000 cm .
1 cm in the scale diagram represents an actual length of 100 m .
That is, an actual length of 100 m is represented by 1 cm in the scale diagram.
$\therefore$ In the scale diagram, an actual length of 250 m is represented by $\frac{250}{100}=2.5 \mathrm{~cm}$.

## Exercise 28.1

(1) The sketches of two flower beds $A B C D$ and $P Q R S T$ are shown in the figure. Complete the table using the given information.


Figure i


Figure ii

| Figure | Scale | Actual length | Lengths in the scale diagram |
| :---: | :---: | :---: | :---: |
| (i) | 1: 1000 | 30 m | ............ |
|  |  | 20 m | $\ldots . . . . .$. |
|  |  | 40 m | ............. |
|  | 1:500 | 30 m | $\ldots . . . . .$. |
|  |  | 20 m | ............ |
|  |  | 40 m | ............. |
| (ii) | 1:10 | 20 m | $\ldots . . . . . . . . .$. |
|  |  | 50 m | $\ldots . . . . . . . . .$. |
|  |  | 30 m | ............. |

(2) (i) Write as a ratio the scale of 4 mm represented by 1 cm .
(ii) The measurements of a small triangular opening which is to be drawn according to the above scale is given in the sketch. Find the length of each side of the triangle in the scale diagram.

sketch

### 28.3 Determining the actual lengths when a scale diagram is given

You learnt in Grade 7 how to find the actual lengths when a scale diagram is given. Now, let us study this further.

## Example 1

The scale diagram of a flower bed $A B C D$ drawn to the scale 1:500 is given here.
(i) Find the actual lengths of all four sides.
(ii) Calculate the actual length of the drain $A C$ that has been cut across the flower bed.

The scale is given as 1:500.

$\therefore 1 \mathrm{~cm}$ in the scale drawing represents an actual length of 500 cm , which is 5 m .
$\therefore$ Actual length of $A B=3 \times 5 \mathrm{~m}=15 \mathrm{~m}$
Actual length of $B C=4 \times 5 \mathrm{~m}=20 \mathrm{~m}$
Actual length of $D C=5 \times 5 \mathrm{~m}=25 \mathrm{~m}$
Actual length of $A D=4.5 \times 5 \mathrm{~m}=22.5 \mathrm{~m}$
(ii) The length of the drain $A C=5 \times 5 \mathrm{~m}=25 \mathrm{~m}$

## Exercise 28.2

(1) A scale diagram of an equilateral triangular flower bed is drawn to the scale $1: 100$.
(i) Find the actual length indicated by 1 cm in the scale diagram.
(ii) Find the actual length of a boundary of the flower bed.
(iii) Find the actual perimeter of the flower bed.

(2) A map of Sri Lanka has been drawn to the scale 1:50000. What is the actual distance in kilometers between two towns which are 4 cm apart in the scale diagram?
(3) The figure shows a scale diagram which has been drawn based on the measurements of a playground. The scale of the drawing is $1: 20000$.
(i) Calculate the actual length of the side $P Q$ of the play ground.
(ii) How much longer is the side $Q R$ than the side $P Q$ of the actual playground?

(4) A scale diagram of a ship drawn to the scale 1: 1000 is shown here. Find the actual length of the ship.

(5) A scale diagram of a car drawn to the scale 1:60 is shown here.
(i) Find the actual length of the car.
(ii) Find the actual diameter of a wheel of
 the car.
(iii) Find the actual breadth of a door.
(7) A scale diagram of an insect drawn to the scale $1: 0.25$ is given in the figure. Find the actual lengths represented by the lengths in the scale diagram.


### 23.4 Drawing scale diagrams

A sketch of a triangular flower bed $A B C$ is shown in the figure.
 Let us select a suitable scale to draw a scale diagram of it. If 1 cm represents 2 m of an actual length, the scale is 1:200.

Let us follow the steps given below to draw the scale diagram.
The actual length represented by 1 cm in the scale diagram $=200 \mathrm{~cm}=2 \mathrm{~m}$

Step 1 - Let us calculate the length of each side of the scale drawing.

An actual length of 10 m is equal to $\frac{10}{2} \mathrm{~cm}=5 \mathrm{~cm}$ in the scale diagram.
An actual length of 8 m is equal to $\frac{8}{2} \mathrm{~cm}=4 \mathrm{~cm}$ in the scale diagram.


An actual length of 9 m is equal to $\frac{9}{2} \mathrm{~cm}=4.5 \mathrm{~cm}$ in the scale diagram.
Step 2 - Using the knowledge gained in the lesson on the construction of triangles, construct the triangle $A B C$ with sides of length $5 \mathrm{~cm}, 4 \mathrm{~cm}$ and 4.5 cm .

## Exercise 28.3

(1) Draw scale diagrams of each of the figures given in the following sketches, to the given scale.


Scale 1: 10
(iii)


Scale 1: 1000
(ii)


Scale 1: 100
(iv)


Scale 1: 2000

## Miscellaneous Exercise

(1) A scale diagram of a side wall of a building is shown in the figure. It has been drawn to the scale $1: 600$.
(i) Find the actual breadth of the wall.
(ii) Calculate the distance to the top of the building from ground level.
(iii) It costs Rs. 45 to paint $1 \mathrm{~m}^{2}$ of the wall. Find the total cost of painting one side of the wall
 completely.
(2) Draw scale diagrams of each of the figures given in the following sketches, to the given scale.
(i)


Scale 1: 200
(ii)


Scale 1: 10000

## Summary

The scale of a scale diagram indicates the actual length that is represented by a unit length in the scale diagram. This scale is often given as a ratio. for every rectilinear plane figure,
(i) a figure drawn to represent the shape of the original figure is called a sketch, and
(ii) when a rectilinear plane figure is drawn such that every measurement of length is increased or decreased by the same ratio, the drawn figure is called a scale diagram of the given figure.

