## Ratios

## By studying this lesson you will be able to,

- represent a ratio as a fraction,
- determine continued ratios, and
- solve problems involving continued ratios.


### 16.1 Ratios

Let us recall what you learnt about ratios in Grade 7.
You learnt that a ratio is a numerical relationship between two or more quantities which are measured in the same unit.

You also learnt when comparing two groups, that a ratio is a numerical relationship between the magnitudes of the two groups.
Consider the following example,
When preparing a concrete mixture, 1 pan of cement, 3 pans of sand and 4 pans of gravel are mixed together.


Cement


Sand


The ratio in which cement, sand and gravel are mixed together when preparing this concrete mixture can be expressed as $1: 3: 4$. This is read as " 1 to 3 to 4 ". Here, 1, 3 and 4 are the terms of the ratio.

By multiplying or dividing each term of a given ratio by a number which is greater than 0 , a ratio equivalent to the given ratio can be obtained.

If the terms of a ratio are whole numbers and if the HCF of these numbers is 1 , then
we say that the ratio is written in its simplest form.

- If the terms of a ratio are whole numbers and if they have a common factor greater than 1 , then it can be expressed in its simplest form by dividing each term of the ratio by the highest common factor of all the terms.
Do the following review exercise to revise the facts you have learnt previously on ratios.


## Review Exercise

(1) Write three equivalent ratios for each ratio given below.
(i) $2: 5$
(ii) $3: 4$
(iii) $9: 6: 3$
(iv) $8: 2: 4$
(2) Write each of the ratios given below in the simplest form.
(i) $6: 15$
(ii) $8: 20$
(iii) $30: 18: 36$
(iv) $40: 16: 64$
(3) Join the ratio in column $A$ to the equivalent ratio in column $B$.

| $A$ | $B$ |  |  |
| :--- | :--- | :--- | :--- |
| 4 | $:$ | 3 | 2 |
| 10 | $:$ | $:$ | 3 |
| 6 | $:$ | 5 |  |
| 2 | $:$ | $7: 9$ | 10 |$): 35: 45$

(4) Rewrite and fill in the blanks.
(i) $3: 4=\square: 8$
(ii) $8: 5=16$ :
(iii) $1: 3=\square: 12$
(iv) $\square: 6=32: 48$
(v) $15: 25=\square: 5$
(vi) $12: \square=36: 15$
(5) The ratio of the price of a pencil to that of a book is $3: 4$. If the price of a pencil is Rs.15, find the price of a book.
(6) The ratio of the mass of Prathapa to that of Nimdiya is $9: 11$. If Nimdiya's mass is 55 kg , find Prathapa's mass.
(7) Saman, Suresh and Kassim are friends. The ratio of their heights is $5: 4: 6$. If Saman's height is 125 cm , calculate the heights of Suresh and Kassim.

### 16.2 Representing a ratio as a fraction

The following example shows how a ratio can be expressed as a fraction by writing an equivalent ratio in which one term is equal to 1 .

$>$ In a race, Dilki ran 30 m in the time that Sayuni took to run 50 m . The ratio of the distance that Dilki ran to the distance that Sayuni ran is $30: 50$. This ratio written in its simplest form is $3: 5$. This means that Dilki ran 3 m in the time that Sayuni took to run 5 m .

- When we divide both the terms in the ratio $3: 5$ by 5 we obtain $\frac{3}{5}: \frac{5}{5}=\frac{3}{5}: 1$.This means that Dilki runs $\frac{3}{5} \mathrm{~m}$ in the time that Sayuni runs 1 m . That is, when the distance run by Dilki is expressed as a fraction of the distance run by Sayuni, it is $\frac{3}{5}$.
- By dividing both terms of the ratio $3: 5$ by 3 , we can in a similar manner express the distance run by Sayuni as a fraction of the distance run by Dilki as $\frac{5}{3}$.
- Since Sayuni runs 5 m in the time that Dilki runs 3 m , the total distance run by them during this period is 8 m . When we divide both terms in the ratio by 8 we obtain $\frac{3}{8}: \frac{5}{8}$. This means that when the distance run by Dilki is expressed as a fraction of the total distance it is $\frac{3}{8}$, and that the distance run by Sayuni is $\frac{5}{8}$ of the total distance.
$>$ Sureni and Pradeepa shared a certain amount of money. Sureni received Rs. 35 while Pradeepa received Rs. 25. The ratio in which the money was shared between Sureni and Pradeepa can be expressed as $35: 25$.
When this is expressed in its simplest form it is $7: 5$.
The total amount shared between them $=$ Rs. $35+25=$ Rs. 60
$\therefore$ The amount received by Sureni as a fraction of the total amount $=\frac{35}{60}=\frac{7}{12}$
We can obtain the above fraction as shown below too.
The ratio in which the money was shared between Sureni and Predeepa $=7: 5$
The amount received by Sureni as a fraction of the whole amount $=\frac{7}{7+5}=\frac{7}{12}$
Similarly,
the amount received by Pradeepa as a fraction of the whole amount $=\frac{5}{12}$


## Example 1



To make a mixed fruit drink, mango juice, pineapple juice and orange juice are mixed in the ratio $2: 3: 1$. Find the fraction of each type of juice in the drink.

The ratio of mango juice to pineapple juice to orange juice $=2: 3: 1$
$\therefore$ The sum of the terms of the ratio $=2+3+1=6$
The fraction of mango juice in the drink $=\frac{2}{6}$
The fraction of pineapple juice in the drink $=\frac{3}{6}$
The fraction of orange juice in the drink $=\frac{1}{6}$

## Exercise 16.1

(1) Sudesh and Rahim shared some money. Sudesh received Rs. 450 while Rahim received Rs. 500.
(i) Write in the simplest form, the ratio in which the money was divided between them.
(ii) Write the amount Sudesh received as a fraction of the amount Rahim received. Express this in its simplest form.
(iii) What fraction of the total amount did Rahim receive?
(2) A stock of dry rations is distributed among the three families $A, B$ and $C$ in the ratio $A: B: C=4: 5: 3$.
(i) Express separately the quantity of dry rations received by each family as a fraction of the whole stock.
(ii) What fraction of the amount that $B$ received is the amount that $A$ received?
(iii) What fraction of the amount that $C$ received is the amount that $A$ received?
(3) In a running event, Amashi ran 70 m in the time that it took Gayani to run 40 m .
(i) Write the ratio of the distance run by Amashi to that run by Gayani, in its simplest form.
(ii) Using the above ratio, write as a fraction,
 the distance run by Gayani in the time that Amashi runs 1 m .
(iii) Write as a fraction, the distance run by Amashi in the time that Gayani runs 1 m .
(iv) Express the distance run by Amashi as a fraction of the total distance run by the two of them.
(v) Express the distance run by Gayani as a fraction of the total distance run by the two of them.
(4) The floor area of the bedroom of a house is $\frac{2}{3}$ the floor area of the sitting room.
(i) What is the ratio of the floor area of the bedroom to that of the sitting room?
(ii) What fraction of the total floor area of the bedroom and the sitting room is the floor area of the sitting room?
(iii) What fraction of the total floor area of the bedroom and the sitting room is the difference between the floor areas of these two rooms?

### 16.3 Dividing in a given ratio

In our day to day life, there are many occasions when we have to share things with each other. Sometimes the sharing is equal while at other times it is not.

Let us recall what we learnt in Grade 7 about dividing something in a ratio.
$A, B$ and $C$ are three people. If Rs. 2000 was divided among them in the ratio $2: 3: 5$, let us calculate how much each person received.

The ratio in which money is divided among them $=2: 3: 5$
The total number of parts $=2+3+5=10$
The amount received by $A$ as a fraction of the whole $=\frac{2}{10}$

$$
\text { The amount received by } \begin{aligned}
A & =\text { Rs. } 2000 \times \frac{2}{10} \\
& =\text { Rs. } 400
\end{aligned}
$$

The amount received by $B$ as a fraction of the whole $=\frac{3}{10}$

$$
\begin{aligned}
\text { The amount received by } B & =\text { Rs. } 2000 \times \frac{3}{10} \\
& =\text { Rs. } 600
\end{aligned}
$$

The amount received by $C$ as a fraction of the whole $=\frac{5}{10}$

$$
\text { The amount received by } \begin{aligned}
C & =\text { Rs. } 2000 \times \frac{5}{10} \\
& =\text { Rs. } 1000
\end{aligned}
$$

## - Dividing the profit when different amounts are invested for the same period of time

Sandun and Sashika started a business at the beginning of a certain year by investing Rs. 30000 and Rs. 40000 respectively. At the end of the year, the profit from the business was Rs. 28000 . They shared it according to the ratio in which they invested money.
Let us consider how to calculate the share of the profit received by each of them.
The ratio in which Sandun and Sashika invested money $=30000: 40000$

$$
=3: 4
$$

The ratio in which the profit should be divided between them $=3: 4$
The total number of parts $=3+4=7$
Sandun's profit as a fraction of the whole $=\frac{3}{7}$
The profit from the business = Rs. 28000
Sandun's share of the profit $=$ Rs. $28000 \times \frac{3}{7}$

$$
\text { = Rs. } 12000
$$

Sashika's profit as a fraction of the whole $=\frac{4}{7}$
Sashika's share of the profit $=$ Rs. $28000 \times \frac{4}{7}$
=Rs. 16000

## - Dividing the profit when different amounts are invested for different periods of time

If people invest different amounts in a business for different periods of time, both the amount invested and the period of investment need to be considered when the profit is shared.
Let us now consider such an example.

Kumudu started a business on January 1st of a certain year by investing Rs. 20000. Sumudu joined the business by investing Rs. 30000 two months later. At the end of the year, the profit from the business was Rs. 36000.
Let us consider how the profit should be divided between the two of them.
Observe that in this case, the investments they made and the periods of investment are both different.

| Name | Amount invested | Period of investment | Amount $\times$ Period |
| :---: | :---: | :---: | :---: |
| Kumudu | Rs. 20000 | 12 months | $20000 \times 12$ |
| Sumudu | Rs. 30000 | 10 months | $30000 \times 10$ |

In such a situation, it is not fair to divide the profit by considering only the investments. Similarly, since the amounts invested are different, it is not fair to consider only the periods of investment either.
We have to consider both the investments and the periods of investment. This is done by basing the ratio in which the profit should be divided on the product of the amount invested and the period of investment (the last column of the above table).
The ratio in which the profits should be divided

$$
\text { between Kumudu and Sumudu }\}=20000 \times 12: 30000 \times 10
$$

$$
=240000: 300000
$$

$$
=4: 5
$$

The sum of the parts $=4+5=9$
The amount Kumudu should receive $=$ Rs. $36000 \times \frac{4}{9}$
= Rs. 16000
The amount Sumudu should receive $=$ Rs. $36000 \times \frac{5}{9}$
$=$ Rs. 20000

## Example 1

Siripala starts a business in January by investing Rs. 30000 . His friend Hussain joins the business two months later by investing Rs. 24000 , and his friend Nadaraja joins the business two months after that by investing Rs. 60000 . Calculate the ratio in which the profit should be divided between them at the end of a year.

| Siripala |  | Hussain |  | Nadaraja |
| :--- | :--- | :--- | :--- | :--- |
| $30000 \times 12$ | $:$ | $24000 \times 10$ | $:$ | $60000 \times 8$ |
| 360000 | $:$ | 240000 | $:$ | 480000 |
| 3 | $:$ | 2 | $:$ | 4 |

(1) The manner in which two people invested money in a joint venture during the same year is shown in the table given below.

| Name | Amount invested | Date of investment | Period of investment | Amount $\times$ Period |
| :---: | :---: | :---: | :---: | :---: |
| Sujith | Rs. 18000 | Jan 01 | ................... |  |
| Vijith | Rs. 20000 | Apr 01 | .......... |  |

(i) Fill in the blanks in the above table.
(ii) Find the ratio in which the profit should be divided between Sujith and Vijith after a year.
(2) Kanthi invested Rs. 10000 and started a dressmaking business on January 01st of a certain year.Two months later Nalani joined the business by investing Rs. 12000.
(i) Calculate the ratio in which the profit should be divided between them at the end of the year.
(ii) If the profit for the year was Rs. 25000 , find the amount received by each of them.
(3) Kamal and Sunil started a business on the 01st of January of a certain year by investing Rs. 24000 and Rs. 30000 respectively. After 4 months Wimal joined the business by investing Rs. 54000 . The profit from the business for the year was Rs. 180000 .
(i) Find the ratio in which the profit should be divided between Kamal, Sunil and Wimal.
(ii) Find separately the amount received by each of them.
(4) Chamara started a spice business by investing Rs. 8000 on the 1st of February. Kumara joined the business by investing Rs. 11000 on the 1st of June of that year. The profit from the business on December 31st was Rs. 45000.
(i) Calculate the ratio in which the profit should be divided between them.
(ii) Find separately the amounts received by Chamara and Kumara.

### 16.4 Continued ratio

A fruit drink is made by mixing pineapple juice, water and mango juice. In this fruit drink, the ratio of pineapple juice to water is $1: 3$ and the ratio of water to mango juice is $3: 2$. Let us find the ratio of pineapple juice to water to mango juice in this drink.

In these two ratios, water is the common substance. It has the same value in both ratios.

$$
\text { The ratio of pineapple juice to water }=1: 3
$$

The ratio of water to mango juice $=3: 2$
In both cases, the term related to water is 3 .
$\therefore$ the ratio of pineapple juice to water to mango juice $=1: 3: 2$
In a concrete mixture, the ratio of gravel to sand is $5: 3$ and of sand to cement is $2: 1$. Let us consider how to find the ratio of gravel to sand to cement in the mixture.


In both these ratios, sand is the common substance. By making the amount of sand equal in both ratios, we can find the continued ratio of the three substances. We use "equivalent ratios" to do this.

The ratio of gravel to sand $=5: 3=5 \times 2: 3 \times 2=10: 6$
The ratio of sand to cement $=2: 1=2 \times 3: 1 \times 3=6: 3$

## Note

In the ratios $5: 3$ and $2: 1$, the terms corresponding to sand are 3 and 2 respectively. The least common multiple of 3 and 2 is 6 . Therefore, equivalent ratios are considered such that the term corresponding to sand in both ratios is equal to 6 .

$$
5: 3=10: 6 \quad 2: 1=6: 3
$$

Therefore, the ratio of gravel to sand to cement is $10: 6: 3$
The ratio of gravel to sand in the concrete mixture is $5: 3$. Therefore, when 10 pans of gravel are used, 6 pans of sand are needed.

The ratio of sand to cement is $2: 1$. Therefore, when 6 pans of sand are used, 3 pans of cement are needed.

Hence the ratio of gravel to sand to cement in the mixture is $10: 6: 3$.

## (2xample 1

When preparing a sweetmeat, flour and sugar are mixed in the ratio $4: 3$ and sugar and coconut are mixed in the ratio $5: 3$. Find the ratio of flour to sugar to coconut in the sweetmeat.

The ratio of flour to sugar $=4: 3$
The ratio of sugar to coconut $=5: 3$


Sugar is common to both ratios. The terms corresponding to sugar in these two ratios are 3 and 5. Equivalent ratios should be written such that the term corresponding to sugar is the least common multiple of 3 and 5 , which is 15 .

The ratio of flour to sugar $=4: 3=4 \times 5: 3 \times 5=20: 15$
The ratio of sugar to coconut $=5: 3=5 \times 3: 3 \times 3=15: 9$
Therefore, the ratio of flour to sugar to coconut $=20: 15: 9$

## Example 2

A certain amount of money was divided among $A, B$ and $C$. The ratio in which it was divided between $A$ and $B$ is $3: 4$ and between $B$ and $C$ is $2: 5$. Find the ratio in which the money was divided among $A, B$ and $C$.
The ratio of $A$ to $B=3: 4$
The ratio of $B$ to $C=2: 5$
$B$ is common to both these ratios. The respective terms for $B$ are 4 and 2 . Their common multiple is 4 .

The ratio of $A$ to $B=3: 4$
The ratio of $B$ to $C=2: 5=2 \times 2: 5 \times 2=4: 10$
$\therefore$ The ratio of $A$ to $B$ to $C$ is $=3: 4: 10$
Exercise 16.3
(1) A fertilizer is produced by combining Nitrogen, Phosphorous and Potassium. The ratio of Nitrogen to Phosphorous is 5:3 and the ratio of Phosphorous to Potassium is 6:1. Find the ratio of Nitrogen to Phosphorous to Potassium in this fertilizer.

(2) Coconut oil, sesame oil and margosa oil are combined together to make medicinal oil. Coconut oil and sesame oil are combined in the ratio $5: 2$ and sesame oil and margosa oil are combined in the ratio $3: 1$. Find the ratio of coconut oil to sesame oil to margosa oil in the medicinal oil.
(3) In a certain farm, there are cattle, goats and hens. The ratio of cattle to goats is $4: 3$ and the ratio of cattle to hens is $2: 7$.
(i) Find the ratio of cattle to goats to hens.
(ii) If there are 105 animals of these three types in the farm, find
 separately the number of cattle, goats and hens in the farm.
(4) In a certain village, Sinhalese, Tamils and Muslims live together. The ratio of Sinhala families to Tamil families is $5: 3$ and the ratio of Tamil families to Muslim families is $4: 1$.
(i) Find the ratio of Sinhalese families to Tamil families to Muslim families.
(ii) How many families are there in the village, if there are 60 Sinhalese families?
(5) Piyadasa, Swaminadan and Nazeer are three friends who set up a joint venture. They shared the profit of their business as follows: Between Piyadasa and Nazeer in the ratio $5: 6$, and between Swaminadan and Nazeer in the ratio $4: 5$.
(i) Find the ratio in which the profit was shared between Piyadasa and Swaminadan.
(ii) If Piyadasa received Rs. 20000 as profit, calculate how much Swaminadan and Nazeer received.

## Miscellaneous Exercise

(1) Ruwani started a sweetmeat business by investing Rs. 5000 at the beginning of a certain year. At the beginning of March of the same year, her neighbours Fathima and Saradha joined the business by investing Rs. 7000 and Rs. 5000 respectively. At the end of the year, the profit from the business was Rs. 54000. Calculate the amount received by each of them if the profit was divided among them based on their investment and the period of investment.

## Summary

When sharing profits of a joint venture, the invested amount and the period of investment are both taken into consideration.
When calculating the ratio in which profits should be shared in a joint venture, the invested amount is multiplied by the period of investment.
[1] When the relationship between three quantities is given by two ratios, we obtain the continued ratio of the three quantities by considering equivalent ratios.

