## Area

By studying this lesson you will be able to,

- derive a formula for the area of a triangle,
- solve problems associated with the area of a triangle,
- find the area of composite plane figures, and
- find the surface area of a cube and a cuboid.


### 20.1 Area

You learnt in Grade 7 that the extent of a surface is called its area.
You also learnt the method of finding the area of a square shaped lamina and of a rectangular shaped lamina.

When the area of a rectangle of length $l$ units and breadth $b$ units is taken as $A$ square units, $A=l b$.


When the area of a square shaped lamina of side length $a$ units is taken as $A$ square units, $A=a^{2}$.


Do the following review exercise to recall these facts.

## Review Exercise

(1) Find the area of each plane figure given below.
(i)

(ii)

(iii)

(iv)

(2) Find the area of the section shaded in pink in each of the plane figures given below.

(ii)

(3) The line $P Q$ is drawn such that it divides the area of the rectangle $A B C D$ into two equal parts. Draw three other figures which demonstrate three other ways in which the line $P Q$ can be drawn such that it divides the area of the rectangle $A B C D$ into two equal parts.

(4) The length and breadth of a rectangular floor of a house are 5 m and 3.5 m respectively. This floor is to be tiled with square shaped tiles of side length 25 cm each, without leaving any space between the tiles.
(i) What is the area of a square shaped tile?
(ii) Find the area of the floor.
(iii) How many floor tiles are required to tile this floor?
(iv) If a tile costs Rs. 275, how much money is needed to buy the required tiles?

### 20.2 Area of a triangle

Let us first identify a base of a triangle and the height of the triangle corresponding to that base.

- Base of a triangle and the height of the triangle corresponding to that base

Any side of the triangle $A B C$ can be considered as one of its bases. The way in which the height of the triangle varies according to the base, is explained below.


When $B C$ is considered as the base of the triangle $A B C$, the length of the base is $a$. In order to find the height of the triangle corresponding to the base $B C$, a perpendicular line has to be drawn from $A$ to $B C$. If this perpendicular meets $B C$ at point $D$, the height of the
 triangle corresponding to the base $B C$ is the length of $A D$.

When $A B$ is considered as the base of the triangle, the perpendicular $C R$ should be drawn from $C$ to $B A$ produced, in order to find the height of the triangle corresponding to the base $A B$. If the length of $C R$ is $r$, the height of the triangle corresponding to the base
 $A B$ is $r$.

According to the above explanation, when $C A$ is considered as the base of the triangle, the height of the triangle corresponding to the base $C A$ is the length of $B Q$ which is $t$.


## - Area of a right angled triangle

## Activity 1

Step 1 - Cut out a rectangular lamina.
Step 2 - Name its vertices as $A, B, C$ and $D$ as shown in the figure.
Step 3 - Join $A$ and $C$, and cut the lamina along this line. By cutting the rectangular lamina along the straight line $A C$, two triangles of equal area are obtained.
Step 4 - Find the area of one of these triangles.


The area of the right angled triangle $A B C$ is half the area of the rectangle $A B C D$.
$\therefore$ The area of the right angled
triangle $A B C\}=\frac{1}{2} \times$ area of the rectangle $A B C D$

$$
\begin{aligned}
& =\frac{1}{2} \times(\text { product of two sides which include a } \\
& \text { right angle) square units } \\
& =\frac{1}{2} \times(B C \times A B)=\frac{1}{2} \times a \times h=\frac{1}{2} a h
\end{aligned}
$$

- Area of a triangle which is not a right angled triangle


## $>$ Finding the area of the acute angled triangle $A B C$ by taking $B C$ as the base

To do this, let us draw the perpendicular $A D$ from the vertex $A$ of the triangle $A B C$ to the side $B C$. Now $A D C$ and $A D B$ are two right angled triangles.
The area of the right angled triangle $A D C=\frac{1}{2} \times x \times h$
The area of the right angled triangle $A D B=\frac{1}{2} \times y \times h$


The area of the triangle $A B C=$ the area of the + the area of the

$$
=\frac{1}{2} x h+\frac{1}{2} y h=\frac{1}{2} h(x+y)
$$

Since $a=(x+y)$,

$$
=\frac{1}{2} h \times a=\frac{1}{2} a h
$$

## Activity 2

Step 1 - Take a rectangular shaped piece of paper and name it $A B C D$ as shown in the figure. Pick any point on the side $A B$ and name it $E$.
Step 2 - Join $D E$ and $C E$. Then the triangle $D E C$ is obtained.
Step 3 - Draw a perpendicular from $E$ to $D C$ and name the point it meets $D C$ as $F$.
Step 4 - Cut the figure along the lines $D E$ and $E C$.


Step 5 - Find the area of the triangle $E C D$.
The area of triangle (1) is equal to the area of triangle (3).
The area of triangle (2) is equal to the area of triangle (4).
$\therefore$ Area of rectangle $A B C D\}=\begin{aligned} & \text { Area of rectangle } \\ & A E F D\end{aligned}+\begin{aligned} & \text { Area of rectangle } \\ & E B C F\end{aligned}$

$$
=\begin{aligned}
& 2 \times \text { area of triangle } \\
& D E F
\end{aligned}+\begin{aligned}
& 2 \times \text { area of triangle } \\
& E C F
\end{aligned}
$$

$\therefore$ Area of rectangle $A B C D=2 \times$ area of triangle $E C D$

$$
\begin{aligned}
\therefore \text { Area of triangle } E C D & =\frac{1}{2} \times \text { area of rectangle } A B C D \\
& =\frac{1}{2} \times D C \times C B \\
& =\frac{1}{2} \times D C \times E F(\text { since } C B=E F)
\end{aligned}
$$

$>$ Finding the area of the obtuse angled triangle $A B C$ by taking $B C$ as the base

The area of triangle $A C D=\frac{1}{2} \times(a+x) \times h$ $\qquad$
The area of triangle $A B D=\frac{1}{2} \times x \times h$ $\qquad$ (2)

$\therefore$ Area of triangle $A B C \quad=$ Area of triangle $A C D$ - area of triangle $A B D$

$$
=\frac{1}{2}(a+x) \times h-\frac{1}{2} \times x \times h
$$

$$
=\frac{1}{2} h(a+x-x)
$$

$$
=\frac{1}{2} h a
$$

$$
=\frac{1}{2} a h
$$

Area of a triangle $=\frac{1}{2} \times$ the length of the base $\times$ the perpendicular height of the triangle of the triangle corresponding to that base
Area of the triangle $=\frac{1}{2} \times$ the length of the base $\times$ height

## Note

When selecting the base of a triangle which is not right angled, the perpendicular can be drawn without producing the base, by selecting the side which is opposite the largest angle of the triangle as the base.

The perpendicular drawn from a vertex of a triangle to the opposite side is called as the altitude and that opposite side is called as the base.


The base of the triangles given above is $B C$. The perpendicular height (altitude) is marked as $h$.
The area of the triangle $A B C=\frac{1}{2} a h$
$\therefore$ The area of a triangle $=\frac{1}{2} \times$ base $\times$ perpendicular height (altitude)

## Example 1

Find the area of the triangle $P Q R$ given in the figure.
The perpendicular is drawn from $Q$ to the side $P R$.
$\therefore$ The base is $P R$.
$\therefore$ The area of the triangle $P Q R=\frac{1}{2} \times 10 \mathrm{~cm} \times 8 \mathrm{~cm}$

$$
=40 \mathrm{~cm}^{2}
$$



## Example 2

Find the value of $x$ according to the information marked in the figure.

When the base is taken as $B C$ and the altitude as $A D$, the area of the triangle $A B C=\frac{1}{2} \times 8 \times 9 \mathrm{~cm}^{2}=36 \mathrm{~cm}^{2}$

When the base is taken as $A B$ and the height is taken as $x$,

the area of the triangle $A B C=\frac{1}{2} \times 15 \times x \mathrm{~cm}^{2}$
Therefore, $\frac{1}{2} \times 15 \times x=36$

$$
\begin{aligned}
15 x & =36 \times 2 \\
x & =\frac{36 \times 2}{15} \\
\therefore x & =4.8 \mathrm{~cm}
\end{aligned}
$$

## Exercise 20.1

(1) Find the area of each of the triangles given below.

(2) The area of the triangular shaped flower bed in the figure is $800 \mathrm{~cm}^{2}$. Find the length marked as $x$.

(3) Find the length marked as $x$ in each of the triangles given below.
(i)

(ii)

(4) The area of the triangle $B C D$ given in the figure is $30 \mathrm{~cm}^{2}$.
(i) Find the value of $h$.
(ii) Find the area of the triangle $A B D$.

(5) The points $A, B, C$ and $D$ are located on the sides of the rectangle $P Q R S$ as indicated in the figure.
(i) Find the area of the rectangle PQRS.
(ii) Find the area of the triangle $A P D$.

(iii) Find the area of the quadrilateral $A B C D$.

### 20.3 The area of composite plane figures

When finding the area of a composite plane figure, first divide it into plane figures of which the area can easily be found. Find the area of each of these plane figures and obtain the sum.


## Example 1

Find the area of the plane figure $A B C D E$ given in the figure.

In this figure, a square and a triangle are obtained by joining $B D$.

The area of $A B D E=8 \mathrm{~cm} \times 8 \mathrm{~cm}=64 \mathrm{~cm}^{2}$ The perpendicular distance $=(13-8) \mathrm{cm}=5 \mathrm{~cm}$ from $C$ to $B D$
$\therefore$ The area of the triangle $B C D=\frac{1}{2} \times 8 \times 5 \mathrm{~cm}^{2}$


$$
=20 \mathrm{~cm}^{2}
$$

$\therefore$ The area of the whole figure $=64+20 \mathrm{~cm}^{2}$

$$
=84 \mathrm{~cm}^{2}
$$

## Exercise 20.2

(1) Find the area of each of the plane figures given below.

(2) Find the area of the shaded section in each of the figures given below.

(4) (i) Copy the rectangle $A B C D$ given in the figure onto a coloured paper and cut and separate out the four marked sections.
(ii) Construct a composite plane figure using all four sections.

(iii) Cut two other rectangular shaped laminas as above and construct two more composite plane figures and paste them in your exercise book.
(iv) Write the relationship between the area of each composite plane figure that was constructed and the area of the original rectangular shaped lamina that was used.

### 20.4 The surface area of a cube and of a cuboid

Let us find the surface area of the cuboid shaped parcel shown in the figure.


The area of face $A_{1}=4 \mathrm{~cm} \times 3 \mathrm{~cm}=12 \mathrm{~cm}^{2}$
The area of face $A_{2}=4 \mathrm{~cm} \times 2 \mathrm{~cm}=8 \mathrm{~cm}^{2}$
The area of face $A_{3}=2 \mathrm{~cm} \times 3 \mathrm{~cm}=6 \mathrm{~cm}^{2}$
$\therefore$ The total surface area $=2 \times 12+2 \times 8+2 \times 6 \mathrm{~cm}^{2} 2 \mathrm{~cm}$

$$
\begin{aligned}
& =24+16+12 \mathrm{~cm}^{2} \\
& =52 \mathrm{~cm}^{2}
\end{aligned}
$$


$\therefore$ The total surface area of the cuboid shaped parcel $=52 \mathrm{~cm}^{2}$
A cuboid of length, breadth and height equal to $x$, $y$ and $z$ units respectively, and its net are shown in the given figures.


By observing these figures, it is clear that the base which is coloured pink and the top surface which is coloured blue are equal in area. This feature can be identified by observing any cuboid shaped object such as a brick too.


Accordingly, a cuboid has three pairs of rectangular shaped faces, where each pair is of equal area.
Let us find the surface area of the cuboid by finding the area of each pair of faces which is equal in area.

The area of the base $=x y$
The area of a lengthwise face $=x z$
The area of a breadth-wise face $=y z$
The total surface area $=2 x y+2 x z+2 y z$

$$
=2(x y+x z+y z)
$$

## Activity 3

(i) Draw a figure of a cube of side length $a$ units and obtain an expression for its surface area in terms of $a$
(ii) Obtain an expression for the surface area of a cuboid of length, breadth and height equal to $a, b$ and $h$ units respectively, in terms of $a, b$ and $h$.

## According to the above activity you must have obtained that;

the surface area of a cube of side length $a$ units is $6 a^{2}$ square units, and if the surface area of a cuboid of length, breadth and height equal to $a, b$ and $h$ units respectively is $A$, then

$$
A=2(a b+b h+a h) \text { square units. }
$$

## Example 1

Find the minimum quantity of cardboard needed to construct a box the shape of a cuboid, of length, width and height equal to $20 \mathrm{~cm}, 15 \mathrm{~cm}$ and 10 cm respectively. $\stackrel{4}{4}$
Here, the minimum quantity of cardboard required is equal to the area of the 6 surfaces of the box.
The area of the 6 surfaces $=2(20 \times 15+20 \times 10+15 \times 10) \mathrm{cm}^{2}$

$$
\begin{aligned}
& =2(300+200+150) \mathrm{cm}^{2} \\
& =2 \times(650) \mathrm{cm}^{2}=1300 \mathrm{~cm}^{2}
\end{aligned}
$$

$\therefore$ The minimum quantity of cardboard needed $=1300 \mathrm{~cm}^{2}$

## Example 2

The height, width and thickness of a door panel are $180 \mathrm{~cm}, 80 \mathrm{~cm}$ and 2 cm respectively. If it costs Rs. 5 to paint $100 \mathrm{~cm}^{2}$ of the panel, find the total amount of money needed to paint the whole panel.
(4)

The surface area of the door panel $=2(180 \times 80+180 \times 2+80 \times 2) \mathrm{cm}^{2}$

$$
\begin{aligned}
& =2(14400+360+160) \mathrm{cm}^{2} \\
& =2(14920) \mathrm{cm}^{2} \\
& =29840 \mathrm{~cm}^{2}
\end{aligned}
$$

The total cost of painting the door panel at Rs. 5 per $100 \mathrm{~cm}^{2}=$ Rs. $\frac{29840}{100} \times 5$

$$
=\text { Rs. } 1492
$$

## Exercise 20.3

(1) Find the surface area of a cube of side length 10 cm .
(2) Find the surface area of a cuboid of length, breadth and height equal to 12 cm , 8 cm and 5 cm respectively.
(3) Find the surface area of each of the cuboid shaped solids shown below.
(i)

(ii)

(iii)

(4) It is required to construct a cube shaped iron box without a lid. If the length of a side is 15 cm , find the minimum amount (in $\mathrm{cm}^{2}$ ) of iron sheet needed to construct the box.
(5) The measurements of a cuboid shaped wooden pole are given in the figure. Find its surface area.

(6) The length, breadth and height of a cuboid shaped closed box are $15 \mathrm{~cm}, 15 \mathrm{~cm}$ and 8 cm respectively.
(i) Draw a sketch of two different faces of this box with its measurements.
(ii) Show that the total surface area of the box is $930 \mathrm{~cm}^{2}$.
(7) Two cube and cuboid shaped wooden blocks are shown in the figure. Anil says that the amount of paint needed to paint these


15 cm
 two blocks are equal. Explain whether you agree or disagree with this statement.
(8) Separately write down the length, width and height of two cuboids of different measurements having the same surface area of $220 \mathrm{~cm}^{2}$.

## Summary

Tue area of a triangle $=\frac{1}{2} \times$ base $\times$ perpendicular height
$\square$ The surface area of a cube of side length $a$ units is $6 a^{2}$ square units.
[1] The total surface area of a cuboid of length, width and height equal to $a, b$ and $h$ units respectively is $2 a b+2 a h+2 b h$ square units or $2(a b+a h+b h)$ square units.
(1)

(i) Which of the above plane figures $A, B, C$ have bilateral symmetry?
(ii) Which of them have rotational symmetry?
(2) Find the values of the angles represented by $x$ and $y$ in each of the following figures.

(3) Simplify.
(i) $\frac{3}{5} \times \frac{20}{27}$
(ii) $1 \frac{3}{7} \times 14$
(iii) $12 \times 2 \frac{3}{8}$
(iv) $4 \frac{1}{6} \times 1 \frac{3}{5}$
(v) $\frac{6}{7} \div \frac{2}{3}$
(vi) $\frac{7}{12} \div 1 \frac{3}{4}$
(vii) $3 \frac{2}{11} \div 2 \frac{1}{7}$
(viii) $16 \div 4 \frac{4}{7}$
(4) Find suitable values for $x, y$ and $z$ in the following chart, where $x$ is the product of three different pairs of numbers

(5) The mass of a box of biscuits is 1.02 kg . Find the mass of 15 such boxes.
(6) The cost of 1 m of cloth is Rs. 52.75 . What is the cost of 12.5 m of this cloth?
(7) The length of a reel of lace is 18.6 m . If it is cut into 6 equal parts, what would be the length of each strip?
(8) What is the maximum number of pieces of rope of length 12.27 m that can be cut from a rope of length 137.43 m ?
(9) A golden thread is pasted around the rectangular wall hanging shown in the figure.
(i) What is the total length of the thread that has been pasted?
(ii) Find the minimum length of the thread needed to create 16 such wall hangings.
(iii) If the price of 1 m of this thread is Rs. 12.80 , find the amount of money needed to buy thread for the above 16 wall hangings.

(10) If $\mathrm{A}: \mathrm{B}=4: 3$ and $\mathrm{B}: \mathrm{C}=6: 5$ find $\mathrm{A}: \mathrm{B}: \mathrm{C}$.
(11) The following table shows the ratios according to which wheat flour, sugar and margarine are mixed to make a type of sweetmeat which is produced by the two companies named $P$ and $Q$.

| Company Ratio | wheat flour : sugar | sugars : margarine |
| :---: | :---: | :---: |
| $P$ | $2: 1$ | $3: 2$ |
| $Q$ | $3: 2$ | $5: 4$ |

(i) Find the ratio of wheat flour : sugar: margarine in the sweetmeat made by company $P$.
(ii) Find the ratio of wheat flour : sugar: margarine in the sweetmeat made by company $Q$.
(iii) With reasons indicate which company produces the sweetmeat that tastes sweeter.
(12) When 7 is added to 3 times the answer that is obtained when 2 is subtract from 5 times the number denoted by $x$, the result is 61 .
(i) Construct an equation using the above information.
(ii) Solve the equation that was constructed.
(13) The mass of a packet of a certain sweetmeat is $m$ grammes. 12 such packets are packed in a box of mass 300 g . The total mass of 3 boxes packed as above is $13 \frac{1}{2} \mathrm{~kg}$. Find the mass of a packet of sweetmeat by constructing an equation and solving it.
(14) Write the following fractions and ratios as percentages.
(i) $\frac{3}{5}$
(ii) $\frac{80}{150}$
(iii) $\frac{1500}{4500}$
(iv) $3: 2$
(iv) $3: 5$
(15) $60 \%$ of the students in a class went on a trip. If the total number of students in the class is 45 , how many students did not go on the trip?
(16) A bank charges interest of Rs. 10750 per annum for a loan of Rs. 75000 . Write the interest as a percentage of the loan amount.
(17) $16 \%$ of the eggs that were being transported in a vehicle cracked due to an unavoidable circumstance. The number of eggs that cracked was 208.
(i) Find the total number of eggs that were being transported.
(ii) How many eggs did not crack?
(18) $A=\{2,3,5,7,11,13\}$
$B=\{$ letters of the word "POLONNARUWA" \}
$C=\{3,6,9,12,15\}$
(i) Fill in the blanks using the suitable symbol from $\in$ and $\notin$

- 5 ..... $A$
- 9 ..... C
- 18 ..... $C$
- $N$..... $B$
- 17 ..... $A$
- B ..... $B$
(ii) Write down the values of $n(A), n(B)$ and $n(C)$.
(19) $D=\{$ even prime numbers greater than 10$\}$
(i) Write down the set $D$.
(ii) What is the value of $n(D)$ ?
(iii) Write the special name given for the set $D$.
(20) (a) The surface area of a cube is $150 \mathrm{~cm}^{2}$. Find the length of an edge of the cube.
(b) (i) Find the surface area of the cuboid shaped block of wood shown in the figure.

(ii) The above cuboid shaped block of wood is cut so that two equal cubes are formed. What is the surface area of each one of them?
(iii) Indicate whether the surface area of each of these cubes is exactly half the surface area of the cuboid, according to the answers to (i) and (ii) above.

