## 18

Percentages

By studying this lesson you will be able to,
- express a fraction as a percentage,
- express a percentage as a fraction,
- to know the relationship between ratios and percentages,
- calculate a percentage from a given quantity, and
- find the total quantity when a percentage and its corresponding amount are given.

18.1 Expressing a fraction as a percentage

You learnt in Grade 7 that the symbol "%" is known as the percentage sign.

The coloured region of the figure is $\frac{1}{4}$ of the whole figure; that is, $\frac{25}{100}$ of the whole figure.
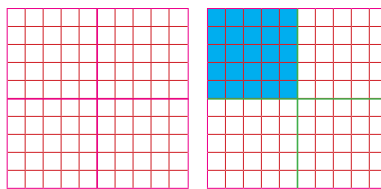
You have learnt that this is 25%, as a percentage of the whole figure. It is read as twenty five percent. Expressing it as such is called, **expressing a portion of a whole as a percentage**.

> A fraction can be expressed as a percentage by writing an equivalent fraction with denominator 100.

We can write the coloured region as a percentage of the whole figure as shown below.

$\frac{1}{4} \times 100\% = 25\%$,

As $\frac{1}{4} = 0.25$, the coloured region of the figure is 0.25 of the whole figure. As a percentage it is, $0.25 \times 100\% = 25\%$.

> By multiplying a given decimal number or fraction by 100%, that decimal number or fraction can be expressed as a percentage.

Example 1

Given that the initial amount is 1, express each of the following quantities as a percentage of the initial amount.

(i) $\frac{3}{8}$ (ii) $\frac{1}{12}$ (iii) 0.068   (iv) $\frac{2}{3}$

(i) $\frac{3}{8} = \frac{3}{8} \times 100 \text{ \%} = 37.5 \text{ \%}$

(ii) $\frac{1}{12} = \frac{1}{12} \times 100 \text{ \%} = \frac{100}{12}\%$

$$= 8\frac{4}{12} \text{ \%}$$

$$= 8\frac{1}{3} \text{ \%}$$

(iii) $0.068 = 0.068 \times 100\% = 6.8 \text{ \%}$

(iv) $\frac{2}{3} = \frac{2}{3} \times 100 \text{ \%} = \frac{200}{3} \text{ \%} = 66\frac{2}{3} \text{ \%}$

Exercise 18.1

Given that the initial amount is 1, express each of the following quantities as a percentage of the initial amount.

(i) $\frac{1}{2}$ (ii) 0.7   (iii) 2.4   (iv) 7.8

(v) 4.025   (vi) 6   (vii) 0.067   (viii) $1\frac{11}{50}$

(ix) $\frac{1}{3}$ (x) $\frac{5}{6}$ (xi) $\frac{9}{11}$ (xii) $1\frac{3}{7}$

18.2 Expressing a percentage as a fraction

Let us consider the following examples in order to learn how to convert a percentage into a fraction.

Example 1

Express each of the following percentages as a fraction.

(i) 20 %   (ii) 125 %   (iii) $33\frac{1}{3}\%$

(i) $20 \text{ \%} = \frac{20}{100} = \frac{1}{5}$

(ii) $125 \text{ \%} = \frac{125}{100} = \frac{5}{4} = 1\frac{1}{4}$

(iii) $33\frac{1}{3}\% = 33\frac{1}{3} \div 100 = \frac{100}{3} \div 100 = \frac{100}{3} \times \frac{1}{100} = \frac{1}{3}$

$5(x - y)$ $\sqrt{64}$ x^0 $1\frac{7}{10}$ $(-1)^1$

Exercise 18.2

Express each of the following percentages as a fraction and simplify it.

(i) 25%   (ii) 40%   (iii) 16%   (iv) 150%

(v) 120%   (vi) 58%   (vii) 32%   (viii) 175%

(ix) $12\frac{1}{3}$ %   (x) $3\frac{1}{3}$ %   (xi) $1\frac{3}{5}$ %   (xii) 2.25%

18.3 Ratios and percentages

"8% of the eggs in the basket are rotten". This means that 100 such eggs would contain 8 rotten eggs. Accordingly, the ratio of the number of rotten eggs to the total number of eggs is 8 : 100. You learnt this in Grade 7.

- **Writing a ratio corresponding to a percentage**

Now let us see how to write the ratio corresponding to the percentage, 30%. 30% can be written as 30 : 100.

$30 : 100 = 30 \div 10 : 100 \div 10 = 3 : 10.$

Accordingly, the ratio corresponding to the percentage 30% is 3 : 10.

- **Writing a percentage corresponding to a ratio**

Now let us see how to express the ratio 1 : 4 as a percentage.

The percentage corresponding to a given ratio can be found by writing the equivalent ratio with 100 as its second term.

$1 : 4 = 1 \times 25 : 4 \times 25 = 25 : 100$

Since the ratio 25:100 can be written as $\frac{25}{100}$, the percentage corresponding to the ratio 1: 4 is 25%.

Example 1

Express 20% as a ratio.

20% can be written as 20 : 100.

Now, 20 : 100 = 20 ÷ 20 : 100 ÷ 20 = 1 : 5.

Accordingly, the ratio corresponding to 20% is 1 : 5.

Note: When writing a ratio, it should be expressed in its simplest form.

Example 2

Express $12\frac{1}{2}\%$ as a ratio.

$$12\frac{1}{2}\% = \frac{12\frac{1}{2}}{100} = 12\frac{1}{2} \div 100 = \frac{25}{2} \times \frac{1}{100} = \frac{25}{200}$$

$\frac{25}{200}$ can be written as 25 : 200. Now,

$$25 : 200 = 25 \div 25 : 200 \div 25$$
$$= 1 : 8$$

Accordingly, the ratio corresponding to $12\frac{1}{2}\%$ is 1 : 8.

Example 3

Express the ratio 2 : 5 as a percentage.

$$2 : 5 = 2 \times 20 : 5 \times 20$$
$$= 40 : 100$$

Accordingly, the percentage corresponding to the ratio 2 : 5 is 40%.

Example 4

Express the ratio 3 : 2 as a percentage.

$$3 : 2 = 3 \times 50 : 2 \times 50$$
$$= 150 : 100$$

Accordingly, the percentage corresponding to the ratio 3 : 2 is 150%.

Example 5

Express the ratio 1 : 3 as a percentage.

$1 : 3 = \frac{1}{3} : 1 = \frac{1}{3} \times 100 : 1 \times 100 = \frac{100}{3} : 100.$

Accordingly, the percentage corresponding to the ratio 1 : 3 is $\frac{100}{3}\%$ $\left(\text{i.e., } 33\frac{1}{3}\%\right)$.

Exercise 18.3

(1) Write the corresponding ratio for each of the following percentages.

    (i) 25%   (ii) 20%   (iii) 45%   (iv) 8%

    (v) 125%   (vi) 300%   (vii) $5\frac{1}{2}\%$ (viii) $33\frac{1}{3}\%$

(2) Write the corresponding percentage for each of the following ratios.

    (i) 1 : 2   (ii) 7 : 20   (iii) 13 : 25   (iv) 27 : 50

    (v) 3 : 2   (vi) 9 : 4   (vii) 6 : 5   (viii) 13 : 10

    (ix) 1 : 7   (x) 3 : 17

(3) 28 males and 22 females participated in a meeting.

    (i) Write the ratio of the males to the total participants, and write the corresponding percentage. Describe what this percentage means in words.

    (ii) Write the ratio of the females to the total participants and write the corresponding percentage.

18.4 Calculating the corresponding percentage when a certain quantity from a total amount is given

Percentages are used when comparing different quantities of a particular kind, or when comparing amounts in different groups. When such comparisons are made, the relevant quantities should be expressed in the same units.

You learnt in Grade 7 how to calculate the relevant percentage when you are given a certain quantity from a total amount.

When a quantity is given, write it as a fraction of the total amount. You can then obtain the corresponding percentage by multiplying the fraction by 100%.

If 30 of the 200 mangoes brought by a vendor to sell were rotten, let us find the percentage of rotten mangoes in the whole stock.

The total number of mangoes brought by the vendor to sell $= 200$

The number of rotten mangoes $= 30$

The number of rotten mangoes as a fraction of the total number of mangoes $= \dfrac{30}{200}$

The percentage of rotten mangoes $= \dfrac{30}{200} \times 100 \%$

$= 15 \%$

Example 1

The distance from town A to town B is 50 km. A man leaving town A, travels 20 km by bus and the rest of the distance by train. Express the distance travelled by bus as a percentage of the total distance.

The distance travelled by bus as a fraction of the total distance $= \dfrac{20}{50}$

The distance travelled by bus as a percentage $= \dfrac{20}{50} \times 100 \%$

$= 40 \%$

Exercise 18.4

(1) Express the first value of each pair given below as a percentage of the second value.

   (i) 200g from 1 kg   (ii) 25 cm from 1 m

   (iii) 750 m from 1 km   (iv) 300 ml from 1 l (v) 20 minutes from 1 hour

(2) If 30 of the 50 students in a class are girls, find the number of girls in the class as a percentage of the total number of students.

(3) If a person who borrowed Rs. 2000, pays Rs. 250 as interest at the end of a year, find the annual interest rate he paid.

(4) If 5 from a lot of 25 fire crackers bought by Prathapa to light on New Year's day did not explode, calculate the number of crackers that exploded as a percentage of the total number of crackers.

(5) If Kareem obtained 36 marks for an assignment marked out of 40, express Kareem's marks as a percentage of the total marks allocated for the assignment.

(6) Mr. Perera's monthly salary is Rs. 30 000. He spends Rs. 15 000 on food, Rs. 3000 on transport and the rest of his salary on other expenses.

   (i) Find the amount spent on food as a percentage of his salary.

  (ii) Find the amount spent on transport as a percentage of his salary.

18.5 Finding the quantity corresponding to a percentage, when the percentage and the total amount are given

The total number of students in a school is 1500. If 48% of the students are boys, let us find the number of boys in the school.

$$\text{The total number of students in the school} = 1500$$
$$\text{The percentage of boys} = 48\ \%$$
$$\text{The number of boys in the school} = 1500 \times \frac{48}{100}$$
$$= 720$$

Example 1

If a man saves 5% from his monthly salary of Rs. 20 000, how much of money does he save?

$$\text{Monthly salary} = \text{Rs. } 20\ 000$$
$$\text{The percentage saved} = 5\ \%$$
$$\text{The amount of money saved} = \text{Rs. } 20\ 000 \times \frac{5}{100}$$
$$= \text{Rs. } 1000$$

Exercise 18.5

(1) If the prevailing price of Rs. 120 per litre of fuel is increased by 10%, by how many rupees will the price of 1 litre of fuel increase?

(2) If the minimum percentage of marks required to pass an examination marked out of 300 is 60%, what is the minimum mark required to pass the examination?

(3) 15% of the workers in an establishment are men. If the total number of workers in the establishment is 800, how many male workers are there?

(4) A person travels 60% of a journey by train, 35% by bus and the rest of the journey by taxi. The total distance of the journey is 140 km.

    (i) Find the distance travelled by train.

    (ii) Find the distance travelled by bus.

(5) Mr. Ranasinghe's monthly salary is Rs. 45 000. He puts aside 30% of his salary for food, 20% for transport and the rest of the salary for other expenses.

    (i) How much money does he put aside for food?

    (ii) How much money does he put aside for transport?

18.6 Finding the total amount, when a certain quantity and its corresponding percentage are given

Let us find the total sum of money, if the value of 10% of the sum is Rs. 250.

$$
\begin{aligned}
10\% \text{ of the sum} &= \text{Rs. } 250 \\
1\% \text{ of the sum} &= \text{Rs. } \frac{250}{10} \\
100\% \text{ of the sum (therefore the total sum)} &= \text{Rs. } \frac{250}{10} \times 100 \\
&= \text{Rs. } 2500
\end{aligned}
$$

Example 1

60% of the students in a class use public transport to travel to school. If the number of students in this class who do not use public transport is 16, find the total number of students in the class.

Percentage of children who do not use public transport = 100% − 60% = 40%

$$
\begin{aligned}
40\% \text{ of the students} &= 16 \\
1\% \text{ of the students} &= \frac{16}{40} \\
100\% \text{ of the students} &= \frac{16}{40} \times 100 \\
\text{Total number of students} &= 40
\end{aligned}
$$

$5(x - y)$ $\sqrt{64}$ $1\frac{7}{10}$ $(-1)^1$

(1) If 30% of a person's salary is Rs. 7200, how much is his salary?

(2) The attendance of the students of a school on a rainy day was 60%. If the number of students who attended school that day was 420, find the total number of students in the school.

(3) After spending 65% of the money he had in hand, if a person had a balance of Rs. 1400, what is the total amount of money he initially had?

(4) A metal alloy is made by mixing iron and zinc. If 36% of the alloy is zinc and the amount of iron in the alloy is 160 g, calculate the total mass of the alloy.

(5) A man gives 5% of the money he obtained by selling his vehicle to a broker. If he is left with Rs. 475 000 there after,

  (i) find the selling price of the vehicle.

  (ii) find the broker fee paid.

(6) 40% of the employees working in a factory are women. If the number of male employees in the factory is 75, how many employees are there in total?

(7) A doctor gave a diet plan to Rajitha to reduce his mass by 9 kg within 6 months. 9 kg is 10% of his total mass.

  (i) How much is Rajitha's mass?

  (ii) 12% of his mass was reduced during the said time period. How much is Rajitha's mass now?

Summary

  📖 A fraction can be expressed as a percentage by writing an equivalent fraction with denominator 100.

  📖 A given fraction or decimal number can be converted into a percentage by multiplying it by 100%.

  📖 A percentage corresponding to a given ratio can be found by writing an equivalent ratio with second term equal to 100.