



By studying this lesson, you will be able to,

- write the reciprocal of a whole number and of a fraction,
- divide a fraction by a whole number and a whole number by a fraction,
- divide a fraction by a fraction,
- divide a whole number by a mixed number,
- divide a mixed number by a whole number,
- divide a fraction by a mixed number, and a mixed number by a fraction, and
- divide a mixed number by a mixed number.

14.1 Reciprocal of a number

Using the previously gained knowledge on multiplying fractions, let us now examine the following.

$$2 \times \frac{1}{2} = \frac{2}{2} = 1$$

$$\frac{1}{3} \times 3 = \frac{3}{3} = 1$$

$$7 \times \frac{1}{7} = \frac{7}{7} = 1$$

$$\frac{2}{5} \times \frac{5}{2} = \frac{10}{10} = 1$$

$$\frac{3}{8} \times \frac{8}{3} = \frac{24}{24} = 1$$

In each of the cases shown above, the product of the two fractions is 1.

As in the above cases, if the product of two numbers is 1, then each is called the **reciprocal** of the other.

Accordingly,

since $2 \times \frac{1}{2} = 1$,

$\frac{1}{2}$ is the reciprocal of 2. Also, 2 is the reciprocal of $\frac{1}{2}$.

Also, since $3 \times \frac{1}{3} = 1$,



$5(x - y)$

$\sqrt{64}$



$1\frac{7}{10}$

$(-1)^1$



$\frac{1}{3}$ is the reciprocal of 3 and 3 is the reciprocal of $\frac{1}{3}$.

Furthermore, since $\frac{2}{5} \times \frac{5}{2} = 1$,

$\frac{2}{5}$ is the reciprocal of $\frac{5}{2}$ and $\frac{5}{2}$ is the reciprocal of $\frac{2}{5}$.

Note

A whole number can also be written as a fraction, taking the whole number as the numerator and 1 as the denominator as in $3 = \frac{3}{1}$.

Number	Reciprocal
2	$\frac{1}{2}$
$\frac{1}{3}$	3
$\frac{2}{5}$	$\frac{5}{2}$
$\frac{3}{8}$	$\frac{8}{3}$

- The numerator of the reciprocal of a fraction is the denominator of that fraction, while its denominator is the numerator of that fraction.
- It is clear that the reciprocal of a fraction is obtained by interchanging its numerator and its denominator.

• Reciprocal of a mixed number

When finding the reciprocal of a mixed number such as $1\frac{1}{2}$, first the mixed number is written as an improper fraction.

Accordingly, $1\frac{1}{2} = \frac{3}{2}$

Since the reciprocal of $\frac{3}{2}$ is $\frac{2}{3}$, the reciprocal of $1\frac{1}{2}$ is $\frac{2}{3}$.

Note:

Since there is no number which when multiplied by 0 (Zero) gives 1, 0 has no reciprocal.



Exercise 14.1

(1) Fill in the blanks using the correct values.

(i) $\frac{3}{4} \times \frac{\square}{3} = 1$

(ii) $\frac{5}{8} \times \frac{8}{\square} = 1$

(iii) $7 \times \frac{\square}{7} = 1$

(iv) $\frac{1}{5} \times \square = 1$

(v) $1\frac{1}{3} \times \frac{3}{4} = \frac{\square}{3} \times \frac{3}{4} = 1$

(vi) $2\frac{1}{2} \times \frac{2}{\square} = \frac{\square}{2} \times \frac{2}{\square} = 1$

(2) Write down the reciprocal of each of the following numbers.

(i) 6

(ii) $\frac{1}{9}$

(iii) $\frac{5}{7}$

(iv) $\frac{8}{3}$

(v) 1

(vi) $3\frac{1}{3}$

(vii) $2\frac{3}{5}$

(viii) $1\frac{5}{9}$

14.2 Dividing a fraction by a whole number

The picture shows a whole cake and $\frac{1}{2}$ a cake.



Suppose we want to share this portion ($\frac{1}{2}$ a cake) equally between Kamal and Amal. Let us consider the share that one person gets from the whole cake, when half the cake is divided equally between them.

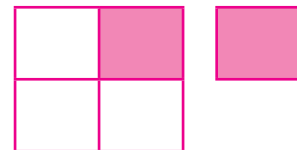
This share is $\frac{1}{2} \div 2$.



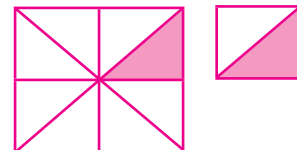
It is clear from the picture that this share is $\frac{1}{4}$ of the whole cake.

Accordingly, $\frac{1}{2} \div 2 = \frac{1}{4}$

The figure on the right hand side shows a square shaped card of which $\frac{1}{4}$ has been coloured.



If the coloured portion of this card is divided into two equal parts, let us find what fraction of the whole square each of the two parts is.





$5(x-y)$

$\sqrt{64}$



$\frac{7}{10}$

$(-1)^1$

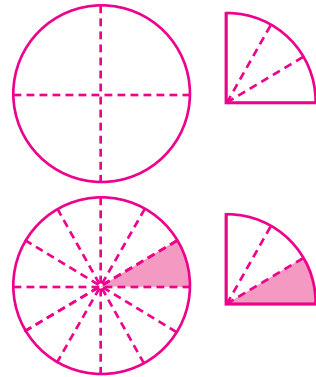


It is clear from the figure that each part is $\frac{1}{8}$ of the whole square.

This can also be written as $\frac{1}{4} \div 2$.

$$\therefore \frac{1}{4} \div 2 = \frac{1}{8}$$

Consider $\frac{1}{4}$ th of the circle shown in the figure. If this is divided further into three equal portions, let us find what fraction of the whole circle each portion is.



It is clear that each portion is $\frac{1}{12}$ th of the whole circle.

$$\therefore \frac{1}{4} \div 3 = \frac{1}{12}$$

Now let us consider each of the above cases one by one.

$$\frac{1}{2} \div 2 = \frac{1}{4}. \quad \text{In addition, } \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}. \quad \therefore \frac{1}{2} \div 2 = \frac{1}{2} \times \frac{1}{2}$$

$$\frac{1}{4} \div 2 = \frac{1}{8}. \quad \text{In addition, } \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}. \quad \therefore \frac{1}{4} \div 2 = \frac{1}{4} \times \frac{1}{2}$$

$$\frac{1}{4} \div 3 = \frac{1}{12}. \quad \text{In addition, } \frac{1}{4} \times \frac{1}{3} = \frac{1}{12}. \quad \therefore \frac{1}{4} \div 3 = \frac{1}{4} \times \frac{1}{3}$$

Dividing a fraction by a number is the same as multiplying the fraction by the reciprocal of that number.

Example 1

Find the value of $\frac{1}{3} \div 2$.

$$\begin{aligned} \frac{1}{3} \div 2 &= \frac{1}{3} \times \frac{1}{2} \text{ (multiplying by the reciprocal of 2)} \\ &= \frac{1}{6} \end{aligned}$$

Example 2

Find the value of $\frac{4}{5} \div 3$.

$$\begin{aligned} \frac{4}{5} \div 3 &= \frac{4}{5} \times \frac{1}{3} \text{ (multiplying by the reciprocal of 3)} \\ &= \frac{4}{15} \end{aligned}$$



Exercise 14.2

(1) Find the value of each of the following.

(i) $\frac{1}{5} \div 4$

(ii) $\frac{3}{4} \div 2$

(iii) $\frac{5}{7} \div 3$

(iv) $\frac{9}{10} \div 5$

• Dividing a whole number by a fraction

Let us now consider how a whole number is divided by a fraction. We can study this through examples.

Example 3

Find the value of $1 \div \frac{1}{3}$.

Let us consider the rectangular lamina shown here as one unit.

This unit has been divided into three equal parts. One of these parts is $\frac{1}{3}$.

Accordingly, there are three $\frac{1}{3}$ portions in one unit.

$$\therefore 1 \div \frac{1}{3} = 3$$

When 1 is multiplied by 3, which is the reciprocal of $\frac{1}{3}$, the same answer is obtained.

$$\therefore 1 \div \frac{1}{3} = 1 \times \frac{3}{1} = 3.$$

Example 4

Find the value of $2 \div \frac{1}{4}$.

Let us explain this by considering two rectangular shaped laminas of the same size. Let us consider each rectangular lamina as one unit.



When a lamina is divided into four equal parts, there are four $\frac{1}{4}$ in one unit.

$\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$ Therefore, there are eight $\frac{1}{4}$ in two units.



Accordingly,

$$2 \div \frac{1}{4} = 8$$

$$2 \div \frac{1}{4} = 2 \times \frac{4}{1} = 8$$



$5(x - y)$

$\sqrt{64}$



$\frac{7}{10}$

$(-1)^1$



Dividing a whole number by a fraction is the same as multiplying the number by the reciprocal of that fraction.

Example 5

Find the value of $3 \div \frac{1}{5}$.

$$\begin{aligned} 3 \div \frac{1}{5} &= 3 \times 5 \text{ (multiplying by the reciprocal)} \\ &= 15 \end{aligned}$$

Exercise 14.3

(1) Find the value of each of the following.

(i) $3 \div \frac{1}{4}$

(ii) $2 \div \frac{2}{5}$

(iii) $4 \div \frac{1}{2}$

(iv) $15 \div \frac{3}{5}$

14.3 Dividing a fraction by a fraction


Consider $\frac{1}{2} \div \frac{1}{4}$.

Here we are trying to find out how many $\frac{1}{4}$ there are in $\frac{1}{2}$ a unit.

Let us illustrate this using a figure.

One unit 

$\frac{1}{2}$ of the above unit 

There are two $\frac{1}{4}$ in $\frac{1}{2}$ a unit. 

Accordingly, $\frac{1}{2} \div \frac{1}{4} = 2$. To obtain this answer, $\frac{1}{2}$ should be multiplied by the reciprocal of $\frac{1}{4}$.

$$\begin{aligned} \text{That is, } \frac{1}{2} \div \frac{1}{4} &= \frac{1}{2} \times \frac{4}{1} \text{ (multiplying by the reciprocal of } \frac{1}{4}) \\ &= \frac{4}{2} = 2 \end{aligned}$$

Dividing a fraction by another fraction is the same as multiplying the first fraction by the reciprocal of the second fraction.



$$5(x-y)$$

$$\sqrt{64}$$



$$\frac{7}{10}$$

$$(-1)^7$$



Example 1

Find the value of $\frac{1}{3} \div \frac{2}{5}$.

$$\begin{aligned} \frac{1}{3} \div \frac{2}{5} &= \frac{1}{3} \times \frac{5}{2} && \text{(multiplying by the reciprocal of } \frac{2}{5} \text{)} \\ &= \frac{5}{6} \end{aligned}$$

Example 2

Find the value of $\frac{3}{7} \div \frac{6}{11}$.

$$\begin{aligned} \frac{3}{7} \div \frac{6}{11} &= \frac{3}{7} \times \frac{11}{6} && \text{(multiplying by the reciprocal of } \frac{6}{11} \text{)} \\ &= \frac{11}{14} \end{aligned}$$

Exercise 14.4

(1) Find the value of each of the following.

(i) $\frac{3}{8} \div \frac{3}{4}$

(ii) $\frac{15}{16} \div \frac{3}{4}$

(iii) $\frac{15}{28} \div \frac{3}{7}$

(iv) $\frac{10}{11} \div \frac{1}{11}$

(v) $\frac{6}{7} \div \frac{3}{7}$

(vi) $\frac{12}{7} \div \frac{3}{7}$

(vii) $\frac{4}{5} \div \frac{8}{9}$

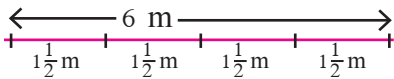
(viii) $\frac{7}{8} \div \frac{7}{10}$

(ix) $\frac{3}{8} \div \frac{2}{5}$

(x) $\frac{2}{3} \div \frac{5}{7}$

14.4 Dividing a whole number by a mixed number

Let us find out how many pieces of wire of length $1\frac{1}{2}$ m can be cut from a wire of length 6 cm.



According to the figure, four pieces can be cut from the wire.

Accordingly, we can write $6 \div 1\frac{1}{2} = 4$.

Now let us simplify the expression $6 \div 1\frac{1}{2}$.

$$\begin{aligned} 6 \div 1\frac{1}{2} &= 6 \div \frac{3}{2} && \text{(writing the mixed number } 1\frac{1}{2} \text{ as an improper fraction)} \\ &= \frac{2}{3} \times \frac{2}{3} && \text{(multiplying by the reciprocal of } \frac{3}{2} \text{)} \\ &= 4 \end{aligned}$$



$5(x - y)$

$\sqrt{64}$



$1\frac{7}{10}$

$(-1)^1$



• Dividing a mixed number by a whole number

Through the following example, let us establish how a mixed number is divided by a whole number.

Example 1

Find the value of $1\frac{1}{2} \div 6$.

$$\begin{aligned} 1\frac{1}{2} \div 6 &= \frac{3}{2} \div 6 \\ &= \frac{3}{2} \times \frac{1}{6} \quad (\text{multiplying by the reciprocal of } 6) \\ &= \frac{1}{4} \end{aligned}$$

14.5 Dividing a fraction by a mixed number

When dividing a fraction by a mixed number, the mixed number is first written as an improper fraction and then the fraction is multiplied by the reciprocal of this improper fraction.

Example 1

Find the value of $\frac{4}{5} \div 1\frac{1}{3}$.

$$\begin{aligned} \frac{4}{5} \div 1\frac{1}{3} &= \frac{4}{5} \div \frac{4}{3} \quad (\text{converting the mixed number into an improper fraction}) \\ &= \frac{4}{5} \times \frac{3}{4} \quad (\text{multiplying by the reciprocal of } \frac{4}{3}) \\ &= \frac{3}{5} \end{aligned}$$

• Dividing a mixed number by a fraction

Here, the mixed number is first written as an improper fraction. This improper fraction is then multiplied by the reciprocal of the fraction by which the mixed number is to be divided.

Example 2

Find the value of $1\frac{1}{3} \div \frac{4}{5}$.

$$\begin{aligned} 1\frac{1}{3} \div \frac{4}{5} &= \frac{4}{3} \times \frac{5}{4} \\ &= \frac{5}{3} \\ &= 1\frac{2}{3} \end{aligned}$$



Exercise 14.5

(1) Find the value of each of the following.

(i) $3 \div 1\frac{1}{2}$

(ii) $7 \div 1\frac{1}{8}$

(iii) $15 \div 1\frac{1}{4}$

(iv) $18 \div 1\frac{2}{25}$

(v) $1\frac{1}{2} \div 3$

(vi) $1\frac{2}{5} \div 14$

(vii) $3\frac{2}{3} \div 22$

(viii) $5\frac{5}{6} \div 21$

(2) Find the value of each of the following.

(i) $\frac{3}{5} \div 2\frac{2}{5}$

(ii) $\frac{6}{7} \div 1\frac{1}{5}$

(iii) $\frac{8}{11} \div 3\frac{1}{5}$

(iv) $\frac{3}{8} \div 2\frac{1}{4}$

(v) $1\frac{4}{5} \div \frac{3}{5}$

(vi) $2\frac{1}{2} \div \frac{5}{7}$

(vii) $10\frac{2}{3} \div \frac{16}{27}$

(viii) $2\frac{3}{5} \div \frac{1}{2}$

(3) Hasim has packed 10 kg of sweetmeats into packets containing $1\frac{1}{4}$ kg each. Find the number of packets that he has made.



(4) A truck can transport $3\frac{1}{2}$ cubes of soil at a time. What is the minimum number of trips that needs to be made to transport 28 cubes of soil?



(5) Chalani needs to cut 21 m of fabric into $1\frac{3}{4}$ m pieces. How many such pieces can Chalani cut from this fabric?

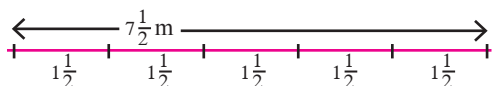


(6) A volume of $31\frac{1}{2}$ l of paint in a barrel was poured equally into 7 containers. Find the amount of paint in each container.



14.6 Dividing a mixed number by a mixed number

Let us find out how many pieces of length $1\frac{1}{2}$ m can be cut from a rope of length $7\frac{1}{2}$ m.



It is clear from the figure that five pieces can be cut from the rope.



$5(x - y)$

$\sqrt{64}$



$1\frac{7}{10}$

$(-1)^n$



This can be written as $7\frac{1}{2} \div 1\frac{1}{2} = 5$.

Let us simplify $7\frac{1}{2} \div 1\frac{1}{2}$.

$$\begin{aligned} 7\frac{1}{2} \div 1\frac{1}{2} &= \frac{15}{2} \div \frac{3}{2} \text{ (converting the mixed number into an improper fraction)} \\ &= \frac{15}{\cancel{2}_1} \times \frac{\cancel{2}_1}{3} \text{ (multiplying by the reciprocal)} \\ &= 5 \end{aligned}$$

When dividing a mixed number by a mixed number, the given mixed numbers are first converted into improper fractions, and the answer is obtained by the method of dividing a fraction by a fraction.

Example 1

Simplify $3\frac{1}{2} \div 1\frac{3}{4}$.

$$\begin{aligned} 3\frac{1}{2} \div 1\frac{3}{4} &= \frac{7}{2} \div \frac{7}{4} \\ &= \frac{\cancel{7}_1}{2} \times \frac{\cancel{4}^2}{\cancel{7}_1} \text{ (multiplying by the reciprocal)} \\ &= 2 \end{aligned}$$

Example 2

Simplify $2\frac{3}{5} \div 1\frac{7}{10}$.

$$\begin{aligned} 2\frac{3}{5} \div 1\frac{7}{10} &= \frac{13}{5} \div \frac{17}{10} \\ &= \frac{13}{\cancel{5}_1} \times \frac{\cancel{10}^2}{17} \\ &= \frac{26}{17} \\ &= 1\frac{9}{17} \end{aligned}$$

Exercise 14.6

(1) Simplify each of the following fractions.

(i) $2\frac{1}{4} \div 2\frac{2}{3}$

(ii) $7\frac{7}{8} \div 3\frac{1}{2}$

(iii) $6\frac{3}{5} \div 4\frac{5}{7}$

(iv) $7\frac{5}{8} \div 8\frac{5}{7}$

(v) $11\frac{1}{2} \div 2\frac{3}{4}$

(vi) $5\frac{1}{3} \div 2\frac{1}{2}$

(2) Fabric of length $2\frac{1}{4}$ m is required to sew a dress. What is the maximum number of such dresses that can be sewn from $56\frac{1}{4}$ m of fabric?





- (3) The distance between two cities is $57\frac{1}{2}$ kilometers. A van spent $1\frac{9}{16}$ hours to travel from one city to the other. If it took the same amount of time to travel each kilometer, find how many kilometers it travelled in an hour?



- (4) Among how many families can $148\frac{1}{2}$ kg of rice be distributed, so that each family gets $8\frac{1}{4}$ kg of rice?



Miscellaneous Exercise

- (1) Simplify the following.

(i) $\frac{4}{5} \times 6$

(ii) $\frac{3}{7} \times 3$

(iii) $\frac{3}{8} \div 4$

(iv) $15 \div \frac{3}{10}$

(v) $8 \times \frac{3}{4}$

(vi) $5\frac{1}{4} \times 5$

(vii) $6\frac{3}{5} \div 3$

(viii) $8 \times 1\frac{1}{5}$

(ix) $7 \div 7\frac{1}{2}$

(x) $\frac{2}{3} \times \frac{7}{8}$

(xi) $\frac{3}{7} \times \frac{2}{3}$

(xii) $\frac{5}{9} \div \frac{7}{10}$

(xiii) $\frac{7}{8} \times \frac{4}{5} \times \frac{3}{7}$

(xiv) $\frac{2}{5} \times 1\frac{3}{7}$

(xv) $\frac{4}{9} \div 2\frac{1}{4}$

(xvi) $1\frac{3}{8} \div 1\frac{1}{7}$



(xvii) $1\frac{1}{2} \times 2\frac{2}{3}$

(xviii) $4\frac{2}{3} \div 1\frac{1}{7}$

(xix) $4\frac{1}{2} \times 3\frac{3}{5} \times 1\frac{1}{3}$

(xx) $3\frac{3}{4} \times 1\frac{2}{5} \times 1\frac{1}{7}$

Summary

-  If the product of two numbers is 1, then each is the reciprocal of the other.
-  Dividing a number by another number is the same as multiplying the first number by the reciprocal of the second number.