22 Volume and Capacity

By studying this lesson you will be able to

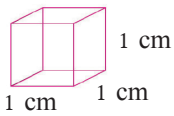
- obtain formulae for the volume of a cube and a cuboid,
- find the volume of a cube and a cuboid by using the formulae,
- solve problems relating to volumes,
- identify what volume and capacity are, and
- estimate capacities.

22.1 Volume

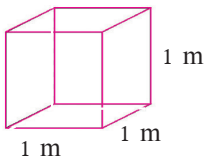
Let us recall the facts you learnt on volume in Grade 7.

The amount of space occupied by an object is called its volume. Cubic centimetre and cubic metre are two units that are used to measure volumes.

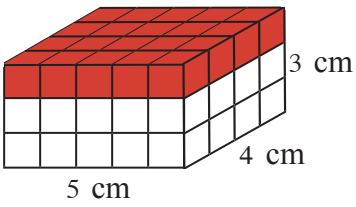
The volume of a cube of side length 1 cm is 1 cubic centimetre (1 cm^3).



The volume of a cube of side length 1 metre is used as a unit to measure larger volumes. Its volume is 1 cubic metre (1 m^3).



The upper layer of the cuboid shown in the figure consists of $5 \times 4 = 20$ small cubes. Since there are three such layers, there are $20 \times 3 = 60$ small cubes.



Therefore, the volume of this cuboid is 60 cm^3.

The volume of a cuboid = length × breadth × height

The volume of a cube = length × breadth × height
= side length × side length × side length

When finding the volume of a cube or a cuboid, the length, breadth and height should be written in the same units.

Do the following review exercise to further recall the above facts.

Review Exercise

(1) Find the volume of a cuboid with length, breadth and height equal to 10 cm, 8 cm and 4 cm respectively.

(2) Find the volume of a cube of side length 6 cm.

(3) The length of a box is 1.8 m, its breadth is 1 m and its height is 70 cm. Find the volume of this box in cubic meters.

(4) The length, breadth and height of a cuboid of volume 120 cm^3 are 8 cm, 5 cm and 3 cm respectively. Write the length, breadth and height of three other cuboids of the same volume.

(5) The area of the base of a cuboid of volume 70 cm^3 is 35 cm^2. Find its height.

(6) If the height and length of a cuboid of volume 70 cm^3 are 4 cm and 5 cm respectively, what is its breadth?

(7) The volume of a cube is 8 cm^3. What is the length of each side?

22.2 Formulae for the volume of a cube and a cuboid

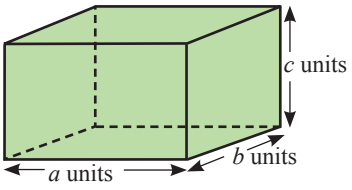
• Formula for the volume of a cuboid

If the volume of a cuboid of length a units, breadth b units and height c units is V cubic units, let us obtain a formula for the volume of the cuboid.

Volume of the cuboid = length × breadth × height

$\therefore V = a \times b \times c$

$V = abc$



If the area of the base of this cuboid is A square units,

$A = a \times b$

$V = a \times b \times c$. Let us substitute A for $a \times b$.

Then $V = A \times c$

Hence, the volume of the cuboid = area of the base × height

If the length, breadth and height of a cuboid are a units, b units and c units respectively, and if the area of its base is A square units and volume is V cubic units, then

$V = abc$ and

$V = Ac$

• Formula for the volume of a cube

As above, let us obtain a formula for the volume of a cube of side length a units and volume V cubic units.

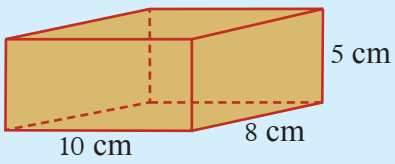
Since the volume of a cube = side length × side length × side length, the formula for the volume of a cube of side length a units and volume V cubic units is,

$$V = a \times a \times a$$
$$V = a^3$$

Example 1

The length, breadth and height of a cuboid are 10 cm, 8 cm and 5 cm respectively.

(i) Find the volume of this cuboid.
(ii) Another cuboid with a square base is of the same volume as that of the above cuboid. Its height is 4 cm. Find the side length of its base.



(i) Since $V = abc$,

the volume of the cuboid $= 10 \text{ cm} \times 8 \text{ cm} \times 5 \text{ cm}$

(ii) $= 400 \text{ cm}^3$

Method I

Since $V = A \times c$,

the area of the base × height = volume

$$A \times 4 = 400$$
$$\therefore A = \frac{400}{4} = 100$$

Since the base is square shaped, the length of a side $= \sqrt{100}$ cm

$= 10$ cm

Method II

Since the base of the cuboid is square shaped, if its length and breadth are taken as a,

its volume $V = a \times a \times c$. Since $V = 400$ and $c = 4$

$$a \times a \times 4 = 400$$
$$a \times a = \frac{400}{4} = 100$$
$$a \times a = 10 \times 10$$
$$\therefore a = 10$$

$\therefore$ the side length of the base $= 10$ cm

If the length of a side of a cube is 1 m, then its length in centimetres is 100 cm.
Therefore, its volume $= 100\ \text{cm} \times 100\ \text{cm} \times 100\ \text{cm}$
$= 1\ 000\ 000\ \text{cm}^3$
That is, $1\ \text{m}^3 = 1\ 000\ 000\ \text{cm}^3$

Note;

Cubic feet and cube are two units of volume which are also in use.

100 cubic feet = 1 cube

Exercise 22.1

(1) The following table has measurements of some cubes and cuboids. Copy the table and fill in the blanks.

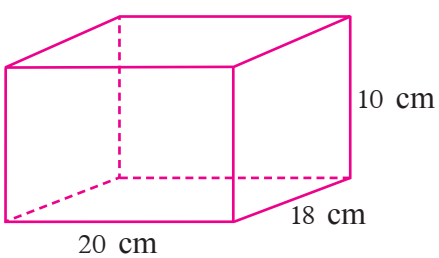
Length	Breadth	Height	Volume
8 cm	6 cm	5 cm	.....
12 cm	.....	10 cm	1200 cm^3
1.5 m	0.5 m	0.6 m	.....
6 m	6 m	.....	216 m^3
$\frac{3}{4}$ m	$\frac{2}{5}$ m	$\frac{2}{3}$ m	.....
1 m	$\frac{1}{2}$ m	40 cm	.....

(2) The area of one face of a cube is 36 cm^2. Find
(i) the length of an edge,
(ii) the volume,
of this cube.

(3) The area of the base of a cuboid is 1300 cm^2. If its volume is 65000 cm^3, find its height in metres.

(4) The volume of a cuboid shaped tank is 3600 cm^3. Its height, breadth and length are three consecutive perfect squares. Find its length, breadth and height. (Write 3600 as a product of prime factors).

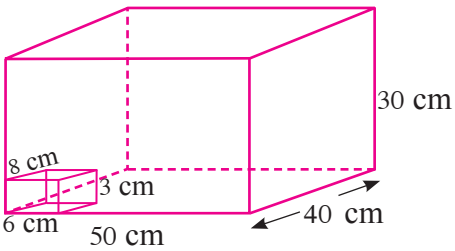
(5) It is required to pack cube shaped wooden blocks of side length 5 cm each in the cuboid shaped box shown in the figure. Find the maximum number of wooden blocks that can be packed in this box.



(6) Find the length, breadth and height of the cuboid shaped box with minimum volume, into which 50 cuboid shaped blocks, each with length, breadth and height equal to 4 cm, 3 cm and 2 cm respectively can be packed.

(7) By melting a solid metal cube of side length 10 cm, 8 identical smaller solid cubes were made without wastage of metal. Find the side length of a small cube.

8) It is necessary to pack soap boxes with measurements 8 cm × 6 cm × 3 cm in a box with measurements 50 cm × 40 cm × 30 cm as shown in the figure. Instructions have been given to pack 10 layers of soap boxes. Find the maximum number of soap boxes that can be packed.

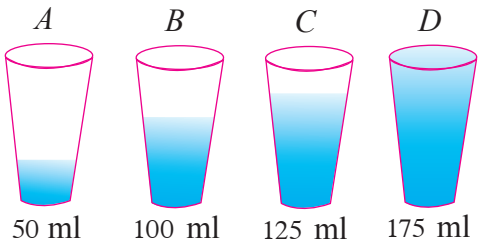


22.3 Capacity

The following figure shows some items that can be observed in day-to-day activities. Each of them indicates a certain amount of milliliters.

In Grade 7, you learnt that millilitres and litres are used to measure liquid amounts and that 1000 ml is equal to 1 l . Since liquids also occupy some amount of space, liquids have a volume.

The figure shows four glass containers A, B, C and D with a certain amount of drink in each of them.



The glasses A, B and C are not completely filled, but glass D is. The volume of drink in glass A is 50 ml and the volume of drink in glass D is 175 ml. The maximum amount of drink that can be poured into glass D is 175 ml. This is the capacity of glass D.

The volume of liquid that is required to fill a container completely is its **capacity**.

Accordingly, it is clear that the space within a container is its capacity.

Litre and millilitre which are the units used to measure liquid volumes are the units used to measure capacity too. Containers used in everyday activities sometimes have their capacity indicated on them. In some cases, the amount of liquid in the container is mentioned.

• The relationship between the units of volume and capacity

There is a close relationship between the units that are used to measure volume and those which are used to measure capacity. The maximum amount of liquid that can be poured into a cube shaped container of side length 1 cm is 1 ml.

$\therefore$ 1 cm × 1 cm × 1 cm = 1 ml

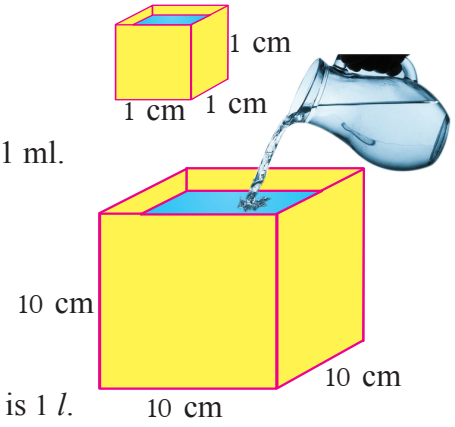
$1\ cm^3 = 1\ ml$

The capacity of a container of volume 1 cm^3 is 1 ml.

Likewise, 10 cm × 10 cm × 10 cm = 1000 ml

$1000\ cm^3 = 1\ l$

The capacity of a container of volume1000 cm^3 is 1 l.



Example 1

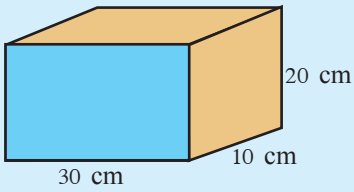
Find the capacity of the cuboid shaped box in the figure.

Volume of the cuboid shaped box $= 30 \times 10 \times 20\ cm^3$

$= 6000\ cm^3$

$\therefore$ Capacity $= 6000$ ml

$= 6\ l$



Example 2

The capacity of a water tank is 6000 l. After filling the tank completely, 800 l of water is used per day for four days and 1200 l of water is used per day for two days. Find the volume of water remaining after these 6 days.

Water used during the first 4 days $= 800\ l \times 4 = 3200\ l$

Water used during the remaining 2 days $= 1200\ l \times 2 = 2400\ l$

$\therefore$ Total volume of water used $= 3200 + 2400\ l$

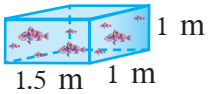
$= 5600\ l$

$\therefore$ Remaining volume of water $= 6000\ l - 5600\ l = 400\ l$

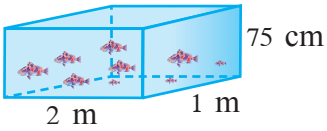
Exercise 22.2

(1) Find the capacity of each fish tank given below in litres.

(i)



(ii)



(2) The capacity of an oil tank is 12 l. There is 3 l 800 ml of oil in it. How much more oil is required to fill the oil tank completely?

(3) The capacity of a container is 150 ml. It is filled completely with a soft drink and then poured into a large bottle. How many litres of soft drink are there in the large bottle if drink is poured into it 10 times in this manner?

(4) A bottle contains 1300 ml of medicinal syrup. From this bottle, 50 ml of syrup is filled into small cups of capacity 65 ml each. Find the maximum number of cups that can be filled in this manner.

(5) A container of capacity 20 *l* is completely filled with milk. From this amount, 8 *l* 800 ml of milk is used to make yoghurt and 10 *l* 800 ml is used to make curd. Find the amount of milk remaining in the container after the above amounts are used.

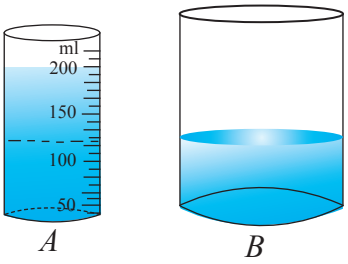
(6) What is the maximum volume of water in millilitres that can be filled into a cube shaped container which is of side length 15 cm?

(7) The base area of a cuboidal shaped container is 800 cm^2. If 4.8 *l* of water is poured into this container, find the height of the water level.

(8) Find the capacity of a cuboidal shaped container with length, breadth and height equal to 4 m, 2.5 m and 0.8 m respectively.

22.4 Estimating the capacity of a container

The water level in container *B* is as shown in the figure, after 200 m*l* of water is poured into it from the calibrated container *A*. Let us estimate the capacity of *B* accordingly.



It can be observed that the height of container *B* is three times the height of the water.

$\therefore$ capacity of container B $= 3 \times 200$ ml

$= 600$ ml

Activity 1

Step 1 - Take a sufficient amount of water as well as calibrated and non-calibrated transparent cylindrical containers from your surroundings (glasses, bottles, plastic cups).

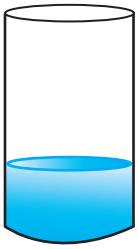
Step 2 - Pour a measured amount of water from a calibrated container into a non-calibrated container. Then examine the height of the water level.

Step 3 - Determine in a suitable way, how many times the height of the water level, the total height of the container is and hence estimate the capacity of the container.

Step 4 - Estimate the capacity of each of the remaining containers in the same manner.

Exercise 22.3

(1) The volume of water in the container in the figure is 150 ml. Estimate the capacity of this container.



(2) 100 lamps have been prepared for a function. 3 litres of oil were required to fill all these lamps completely. Estimate the capacity of a lamp.

(3) A household usually needs 275 litres of water per day. Estimate the minimum capacity of a tank which can store the water required for this house for a week.

Summary

- The volume of a cuboid of length, breadth and height equal to a units, b units and c units respectively is $a \times b \times c$ cubic units. If its volume is V cubic units, then
$$V = abc$$
- The volume of a cube of side length a units is a^3 cubic units. If the volume is V cubic units, then
$$V = a^3$$
- The volume of liquid that is required to fill a container completely is its capacity.