26 Construction of triangles

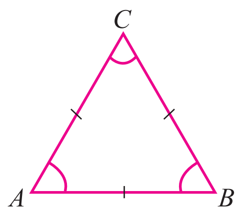
By studying this lesson you will be able to,
- identify that the sum of the lengths of any two sides of a triangle is greater than the length of the remaining side, and
- construct a triangle when the lengths of all the sides are given.

26.1 Introduction

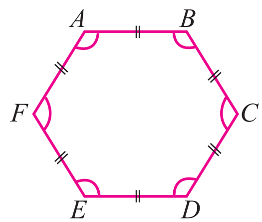
In geometry, we need to draw and construct plane figures. When constructing a plane figure, we have to construct it according to the given conditions.

In Grade 7 you learnt to construct a straight line segment of given length, an equilateral triangle of given side length, and a regular hexagon by means of equilateral triangles or a circle.

A straight line segment



An equilateral triangle



A regular hexagon

- Let us recall the steps that need to be performed to construct an equilateral triangle.
  - Construct a straight line segment.
  - Taking the same length as that of the straight line segment onto the pair of compasses, construct an arc by placing the point of the pair of compasses at one end of the above line segment.
  - Construct an arc from the other end point using the same length as above, such that it intersects the earlier arc.
  - Join the intersection point of the arcs to the end points of the straight line segment.

- A regular hexagon can be constructed by performing the following steps.
  - Construct a circle.
  - Divide the circle into 6 equal parts by intersecting the circle with arcs of the same length as the radius of the circle
  - Join the points of intersection.

Do the following review exercise to recall these facts which you learnt in Grade 7.

Review Exercise

(1) Construct the straight line segment AB of length 7.9 cm.

(2) Construct an equilateral triangle of side length 5.4 cm.

(3) (i) Construct a circle of radius 4 cm and center O.

(ii) Construct the regular hexagon $ABCDEF$ of side length 4 cm such that its vertices lie on the above constructed circle.

(4) Construct a regular hexagon of side length 5 cm.

26.2 Identifying the condition for three line segments to be the sides of a triangle

The figure represents a paddy field bounded by AB, BC and CA which are straight paths around it. Nimali is at A and her puppy is at B. Nimali has two routes to get from A to her puppy at B. Identify the two routes and determine which is shorter.

It is easy to establish that the shorter route is along AB. This means that the sum of the distances AC and CB is greater than the distance AB.

Do the following activity to find a condition to determine whether three line segments of given length can be the sides of a triangle.

Activity 1

Step 1 - Take pieces of ekel of length 3 cm, 4 cm, 5 cm, 7 cm and 9 cm respectively.

Step 2 - Pick any three pieces of ekel, place them on a table and see whether a triangle can be formed with the three pieces, such that their endpoints meet.

Step 3 - Complete a row of the following table by first noting down the lengths of the three pieces of ekel you picked.

Step 4 - Repeat the above steps several times over.

Lengths of the three piece of ekel (in cm)	The sum of the lengths of two pieces (in cm)	Length of the third piece (in cm)	Relationship between the values in the second and the third columns	If a triangle can be formed place a ✓. If not, place a ×.
3, 4, 5	7 9 8	5 3 4	$7 > 5$ $9 > 3$ $8 > 4$	✓
3, 4, 9	7 13 12	9 3 4	$7 < 9$ $13 > 3$ $12 > 4$	×
3, 7, 9				
4, 5, 7				

According to the table you have completed, it is clear that it is not always possible to construct a triangle with the three pieces of ekel that were selected.

However, it is possible to construct a triangle with three pieces of ekel, if the sum of the lengths of any two of them is greater than the length of the third.

It is clear that the sum of the lengths of any two sides of a triangle is greater than the length of the remaining side, and that given three straight line segments, if the sum of the lengths of any two of them is less than that of the third, then we cannot construct a triangle with line segments of these lengths as the sides.

Exercise 26.1

(1) Choose the triples from the following groups that can be the lengths of the sides of a triangle.

(a) For each triple that was selected, write the reason for your choice.
(b) For the triples that were not selected, write the reason for not selecting them.

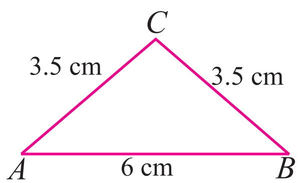
(i) 5 cm, 6 cm, 7 cm (ii) 4 cm, 4 cm, 4 cm (iii) 4 cm, 4 cm, 8 cm
(iv) 3 cm, 3 cm, 7 cm (v) 5 cm, 5 cm, 8 cm (vi) 6 cm, 4 cm, 10 cm

26.3 Construction of triangles

In Grade 7 you learnt how to construct an equilateral triangle. Let us consider how to construct an isosceles triangle.

- ### Construction of an isosceles triangle

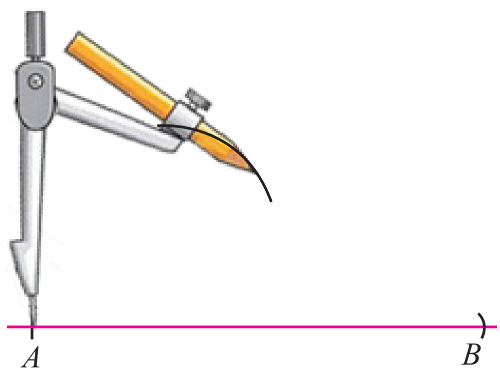
Let us construct the isosceles triangle ABC with $AB = 6$ cm, and BC and CA equal to 3.5 cm each.
Let us draw a sketch of the triangle first.



Sketch

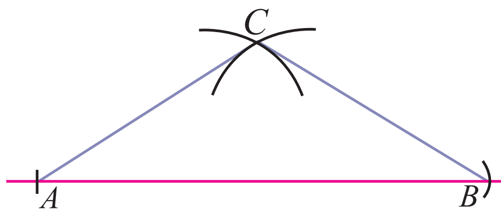
Step 1 - Construct a straight line segment AB of length 6 cm using a pair of compasses and a ruler.

Step 2 - Set the pair of compasses so that its point and the pencil point are at a distance of 3.5 cm apart. Place the point of the pair of compasses on A and construct an arc as shown in the figure.



Step 3 - Place the point of the pair of compasses at the point B, and without changing the length on the pair of compasses, construct another arc such that it intersects the first arc. If the arcs do not intersect, place the point of the pair of compasses again at A and lengthen the first arc sufficiently until the two arcs intersect. Name the point of intersection of the two arcs as C.

Step 4 - Join AC and BC.



Step 5 - After completing the triangle ABC by drawing the straight line segments AC and BC, measure the magnitudes of the interior angles by using the protractor, and write them down.

By this we can establish the fact that we have constructed an isosceles triangle with side lengths 6 cm, 3.5 cm and 3.5 cm.

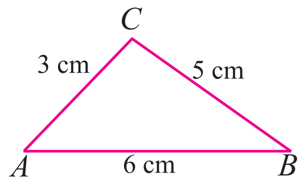
(i) Construct an isosceles triangle of side lengths 7.6 cm, 5.2 cm and 5.2 cm.
(ii) Measure and write down the magnitudes of the angles.
(iii) Write what type of triangle this is according to the angles.

Construction of a scalene triangle

Let us now construct a scalene triangle.

If all three sides of a triangle are different in length, then it is called a scalene triangle.

Let us construct a scalene triangle ABC, with side lengths AB = 6 cm, BC = 5 cm and AC = 3 cm

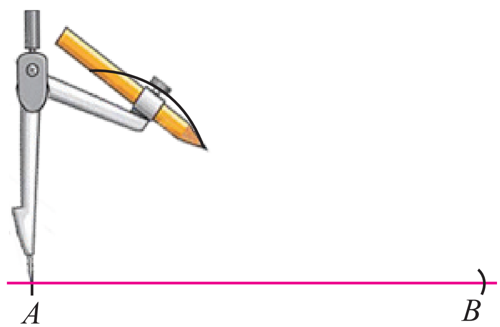


Sketch

Let us draw a sketch of the triangle first.

Step 1 - Construct a straight line segment AB of length 6 cm using a pair of compasses and a ruler.

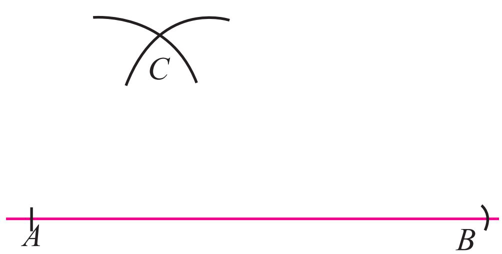
Step 2 - Set the pair of compasses so that its point and the pencil point are at a distance of 3 cm apart. Place the point of the pair of compasses on A and construct an arc as shown in the figure.



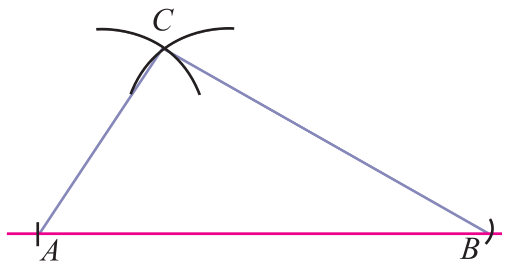
Step 3 - Set the pair of compasses so that its point and the pencil point are at a distance of 5 cm apart.

Place the point of the pair of compasses on the point B and construct another arc so that it intersects the first arc.

If the two arcs do not intersect, place the point of the pair of compasses again on A and lengthen the first arc sufficiently until the two arcs intersect.
Name the point of intersection of the two arcs as C.



Step 4 - Join AC and BC



Step 5 - After completing the triangle ABC by drawing the straight line segments AC and BC, measure the magnitudes of the interior angles by using the protractor, and write them down.

You have now constructed the triangle ABC of side lengths 3 cm, 5 cm and 6 cm.

$C\hat{A}B = 55^o$, $A\hat{B}C = 30^o$ and $B\hat{C}A = 95^o$. Therefore, $C\hat{A}B + A\hat{B}C + A\hat{C}B = 180^o$.

This triangle is a scalene triangle according to the lengths of the sides.

➤ (i) In the triangle PQR, $PQ = 4$ cm, $QR = 3$ cm and $PR = 5$ cm. Construct this triangle.

(ii) Measure and write down the magnitude of the largest angle of this triangle. Write what type of triangle this is according to its angles.

Exercise 26.2

(1) (i) Construct two equilateral triangles, one of side length 4 cm and the other of side length 5.7 cm.

(ii) Measure and write down the magnitudes of the angles of the two triangles.

(2) (i) Construct triangles with the given side lengths by using a pair of compasses and a ruler.

(ii) Show that the sum of the angles of each of the triangles you constructed is equal to 180^o by measuring them.

(iii) Categorize the triangles according to the largest angle.

(a) 6 cm, 8 cm, 10 cm

(b) 4.5 cm, 6 cm, 7.5 cm

(c) 5 cm, 5 cm, 4 cm

Summary

- To construct a triangle when the lengths of the three sides are given, the following steps are performed.
  - Constructing a straight line segment of length equal to the length of one of the sides of the triangle.
  - Constructing an arc of length equal to the length of another side of the triangle by placing the point of the pair of compasses at one end point of the above straight line segment.
  - Constructing another arc from the other end point of the straight line segment, of length equal to the length of the remaining side, so that it intersects the above drawn arc.
  - Joining the point of intersection of the two arcs to the end points of the straight line segment.
- The sum of the lengths of any two sides of a triangle is greater than the length of the remaining side.