25 The number line and Cartesian Plane

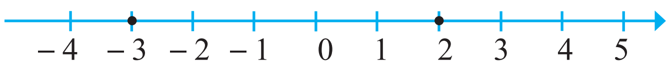
By studying this lesson you will be able to,

- represent fractions and decimal numbers with one decimal place on a number line,
- compare fractions and decimals using the number line,
- represent on a number line, the values that the unknown an inequality with one unknown can take,
- identify a point on a coordinate plane by considering the x - and y - coordinates, and
- identify the nature of the coordinates of the points that lie on a line which is parallel to an axis of the coordinate plane.

25.1 Introduction

In Grade 7 you learnt how to represent an integer on a number line.

Let us find out which of the two numbers 2 and –3 is greater.

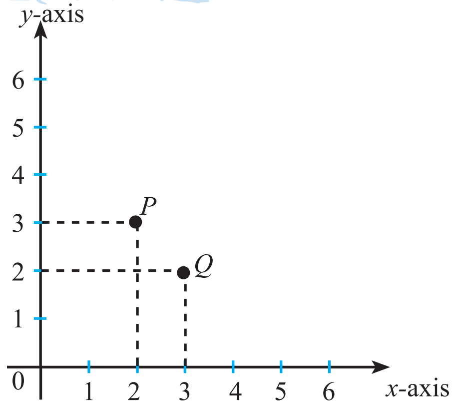


The numbers –3 and 2 have been marked on the above number line.

Any number that is on the right hand side of a given number on the number line is greater than the given number. This property is applicable to the whole number line. Therefore this rule can be applied when comparing two integers using the number line.

Since 2 is on the right hand side of (-3) on the number line, 2 is greater than (-3). This can be written either as $2 > (-3)$ or as $(-3) < 2$.

You have also learnt previously how a Cartesian plane which consists of two number lines drawn perpendicular to each other is used to specify the location of a point on a plane.



- The two number lines which intersect each other perpendicularly are called the x-axis and the y-axis.
- The point of intersection of the x and y axes is called the origin.
- 0 of the two number lines is located at the origin.

- The line drawn from the point P, perpendicular to the x-axis, meets the x-axis at 2. The line drawn from the point P, perpendicular to the y-axis meets the y-axis at 3.

Accordingly, the x-coordinate of point P is 2 and the y-coordinate of P is 3. The coordinates of point P are written as (2,3) by writing the x-coordinate first and the y-coordinate second within brackets. This is written in short as $P(2,3)$.The coordinates of the point Q in the Cartesian plane are (3,2).

Review Exercise

(1) (i) Write all the integers that lie between -3 and 5.
(ii) Mark these integers on a number line.
(iii) Of the numbers mentioned in (i) above, write the least and the greatest numbers.

(2) Write the integers $7, -8, 0, -3, 5, -4$ in ascending order.

(3) Choose the appropriate symbol from $>$ and $<$ and fill in the blanks.

(i) $5 \;....\; -2$	(ii) $3 \;....\; 0$	(iii) $-5 \;....\; 0$
(iv) $-10 \;....\; -1$	(v) $5 \;....\; -7$	(vi) $0 \;....\; -3$

(4) Draw a Cartesian plane and mark the following points on it.

(i) A (3,1)	(ii) B (0,5)	(iii) C (3,0)
(iv) D (2,3)	(v) E (4,1)	(vi) F (3,4)

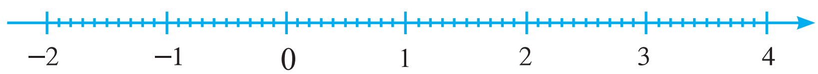
25.2 Representing fractions and decimals on a number line

Fractions and decimals which are not integers can also be represented on a number line. Such a number is located between two consecutive integers on the number line. For example, 1.5 is located between 1 and 2 on the number line, and $-\frac{2}{3}$ is located between -1 and 0.

Do the following activity to learn how to represent on a number line, fractions and decimals that lie between two consecutive integers.

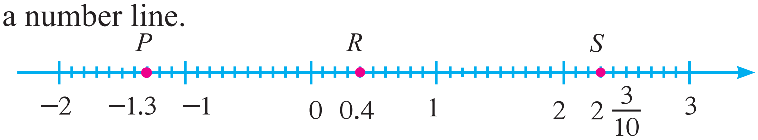
Activity 1

On your square ruled exercise book, draw a number line marked from −2 to 4, taking 1 unit to be 5 squares, as shown below. Divide one unit into 10 equal parts by dividing each square of the exercise book into two equal parts.



- Mark a point on the number line which lies exactly between the two consecutive integers 2 and 3, and name it *P*.
- What is the value of *P*?
- Name the numbers $-\frac{1}{2}$, 1.5 and 0.5 which are on the number line as *Q*, *R* and *S* respectively.
- Mark another point which is not an integer and which does not lie exactly between two consecutive integers on the number line, and write its value.

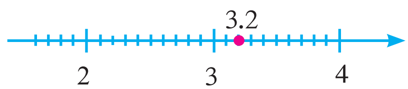
The figure below shows several numbers which are not integers that have been marked on a number line.

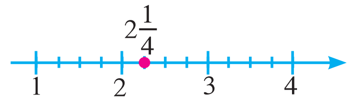


When dividing a unit on the number line into several equal parts to represent a particular number, it is necessary to be careful to select the number of equal parts appropriately, depending on the number that is to be represented.

It is suitable to divide one unit into 10 equal parts when representing a decimal number with one decimal place, and to divide one unit into parts equaling the number in the denominator when representing a fraction.

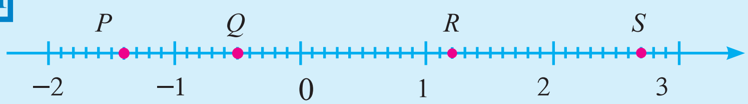
For example, it is suitable to divide one unit into 10 equal parts to represent 3.2 and to divide one unit into 4 equal parts to represent $2\frac{1}{4}$.





Fractions and decimal numbers can be compared using a number line in the same way that integers are compared.

Example 1



(i) Write the numbers that are represented by the points P, Q, R and S on the number line shown in the figure.

(ii) Write these numbers in ascending order.

(i) -1.4 , $-\frac{1}{2}$, 1.2, 2.7

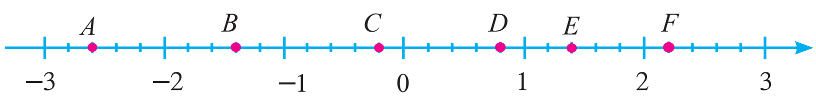
(ii) $-\frac{1}{2} = -0.5$. $-1.4 < -0.5 < 1.2 < 2.7$

$\therefore$ When these numbers are arranged in ascending order we obtain

$-1.4, -\frac{1}{2}$, 1.2 , 2.7.

Exercise 25.1

(1) Write the numbers that are represented by the points A, B, C, D, E and F on the given number line.



(2) (i) Mark the numbers 1.8, 3.5, 2.6 and 4.1 on a number line.
(ii) Mark the numbers 13.2, 14.7, 15.5 and 16.3 on a number line.

(3) Arrange each of the following groups of numbers in ascending order using a number line.

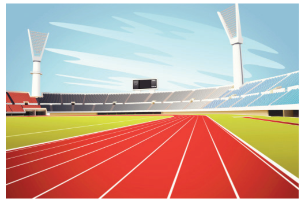
(i) -2, $1\frac{1}{2}$, -1.5, -3

(ii) 2.5, -0.5, -5.2, $3\frac{1}{4}$

(iii) $1\frac{1}{4}$, 0, $-2\frac{2}{5}$, -4.1

(iv) 2.7, -10.5, $5\frac{1}{4}$, -1.3

25.3 Representing inequalities containing an algebraic term on a number line



According to the rules of a certain competition, only children of height greater than 120 cm are allowed to participate. If the height of a competitor is denoted by h cm, this means that $h > 120$. Accordingly, anyone of height greater than 120 cm such as 121 cm, 125 cm or 127 cm can participate in the competition.

$x > 2$ is an inequality. This means that the values that x can take are only those which are greater than 2. On the other hand the inequality $x \geq 2$ means that the values that x can take are those which are greater than or equal to 2.

- The symbol $>$ is used to denote 'greater than',
- the symbol $<$ is used to denote 'less than',
- the symbol $\geq$ is used to denote 'greater than or equal', and
- the symbol $\leq$ is used to denote 'less than or equal'.

Accordingly, $8 > x$ can also be written as $x < 8$, and $2 \geq y$ can also be written as $y \leq 2$.

Therefore, $h > 120$ means that the values that h can take are only those values which are greater than 120.

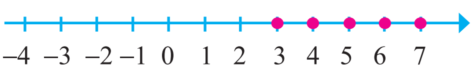
The set of all values that the algebraic term in an inequality with one algebraic term can take is called the set of solutions of the inequality.

- The integral solutions of the inequalities (i) $x > 2$ and (ii) $x \geq 2$ are represented on the number lines given below.

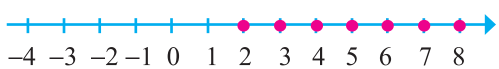
The integers belonging to the set of integral (whole number) solutions of the inequality $x > 2$ are 3, 4, 5, 6, ...

The integers belonging to the set of integral (whole number) solutions of the inequality $x \geq 2$ are 2, 3, 4, 5, 6, ...

(i) $x > 2$ where x is an integer



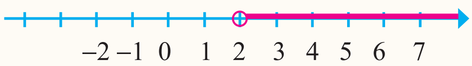
(ii) $x \geq 2$ where x is an integer



- However, when all the solutions of $x > 2$ or $x \geq 2$ are represented on a number line, we obtain a section of the number line.

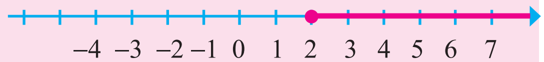
(i) $x > 2$

The set of all solutions of the inequality $x > 2$ is the set of all the numbers greater than +2. This includes all the fractions and decimals which are greater than 2 too. Therefore the solutions of this inequality are marked as follows.



Since 2 does not belong to the set of all solutions of this inequality, the point on the number line representing 2 is not shaded. An un-shaded circle is drawn on 2. Since all the numbers greater than 2 belong to this set, it is represented by a dark line drawn to the right of 2 as shown above.

(ii) $x \geq 2$



Since 2 belongs to the set of all solutions of the inequality, a shaded circle is drawn on 2 as shown in the figure.

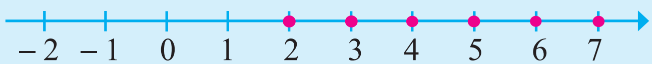
Example 1

(1) Mark the set of integral solutions of the inequality $x > 1$ on a number line.

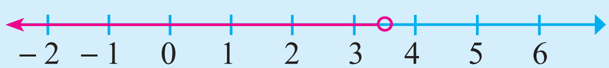
(2) Represent the set of all solutions of each of the following algebraic inequalities on a separate number line.

(a) $x < 3\frac{1}{2}$ (b) $x > 3\frac{1}{2}$ (c) $x \leq 3\frac{1}{2}$ (d) $x \geq 3\frac{1}{2}$

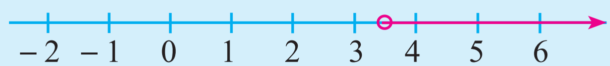
(i)



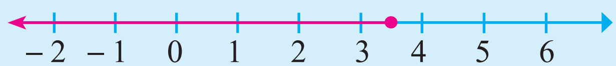
(ii) (a)



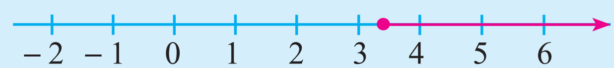
(b)



(c)



(d)



Exercise 25.2

(1) Mark the set of integral solutions of each of the following inequalities on a separate number line.

(i) $x > 0$ (ii) $x < 3.5$ (iii) $x \geq -2\frac{1}{2}$

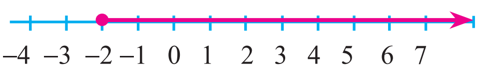
(2) Represent the set of all solutions of each of the following algebraic inequalities on a separate number line.

(i) $-\frac{1}{2} \geq m$ (ii) $2.5 \leq m$ (iii) $1.5 < m$

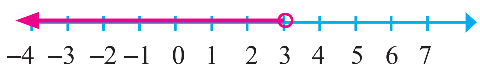
25.4 More on representing inequalities on a number line

➢ To find the values which satisfy both the inequalities $x \geq -2$ and $x < 3$ at the same time, let us first represent the solutions of the two inequalities on separate number lines.

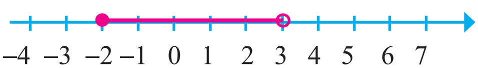
(i) $x \geq -2$



(ii) $x < 3$



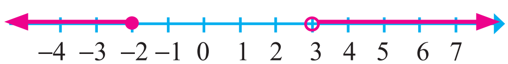
Now let us represent the values of x which satisfy both these inequalities on a number line.



When two inequalities are combined in this manner, by writing it as, $x \geq -2$ **and** $x < 3$, we express the fact that both inequalities have to be satisfied simultaneously.

We can express the region on the number line consisting of all the values that satisfy both these inequalities as $-2 \leq x < 3$.

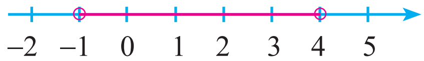
➢ Now let us represent the values of x which satisfy at least one of the two inequalities $x \leq -2, x > 3$ on a number line.



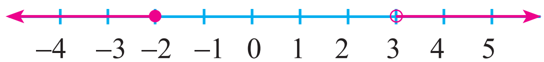
Any number in the shaded region of the number line satisfies at least one of the given inequalities.

When two inequalities are combined in this manner, by writing it as $x \leq -2$ **or** $x > 3$, we express the fact that at least one of the two inequalities should be satisfied.

The values in the shaded region of the number line satisfy both the inequalities $x > -1$ and $x < 4$. This region can be expressed algebraically by the inequality $-1 < x < 4$.



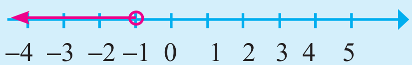
The figure given below shows the values satisfying $x \leq -2$ or $x > 3$ represented on a number line.



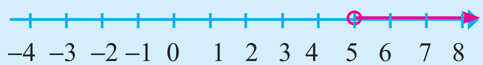
Example 1

(i) Indicate the values of x which satisfy both of the inequalities $x < -1$ and $x > 5$ on a number line.

$x < -1$

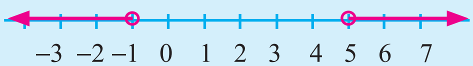


$x > 5$



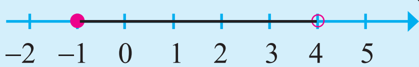
No number satisfies both these inequalities at the same time. Therefore the set of values that satisfy both the inequalities $x < -1$ and $x > 5$ is the empty set.

(ii) Represent the values of x which satisfy at least one of the two inequalities $x < -1$ and $x > 5$ on a number line.



Example 2

Write the inequality represented on the number line in algebraic form.



$-1 \leq x < 4$

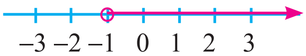
Exercise 25.3

(1) Represent each of the following inequalities on a separate number line.

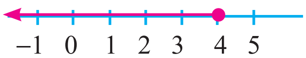
(i) $-2 < x < 3$ (ii) $-3 < x \leq 2$ (iii) $0 \leq x < 6$

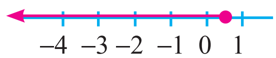
(iv) $-1 \leq x \leq 4$ (v) $x \leq -1$ or $x \geq 5$ (vi) $x \leq -1$ or $x \geq 4$

(2) Write the inequality represented on each of the following number lines in algebraic form.

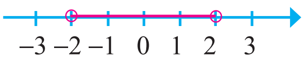
(i) 

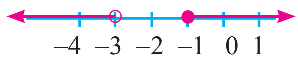
(ii) 

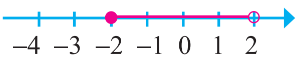
(iii) 

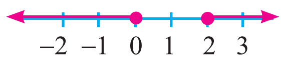
(iv) 

(3) For each of the following cases, write the inequalities represented on the number line.

(i) 

(ii) 

(iii) 

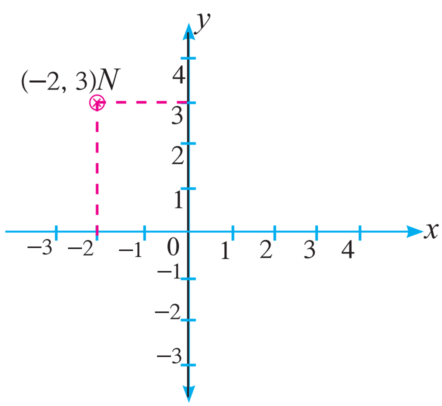
(iv) 

(4) Write the set of integral values that satisfy both the inequalities $x > -1$ and $x < 5$.

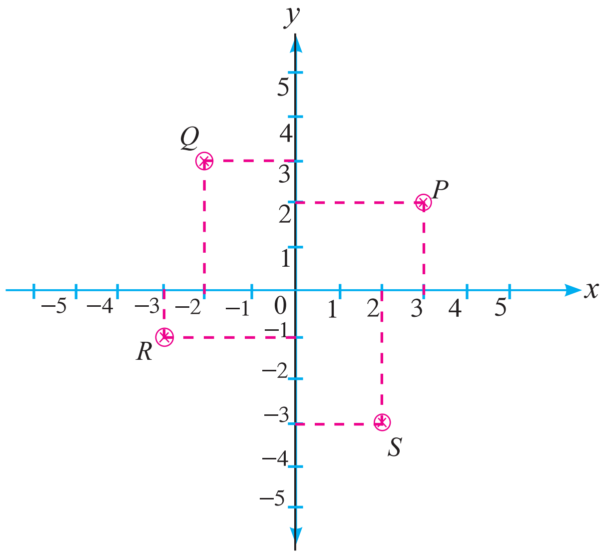
25.5 Plotting points on a Cartesian plane

You have learnt previously how points with coordinates equal to either 0 or positive integers are marked on a Cartesian plane. Let us now study how coordinates with negative numbers also are marked on a Cartesian plane. Let us consider how the point $N(-2,3)$ is marked on a Cartesian plane.

The point N with coordinates $(-2, 3)$ on the Cartesian plane is the intersection point of the line drawn perpendicular to the x-axis through the point -2 on the x-axis and the line drawn perpendicular to the y-axis through the point 3 on the y-axis.



Now let us consider how the coordinates of points on the Cartesian plane are identified. The line drawn from point R perpendicular to the x-axis, meets the x-axis at –3. The line drawn from point R perpendicular to the y-axis, meets the y-axis at –1. Accordingly, the x-coordinate of R is –3 and the y-coordinate of R is –1. Therefore the coordinates of R are written as $(-3, -1)$.



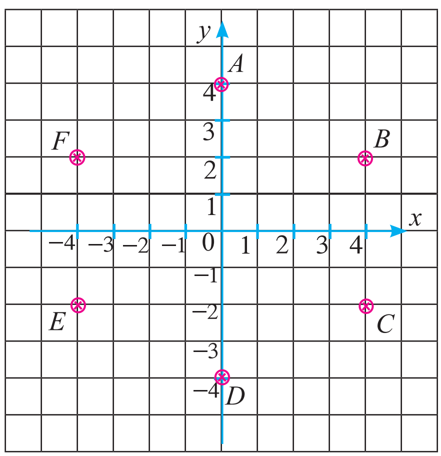
Point	x - coordinate	y - coordinate	coordinates
P	3	2	(3, 2)
Q	−2	3	(−2, 3)
R	−3	−1	(−3, −1)
S	2	−3	(2, −3)

Exercise 25.4

(1) Mark each of the following points on a Cartesian plane where the x-axis and the y-axis are marked from –5 to 5.

A (2, −5) , B (−3, 4) , C (−3, −3) , D (−4, −1) , E (−2, 0) , F (0, −4)

(2) Write the coordinates of the points which are marked on the Cartesian plane given below.



(3) Mark the points with the following coordinates on a Cartesian plane where the x-axis and the y-axis are marked from -5 to 5. Identify the figure that is obtained by joining all the points in the given order.

$(0, 4)$, $(1, 1)$, $(4, 0)$, $(1, -1)$, $(0, -4)$, $(-1, -1)$, $(-4, 0)$, $(-1, 1)$, $(0, 4)$

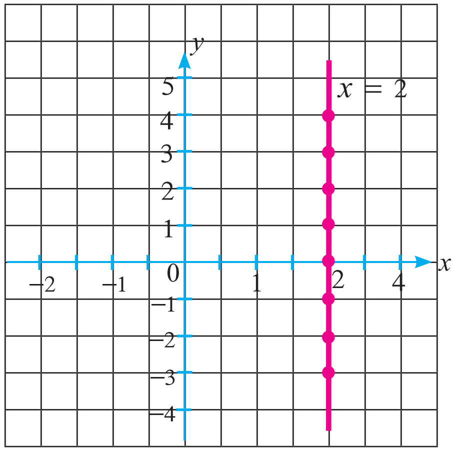
25.6 Straight lines parallel to the two axes

Observe each of the following coordinates carefully.

$(2, 4)$, $(2, 3)$, $(2, 2)$, $(2, 0)$, $(2, -1)$, $(2, -2)$, $(2, -3)$

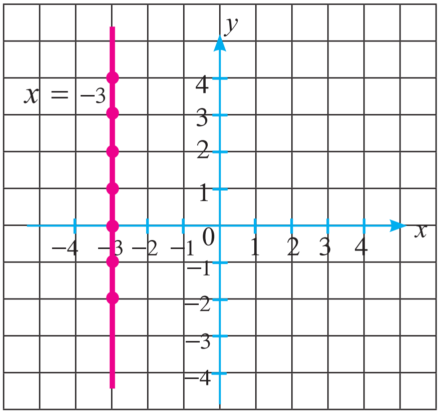
The x-coordinate of each of these pairs is 2.

When the points with these coordinates are marked on the Cartesian plane, they are as follows.

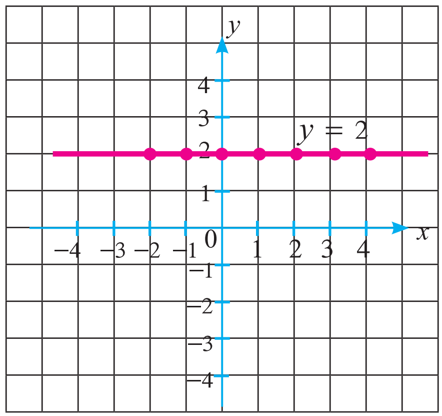


All these points lie on the straight line which is parallel to the y-axis and intersects the x-axis at the point 2. That is, they all lie on the straight line given by $x = 2$. Furthermore, the x-coordinate of every point on this line is equal to 2.

➤ $x = -3$ is the straight line on which all the points with x-coordinate equal to -3 lie.



➤ The straight line given by the equation $y = 2$ is shown in the Cartesian plane given below. This line is parallel to the x-axis and intersects the y-axis at 2.



Example 1

(a) (i) Write the coordinates of 5 points which lie on the straight line given by $x = -3$.

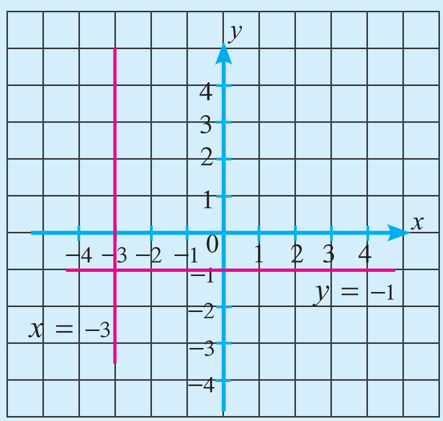
(ii) Write the coordinates of 5 points which lie on the straight line given by $y = -1$.

(b) Draw the straight lines given by $x = -3$ and $y = -1$ on the same Cartesian plane.

(a) (i) The points with coordinates $(-3, -1)$, $(-3, 0)$, $(-3, 1)$, $(-3, 2)$, and $(-3, 3)$ lie on the straight line given by $x = -3$.

(ii) The points with coordinates $(-3, -1)$, $(-2, -1)$, $(-1, -1)$, $(0, -1)$, and $(2, -1)$ lie on the straight line given by $y = -1$.

(b)



Exercise 25.5

(1) Copy each of the following statements in your exercise book. Place a "✓" next to the correct statements and a "×" next to the incorrect statements.

(i) (0, 5) are the coordinates of a point that lies on the straight line given by $x = 5$. ( )

(ii) The straight line given by $y = 3$ is parallel to the x-axis. ( )

(iii) The coordinates of the point of intersection of the straight lines given by $x = 2$ and $y = 1$ are (2, 1). ( )

(iv) The straight line given by $y = 0$ is identical to the x-axis of the Cartesian plane. ( )

(v) From among the ordered pairs, (3, 1), (−2, 1), (1,1) and (0,1), the pair which is not the coordinates of a point which lies on the straight line given by $y = 1$ is (1 , −1). ( )

(2) (i) Draw the straight lines $x = 3$ and $y = -3$ on the same Cartesian plane.

(ii) Write the coordinates of the point of intersection of the two lines.

(3) (i) Draw a Cartesian plane with both the x-axis and the y-axis marked from −5 to 5.

(ii) On this Cartesian plane, draw the four straight lines which are the graphs of the following equations.

(a) $y = 2$ (b) $y = -2$ (c) $x = 4$ (d) $x = -2$

(iii) What is the special name that is given to the figure which is obtained by the intersection of these lines?

(iv) Write the coordinates of the points of intersection of each pair of lines which intersect.

(v) Draw the axes of symmetry of the closed plane figure that was obtained in (iii) above and write their equations.

Miscellaneous Exercise

(1) Represent the set of integral solutions of the inequality $-2 \leq x \leq 3$ on a number line.

(2) (i) Mark the points $A(-1,\ 1)$, $B(\ 2,\ 1)$ and $C(\ 1,\ -1)$ on a Cartesian plane which has both axes marked from −3 to 3.

(ii) Mark the point D on the Cartesian plane such that $ABCD$ forms a parallelogram and write it coordinates.

(iii) Write the equations of the sides AB and DC of the parallelogram.

(3) Arrange each of the following groups of numbers in ascending order using a number line.

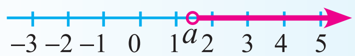
(i) $-5,\ -1\frac{3}{4},\quad -3\frac{1}{3},\quad -0.2$ (ii) $3.8,\ -5\frac{1}{2},\quad 0.5,\quad -7.5$

(iii) $1.2,\ -0.3,\ 1\frac{2}{5},\ 2$ (iv) $-1\frac{3}{4},\ -2,\ 1\frac{5}{8},\ 0$

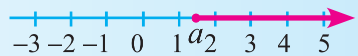
Summary

- Fractions and decimal numbers can be represented on a number line as numbers which lie between integers.
- The inequalities (i) $x > a$ and (ii) $x \geq a$ can be represented on a number line as follows.

(i)

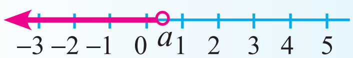


(ii)

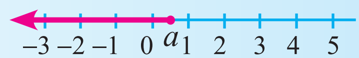


- The inequalities (i) $x < a$ and (ii) $x \leq a$ can be represented on a number line as follows.

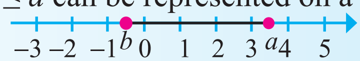
(i) $x < a$



(ii) $x \leq a$



- The inequality $b \leq x \leq a$ can be represented on a number line as follows.



- All points on a straight line of the form $x = a$, which is parallel to the y - axis has 'a' as their x-coordinate.
- All points on a straight line of the form $y = b$, which is parallel to the x-axis has b' as their y-coordinate.