17 Equations

By studying this lesson you will be able to,

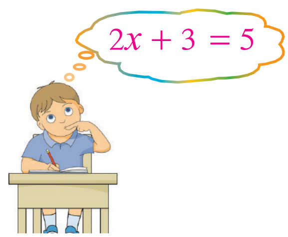
- construct simple equations in one unknown where the coefficient of the unknown is a fraction,
- construct simple equations using one pair of brackets,
- solve simple equations, and
- check the accuracy of the solution of a simple equation.

17.1 Equations

You have learnt that, when the value represented by an algebraic expression is equal to the value of a given number, this can be expressed as, “algebraic expression = number”.

You have also learnt that, when the value represented by an algebraic expression is equal to the value represented by another algebraic expression, this can be expressed as,

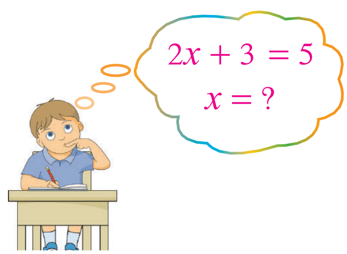
"first algebraic expression = second algebraic expression".



Relationships of the above forms are called **equations**.

$2x + 3 = 5$ is an equation. It has only one unknown term x, of which the index is one. Such equations are called simple equations.

Finding the value of the unknown term for which the value of the left hand side of the equation is equal to the value of the right hand side, is called **solving the equation**.



The value obtained for the unknown by solving the equation, is called **the solution of the equation**. Simple equations have only one solution.

The above equation $2x + 3 = 5$ denotes the following; "5 is obtained by adding 3 to twice the value of the unknown term".

Let us recall the method of solving this equation.

$$
\begin{aligned}
2x + 3 &= 5 \\
2x + 3 - 3 &= 5 - 3 \quad \text{(subtract 3 from both sides, since } 3 - 3 = 0) \\
2x &= 2 \\
\frac{2x}{2} &= \frac{2}{2} \quad \text{(divide both sides by 2, since } \frac{2}{2} = 1) \\
\therefore x &= 1
\end{aligned}
$$

$$
\boxed{\begin{aligned} 2x + 3 &= 5 \\ 2x &= 5 - 3 \end{aligned}}
$$

Let us check the accuracy of the solution that we obtained.

When the value obtained for the unknown is substituted in the equation, if we get the same value on the left hand side of the equation as that of the right hand side, then the solution is correct.

When $x = 1$, the left hand side of the equation is, $2x + 3 = 2 \times 1 + 3$

$= 2 + 3$

$= 5$

The right hand side of the equation $= 5$

That is, the left hand side = the right hand side.

Therefore $x = 1$ is the correct solution of the equation $2x + 3 = 5$.

- We obtain the same value on the two sides of an equation when we subtract the same number from both sides.
- We obtain the same value on the two sides of an equation when we add the same number to both sides.
- We obtain the same value on the two sides of an equation when we divide both sides by the same non-zero number.
- We obtain the same value on the two sides of an equation when we multiply both sides by the same number.

Do the following review exercise to recall what you have learnt about constructing simple equations and solving them.

Review Exercise

(1) Construct a simple equation for each of the statements given below and solve it.

(i) When 5 is added to the value of x, the result is 12.
(ii) When 3 is subtracted from the value of a, the result is 8.
(iii) Shashi's age is denoted by x. Her sister who is 2 years older to her is 12 years old.
(iv) I have an amount of money denoted by x rupees. Twice this amount is 60 rupees.
(v) When 5 is subtracted from three times the value of x, the result is 1.
(vi) My father is 44 years old today. His age is 5 years more than 3 times my age. (Take my age today as y years.)

(2) Solve each of the following equations.

(i) $x + 10 = 15$ (ii) $x - 5 = 25$ (iii) $5x = 20$

(iv) $2x + 3 = 13$ (v) $4x - 1 = 19$ (vi) $3x + 22 = 13$

17.2 More on the construction of simple equations

- ### Construction of simple equations where the coefficient of the unknown is a fraction

Simple equations where the coefficient of the unknown is a whole number have been constructed earlier. Now let us consider how simple equations where the coefficient of the unknown is a fraction are constructed.

My brother's age is 3 years more than one fourth of my age. He is 6 years old now. Let us construct an equation by using this information.

Let x be my age in years.

Then, one fourth of my age $= \frac{1}{4} \times x = \frac{x}{4}$

Since my brother's age is 3 years more than one fourth my age,

My brother's age $= \frac{x}{4} + 3$

Since my brother is 6 years old, $\frac{x}{4} + 3 = 6$

- ### Constructing simple equations using one pair of brackets

By adding a certain amount of money to the 8 rupees I gave Kasun, he bought 26 olives at the price of two fruits per rupee.
Let us construct a simple equation with this information to find the amount Kasun spent to buy the olives.

Let x rupees be the amount Kasun spent.

Then the total amount spent to buy the olives $= x + 8$ rupees

$\left.\begin{array}{l}\text{The total number of olives that can be bought}\\ \text{for } x + 8 \text{ rupees at the price of two fruits per rupee}\end{array}\right\} = 2(x + 8)$

It is necessary to use brackets here because the total amount, which is $x + 8$ rupees, needs to be multiplied by 2.We express the fact that the sum of the two terms x and 8 is multiplied by 2, as $2 \times (x + 8)$, by using brackets.
The number of olives that were bought is 26. Therefore,

$2\,(x + 8) = 26$

Example 1

Nimali plucked some mangoes from the tree in her garden, saved 16 for herself, and by selling the rest at 25 rupees each, made 875 rupees.
Construct a simple equation with this information, to find the total number of mangoes that Nimali plucked.

Let x be the total number of mangoes that were plucked.

The number of mangoes that were sold $= x - 16$

To find the amount of money she received by selling the mangoes at 25 rupees per fruit, $(x - 16)$ should be multiplied by 25.

This amount is $25\,(x - 16)$.

As the amount received by selling the mangoes is 875 rupees,

$25(x - 16) = 875$.

Exercise 17.1

(1) Construct a simple equation for each of the following statements.

(i) When 5 is added to half the value of x, the result is 8.

(ii) A parcel has one book of value x rupees and another book worth 50 rupees. The value of the books in 5 such parcels is 750 rupees.

(iii) Raj's brother's age is one year less than one third of Raj's age. Raj's brother is 3 years old.

(iv) Rashmi has 200 rupees which is 5 times the amount that is obtained when 10 rupees is deducted from twice the amount that Vishmi has.

(v) When 5 is subtracted from half of a certain number, the result is 2.

17.3 Solving equations in one unknown when the coefficient of the unknown is a fraction

Let us consider the method of solving equations in one unknown when the coefficient of the unknown is a fraction.

Let us solve the equation $\frac{x}{2} = 3$.

$\frac{x}{2} \times 2 = 3 \times 2$ (multiply both sides by 2)

$\frac{x \times \cancel{2}^{1}}{\cancel{2}_{1}} = 6$

$\therefore\ x = 6$

Example 1

Solve $\frac{2}{3}x - 1 = 3$.

$$\frac{2x}{3} - 1 = 3$$

$$\frac{2x}{3} - 1 + 1 = 3 + 1 \quad \text{(add 1 to both sides)} \ (-1 + 1 = 0)$$

$$\frac{2x}{3} = 4$$

$$\frac{2x}{3} \times 3 = 4 \times 3 \quad \text{(multiply both sides by 3)} \ \left(\frac{2}{\cancel{3}} \times \frac{\cancel{3}}{1} = 2\right)$$

$$2x = 12$$

$$\frac{2x}{2} = \frac{12}{2} \quad \text{(divide both sides by 2)}$$

$$x = 6$$

Now let us check the accuracy of the solution $x = 6$.

When $x = 6$, the left hand side $= \frac{2x}{3} - 1 = \frac{2 \times 6}{3} - 1$

$= \frac{12}{3} - 1$

$= 4 - 1$

$= 3$

Right hand side $= 3$

That is, the left hand side = the right hand side

$\therefore$ $x = 6$ is the correct solution of the equation $\frac{2x}{3} - 1 = 3$.

Example 2

Solve $2 - \frac{3}{10}a = 5$.

$$2 - \frac{3}{10}a - 2 = 5 - 2 \ \text{(subtract 2 from both sides)}$$

$$-\frac{3}{10}a = 3$$

$$-\frac{3a}{\cancel{10}_1} \times \cancel{10}^1 = 3 \times 10 \ \text{(multiply both sides by 10)}$$

$$-3a = 30$$

$$\frac{-3a}{(-3)} = \frac{30}{(-3)} \ \text{(divide both sides by } (-3))$$

$$a = -10$$

Exercise 17.2

(1) Solve each of the following equations. Check the accuracy of the solution.

(i) $\dfrac{x}{5} = 2$ (ii) $\dfrac{a}{3} + 1 = 3$ (iii) $\dfrac{p}{4} - 1 = 2$

(iv) $\dfrac{2x}{5} = 7$ (v) $3 - \dfrac{2y}{5} = 1\dfrac{4}{5}$ (vi) $\dfrac{5m}{2} - 2 = \dfrac{1}{2}$

17.4 Solving equations having one pair of brackets

Let us solve the equation $2\,(x + 3) = 10$.

Method I

$$2\,(x + 3) = 10$$

$$\frac{\cancel{2}^{1}(x + 3)}{\cancel{2}_{1}} = \frac{10}{2} \quad \text{(divide both sides by 2)}$$

$$x + 3 = 5$$

$$x + 3 - 3 = 5 - 3 \quad \text{(subtract 3 from both sides)}$$

$$\therefore\ x = 2$$

Method II

$$2\,(x + 3) = 10$$

$$2x + 6 = 10$$

$$2x + 6 - 6 = 10 - 6$$

$$2x = 4$$

$$\frac{2x}{2} = \frac{4}{2}$$

$$\therefore\ x = 2$$

We can check the accuracy of the solution by substituting $x = 2$ in the equation $2\,(x + 3) = 10$.

Example 1

Solve $10(1-2x)+1=6$.

$$10(1-2x)+1=6$$

$$10(1-2x)+1-1=6-1 \quad \text{(subtract 1 from both sides)}$$
$$10(1-2x)=5$$
$$\frac{10(1-2x)}{10}=\frac{5}{10} \quad \text{(divide both sides by 10)}$$
$$1-2x=\frac{1}{2}$$
$$1-2x-1=\frac{1}{2}-1 \quad \text{(subtract 1 from both sides)}$$
$$-2x=-\frac{1}{2}$$
$$\frac{-2x}{-2}=-\frac{1}{2}\div(-2) \quad \text{(divide both sides by –2)}$$
$$x=\frac{(-1)}{2}\times\frac{1}{(-2)}=\frac{(-1)}{(-4)}$$
$$\therefore\ x=\frac{1}{4}$$

Let us check the accuracy of the solution.

When $x=\frac{1}{4}$, the left hand side $=10\ (1-2x)+1$

$$=10\ (1-2\times\frac{1}{4})+1$$
$$=10\ (1-\frac{1}{2})+1$$
$$=10\times\frac{1}{2}+1$$
$$=5+1$$
$$=6$$

That is, the left hand side = the right hand side

$\therefore\ x=\frac{1}{4}$ is the solution.

Exercise 17.3

(1) Solve each of the following equations. Check the accuracy of the solution.

(i) $2\ (x+3)=8$ (ii) $3\ (p-2)=9$ (iii) $2\ (2x-1)=6$

(iv) $5\ (1-3x)=20$ (v) $2\ (3-4x)-1=-19$ (vi) $10\ (2x+1)-5=25$

(vii) $2(\frac{x}{3}-1)=(-6)$ (viii) $2\ (\frac{5x}{2}+1)=-18$ (ix) $2-\frac{3x}{4}=(-7)$

(x) $\dfrac{(x-2)}{5} = 2$ (xi) $\dfrac{(3-x)}{2} - 1 = \dfrac{3}{7}$ (xii) $\dfrac{(2p-1)}{3} + 2 = \dfrac{5}{9}$

(2) An envelope contains x number of 10 rupee notes and five 20 rupee notes. The total amount of money in 5 such envelopes is 750 rupees.

(i) Construct an equation with the information given above.
(ii) By solving the equation, find the number of 10 rupee notes in one envelope.

Miscellaneous Exercise

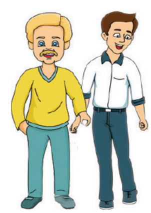
(1) Let x be a positive integer. When 12 is added to twice the integer right after x, the result is 38.

(i) Express the positive integer right after x in terms of x.
(ii) Construct an equation in terms of x.
(iii) Find the positive integer denoted by x by solving the above equation.

(2) A factory worker receives p rupees as his daily wage and an additional 100 rupees as an allowance for each day he works. He worked 20 days during a certain month and received 20 000 rupees in total. What is his daily wage?

(3) A father's age is a years and his son's age is 31 years. 5 years ago the son's age was one year more than $\frac{1}{2}$ the age of his father at that time.

(i) What was the son's age 5 years ago?
(ii) Write the father's age 5 years ago in terms of a.
(iii) Construct an equation in a using the above information.
(iv) Find the father's present age by solving the equation.



Summary

- When an algebraic expression is equated to a number or another algebraic expression, such a relationship is known as an equation.
- The solution to an equation is the value of the unknown term that satisfies the equation.