



By studying this lesson, you will be able to,

- multiply a fraction by a whole number,
- multiply a fraction by a fraction,
- multiply a fraction by a mixed number, and
- multiply a mixed number by a mixed number.

13.1 Fractions

Let us first recall what you have learnt about fractions in Grades 6 and 7. Let us take the area of the figure given below as a unit.



This unit has been divided into five equal parts of which three parts are coloured. You have learnt that the coloured area is $\frac{3}{5}$ of the whole area.

You have also learnt that if a unit is divided into equal parts, then one part or several of these parts is called a fraction of the unit. A portion of a collection is also considered as a fraction of that collection.

In addition, you have learnt that fractions such as $\frac{3}{5}$, $\frac{1}{2}$ and $\frac{2}{3}$ which are less than one and greater than zero are called proper fractions.

A number which has been written by adding together a whole number and a proper fraction is called a **mixed number** or an **improper fraction**, depending on how it has been represented.

Some examples of mixed numbers are $1\frac{1}{2}$, $2\frac{1}{3}$ and $4\frac{2}{5}$.

In the mixed number $4\frac{2}{5}$, the whole part is 4 and the fractional part is $\frac{2}{5}$.

Some examples of improper fractions are $\frac{3}{2}$, $\frac{5}{3}$ and $\frac{11}{7}$.

The numerator of an improper fraction is greater than or equal to the denominator.

A fraction **equivalent** to a given fraction can be obtained by multiplying the numerator and the denominator of the fraction by the same non-zero number.



$5(x - y)$

$\sqrt{64}$



$1\frac{7}{10}$

$(-1)^1$



A fraction **equivalent** to a given fraction can also be obtained by dividing the numerator and the denominator by a non - zero common factor of the numerator and the denominator.

• Representing a mixed number as an improper fraction

By following the steps given below, a mixed number can be represented as an improper fraction.

- Multiply the whole number part of the mixed number by the denominator of the fractional part, and add it to the numerator of the fractional part.
- The denominator of the improper fraction is the same as the denominator of the fractional part of the mixed number.

• Representing an improper fraction as a mixed number

You learnt in Grade 7 how to represent an improper fraction as a mixed number.

Let us represent $\frac{7}{4}$ as a mixed number.

Method I

$$\begin{aligned}\frac{7}{4} &= \frac{4 + 3}{4} \\ &= \frac{4}{4} + \frac{3}{4} \\ &= 1 + \frac{3}{4} = 1\frac{3}{4}\end{aligned}$$

Method II

$$\frac{7}{4} = 7 \div 4 \qquad 1\frac{3}{4}$$

The quotient and remainder of $7 \div 4$ are 1 and 3 respectively. The quotient is the whole number part of the mixed number and the remainder is the numerator of the fractional part.

The denominator of the fractional part of the mixed number is the same as the denominator of the improper fraction.

$$\therefore \frac{7}{4} = 1\frac{3}{4}$$

You have learnt how to add and subtract fractions in Grades 6 and 7.

Do the following review exercise to recall what you have learnt previously about fractions.



Review Exercise

(1) Choose the appropriate value from the brackets and fill in the blanks.

(i) $\frac{3}{4}$ is $\frac{1}{4}$ ths (two, three, five)

(ii) $\frac{2}{5}$ is two ($\frac{1}{3}$ rds, $\frac{1}{2}$ s, $\frac{1}{5}$ ths)

(iii) Four $\frac{1}{7}$ ths is ($\frac{4}{7}$, $\frac{4}{5}$, $\frac{4}{9}$)

(2) Write down two equivalent fractions for each fraction given below.

(i) $\frac{3}{4}$

(ii) $\frac{2}{5}$

(iii) $\frac{6}{10}$

(iv) $\frac{8}{24}$

(3) Represent each mixed number given below as an improper fraction.

(i) $1\frac{1}{5}$

(ii) $3\frac{3}{5}$

(iii) $6\frac{1}{6}$

(4) Represent each improper fraction given below as a mixed number.

(i) $\frac{14}{5}$

(ii) $\frac{18}{7}$

(iii) $\frac{37}{3}$

(5) Simplify the following.

(i) $\frac{2}{5} + \frac{1}{5}$

(ii) $\frac{1}{3} + \frac{1}{2}$

(iii) $\frac{3}{5} + \frac{1}{3}$

(iv) $\frac{7}{12} + \frac{1}{8}$

(v) $\frac{1}{6} + \frac{5}{8}$

(vi) $\frac{11}{15} + \frac{2}{10}$

(vii) $1\frac{1}{2} + 4\frac{3}{8}$

(viii) $2\frac{1}{4} + 3\frac{5}{9}$

(6) Simplify the following.

(i) $\frac{6}{7} - \frac{2}{7}$

(ii) $\frac{7}{10} - \frac{2}{5}$

(iii) $\frac{1}{3} - \frac{2}{7}$

(iv) $1 - \frac{1}{5}$

(v) $\frac{7}{8} - \frac{5}{6}$

(vi) $3\frac{7}{8} - 1\frac{1}{2}$

(vii) $3 - 1\frac{5}{8}$

(viii) $2\frac{2}{5} - 1\frac{3}{20}$



$5(x - y)$

$\sqrt{64}$



$\frac{7}{10}$

$(-1)^1$



13.2 Multiplying a fraction by a whole number

The figure depicts a cake, which is divided into five equal parts.



We know that one part of the entire cake is $\frac{1}{5}$ of the cake.
Let us take 3 such parts.



Let us consider how much the total of these three parts is from the entire cake. For this, we have to add these three quantities.

$$\text{It is } \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{3}{5}$$

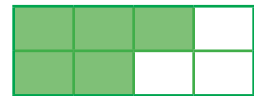
You have learnt previously that addition of the same number repeatedly can be represented as a multiplication.

For example, $2 + 2 + 2 = 2 \times 3 = 6$

Accordingly, we can write $\frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{1}{5} \times 3$

Therefore, $\frac{1}{5} \times 3 = \frac{3}{5}$. That is, three $\frac{1}{5}$ is equal to $\frac{3}{5}$.

- The figure depicts a rectangle which has been divided into eight equal parts. One part is $\frac{1}{8}$ of the entire figure.



Let us consider the sum of 5 such parts.

$$\text{It can be written as } \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{5}{8}$$

That is, five $\frac{1}{8}$ s is equal to $\frac{5}{8}$

$$\frac{1}{8} \times 5 = \frac{5}{8}$$



$$5(x-y)$$

$$\sqrt{64}$$



$$\frac{7}{10}$$

$$(-1)^7$$

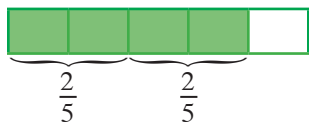


Accordingly,

$$\frac{1}{3} \times 1 = \frac{1}{3}, \quad \frac{1}{3} \times 2 = \frac{2}{3}, \quad \frac{1}{10} \times 7 = \frac{7}{10}$$

- Now let us consider a multiplication of the form $\frac{2}{5} \times 2$.

Let us represent this by a figure.



This can be written as $\frac{2}{5} + \frac{2}{5} = \frac{4}{5}$

When this sum is written as a product we obtain,

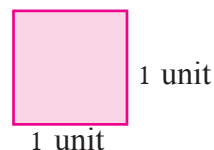
$$\frac{2}{5} \times 2 = \frac{4}{5}$$

When a fraction is multiplied by a whole number, the numerator of the resultant fraction is the product of the whole number and the numerator of the given fraction, and its denominator is the same as that of the given fraction.

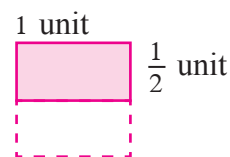
● Multiplying a whole number by a fraction

You have learnt that the area of a square shaped lamina of length 1 unit and breadth 1 unit is 1 square unit.

That is, the area of the square shaped lamina = 1 unit \times 1 unit
= 1 square unit



Now let us find the area of a rectangular shaped lamina which is of length 1 unit and breadth $\frac{1}{2}$ a unit using two methods.



Method I

Since the area of this rectangular shaped lamina is $\frac{1}{2}$ the area of the square of area 1 square unit, the area of the rectangular lamina is $\frac{1}{2}$ square units.



$5(x - y)$

$\sqrt{64}$



$\frac{7}{10}$

$(-1)^1$

**Method II**

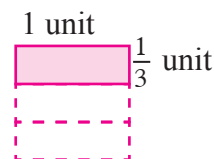
Since the length of this lamina is 1 unit and its breadth is $\frac{1}{2}$ a unit,

area of the lamina = (length \times breadth) square units

$$= 1 \times \frac{1}{2} \text{ square units}$$

$$\therefore 1 \times \frac{1}{2} = \frac{1}{2}$$

Furthermore, the area of the rectangular shaped lamina in the figure which is of length 1 unit and breadth $\frac{1}{3}$ units is $\frac{1}{3}$ square units.



That is, $1 \times \frac{1}{3} = \frac{1}{3}$

You have learnt in the previous section that $\frac{1}{3} \times 1 = \frac{1}{3}$

$$\therefore \frac{1}{3} \times 1 = 1 \times \frac{1}{3}$$

Similarly,

$$\frac{2}{7} \times 3 = \frac{6}{7} \quad \text{and} \quad 3 \times \frac{2}{7} = \frac{6}{7}$$

$$\frac{4}{11} \times 2 = \frac{8}{11} \quad \text{and} \quad 2 \times \frac{4}{11} = \frac{8}{11}$$

$$\frac{2}{13} \times 5 = \frac{10}{13} \quad \text{and} \quad 5 \times \frac{2}{13} = \frac{10}{13}$$

$$\therefore \frac{2}{7} \times 3 = 3 \times \frac{2}{7}$$

$$\therefore \frac{4}{11} \times 2 = 2 \times \frac{4}{11}$$

$$\therefore \frac{2}{13} \times 5 = 5 \times \frac{2}{13}$$

When multiplying a fraction by a whole number, and when multiplying the same whole number by the same fraction we obtain the same answer.

Example 1

(i) Simplify $\frac{3}{7} \times 2$.

$$\frac{3}{7} \times 2 = \frac{3 \times 2}{7}$$

$$= \frac{6}{7}$$

Example 2

(ii) Simplify $\frac{3}{8} \times 5$.

$$\frac{3}{8} \times 5 = \frac{3 \times 5}{8}$$

$$= \frac{15}{8}$$

$$= 1\frac{7}{8}$$

Example 3

(iii) Simplify $4 \times \frac{2}{5}$.

$$4 \times \frac{2}{5} = \frac{4 \times 2}{5}$$

$$= \frac{8}{5}$$

$$= 1\frac{3}{5}$$



$5(x-y)$

$\sqrt{64}$



$\frac{7}{10}$

$(-1)^7$



Exercise 13.1

- (1) Express the product of each of the following in its simplest form (If the answer is an improper fraction, express it as a mixed number).

(i) $\frac{1}{6} \times 5$

(ii) $\frac{3}{10} \times 3$

(iii) $6 \times \frac{2}{13}$

(iv) $\frac{3}{7} \times 5$

(v) $\frac{2}{7} \times 9$

(vi) $\frac{1}{10} \times 17$

(vii) $5 \times \frac{7}{9}$

(viii) $\frac{3}{4} \times 12$

(ix) $\frac{2}{5} \times 10$

(x) $\frac{7}{8} \times 1$

(xi) $\frac{2}{3} \times 0$

(xii) $4 \times \frac{3}{5}$

(xiii) $3 \times \frac{1}{4}$

(xiv) $\frac{5}{6} \times 8$

(xv) $10 \times \frac{3}{5}$

- (2) A vehicle that travels at a constant speed, journeys $\frac{3}{4}$ kilometers in a minute. How far does it travel in 8 minutes?



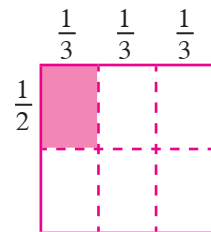
- (3) A machine produces 600 plastic cups in an hour. How many cups does it produce in $\frac{2}{3}$ hours?



13.3 Multiplying a fraction by a fraction

The figure shows a square shaped lamina of side length 1 unit. It is divided into 6 equal parts, of which one part is shaded as in the figure.

Since the shaded part is $\frac{1}{6}$ of the whole area of the lamina, its area is $\frac{1}{6}$ square units.



Also, the shape of the shaded part is rectangular. Its length is $\frac{1}{2}$ the length of the square lamina and its breadth is $\frac{1}{3}$ the breadth of the square lamina.

The area of the rectangular shaped lamina is calculated by multiplying its length by its breadth.



$5(x - y)$

$\sqrt{64}$



$1\frac{7}{10}$

$(-1)^1$

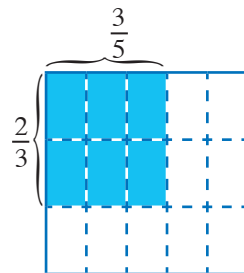


Therefore, the area of the shaded part can be written as $\frac{1}{2} \times \frac{1}{3}$ square units. Since this is equal to $\frac{1}{6}$ square units,

$$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$$

The figure shows a square shaped lamina of side length 1 unit. It is divided into 15 equal parts.

Let us find the area of the shaded part using two different methods.



Method I

Since the area of the shaded part is $\frac{6}{15}$ of the area of the whole lamina, the area of this part is $\frac{6}{15}$ square units.

Method II

The length of the shaded part of the rectangular shape = $\frac{2}{3}$ of the length of the square (that is, $\frac{2}{3}$ units)

Its breadth = $\frac{3}{5}$ of the length of the square (that is, $\frac{3}{5}$ units).

\therefore The area of the shaded part is $\frac{3}{5} \times \frac{2}{3}$ square units.

$$\therefore \frac{3}{5} \times \frac{2}{3} = \frac{6}{15}$$

Let us consider the above two cases.

$$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6} \quad \left(\frac{1 \times 1}{2 \times 3} = \frac{1}{6} \right)$$

$$\frac{3}{5} \times \frac{2}{3} = \frac{6}{15} \quad \left(\frac{3 \times 2}{5 \times 3} = \frac{6}{15} \right)$$

When two fractions are multiplied,

- the numerator of the resultant fraction is the product of the two numerators.
- the denominator of the resultant fraction is the product of the two denominators.

**Note**

- When any fraction is multiplied by zero, the result is zero.

$$\frac{1}{2} \times 0 = \frac{1}{2} \times \frac{0}{1} = \frac{1 \times 0}{2 \times 1} = \frac{0}{2} = 0$$

- When any fraction is multiplied by 1, the result is the same fraction.

$$\frac{1}{2} \times 1 = \frac{1}{2} \times \frac{1}{1} = \frac{1 \times 1}{2 \times 1} = \frac{1}{2}$$

Example 1

Simplify

$$\begin{aligned} \text{(i)} \quad \frac{4}{7} \times \frac{2}{3} \\ \frac{4}{7} \times \frac{2}{3} &= \frac{4 \times 2}{7 \times 3} \\ &= \frac{8}{21} \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad \frac{3}{8} \times \frac{4}{5} \times \frac{1}{2} \\ \frac{3}{8} \times \frac{4}{5} \times \frac{1}{2} &= \frac{3 \times 4 \times 1}{8 \times 5 \times 2} = \frac{12}{80} \\ &= \frac{12 \div 4}{80 \div 4} \text{ (equivalent fraction)} \\ &= \frac{3}{20} \end{aligned}$$

Note

$$\frac{3}{8} \times \frac{4}{5} = \frac{12}{40}$$

In the fraction $\frac{12}{40}$, since 4 is a common factor of both the numerator and the denominator, let us divide the numerator as well as the denominator by 4.

$$\frac{12}{40} = \frac{12 \div 4}{40 \div 4} = \frac{3}{10}$$

This is written as $\frac{12^3}{40_{10}} = \frac{3}{10}$.

$$\frac{3}{8} \times \frac{4}{5} = \frac{12^3}{40_{10}} = \frac{3}{10}$$

Also,

$$\frac{3}{8} \times \frac{4}{5} = \frac{3 \times 4}{8 \times 5} = \frac{3 \times 4}{2 \times 4 \times 5}$$

Now, since 4 is the greatest common factor of the numerator and the denominator, by dividing the numerator and the denominator by 4 we obtain,

$$\frac{3 \times \cancel{4}^1}{2 \times \cancel{4}_1 \times 5} = \frac{3}{10}$$

When simplifying $\frac{3}{8} \times \frac{4}{5}$, it is easy to first divide the numerator and the denominator by their common factors.

$$\frac{3}{\cancel{8}_2} \times \frac{\cancel{4}^1}{5} = \frac{3 \times 1}{2 \times 5} = \frac{3}{10}$$



$5(x - y)$

$\sqrt{64}$



$\frac{7}{10}$

$(-1)^1$

**Exercise 13.2**

(1) Simplify the following.

(a) (i) $\frac{1}{2} \times \frac{1}{4}$

(ii) $\frac{2}{3} \times \frac{1}{5}$

(iii) $\frac{3}{4} \times \frac{5}{7}$

(iv) $\frac{3}{5} \times \frac{2}{7}$

(v) $\frac{3}{8} \times \frac{2}{5}$

(vi) $\frac{7}{10} \times \frac{3}{14}$

(vii) $\frac{5}{12} \times \frac{4}{7}$

(viii) $\frac{6}{7} \times \frac{14}{15}$

(b) (i) $\frac{6}{7} \times \frac{3}{8}$

(ii) $\frac{3}{5} \times \frac{2}{3}$

(iii) $\frac{2}{11} \times \frac{3}{4}$

(iv) $\frac{3}{10} \times \frac{5}{6}$

(v) $\frac{3}{4} \times \frac{2}{3}$

(vi) $\frac{5}{12} \times \frac{3}{10}$

(vii) $\frac{1}{2} \times \frac{1}{4} \times \frac{3}{5}$

(viii) $\frac{2}{3} \times \frac{5}{8} \times \frac{3}{10}$

13.4 Multiplying a fraction by a mixed number

Let us now consider how to multiply a fraction by a mixed number.

Let us multiply $\frac{3}{5}$ by $1\frac{1}{2}$.

That is, let us find the value of $\frac{3}{5} \times 1\frac{1}{2}$.

Let us first represent the mixed number as an improper fraction.

$$\begin{aligned} \frac{3}{5} \times 1\frac{1}{2} &= \frac{3}{5} \times \frac{3}{2} \\ &= \frac{3 \times 3}{5 \times 2} \\ &= \frac{9}{10} \end{aligned}$$

When simplifying fractions which include mixed numbers, multiplication is made easier by first converting the mixed numbers into improper fractions.

Example 1

Simplify $\frac{2}{3} \times 1\frac{1}{4}$.

$$\begin{aligned} \frac{2}{3} \times 1\frac{1}{4} &= \frac{2}{3} \times \frac{5}{4_2} \quad (\text{divide 2 and 4 by 2}) \\ &= \frac{1 \times 5}{3 \times 2} \\ &= \frac{5}{6} \end{aligned}$$

Example 2

Simplify $1\frac{3}{5} \times \frac{3}{4}$.

$$\begin{aligned} 1\frac{3}{5} \times \frac{3}{4} &= \frac{8}{5} \times \frac{3}{4_1} \quad (\text{divide 4 and 8 by 4}) \\ &= \frac{2 \times 3}{5 \times 1} \\ &= \frac{6}{5} \\ &= 1\frac{1}{5} \end{aligned}$$



Exercise 13.3

(1) Simplify the following.

$$(i) \frac{2}{3} \times 1\frac{1}{3}$$

$$(ii) \frac{3}{5} \times 1\frac{1}{4}$$

$$(iii) \frac{5}{8} \times 1\frac{2}{3}$$

$$(iv) \frac{7}{10} \times 2\frac{1}{7}$$

$$(v) \frac{1}{6} \times 2\frac{1}{5}$$

$$(vi) \frac{3}{5} \times 3\frac{1}{9}$$

$$(vii) \frac{7}{10} \times 33\frac{1}{3}$$

$$(viii) \frac{5}{12} \times 3\frac{3}{11}$$

$$(ix) 2\frac{1}{2} \times \frac{1}{5}$$

$$(x) 3\frac{3}{4} \times \frac{7}{10}$$

$$(xi) \frac{2}{5} \times \frac{1}{2} \times 2\frac{1}{2}$$

$$(xii) \frac{3}{4} \times \frac{2}{5} \times 1\frac{1}{6}$$

(2) If a vehicle travels a distance of $12\frac{1}{2}$ km on 1 l of fuel, find the distance it travels on $\frac{3}{4}$ l of fuel.

(3) Aheli reads a certain book for $1\frac{3}{4}$ hours each day. She finishes reading the book in 7 days. Find in hours, the time she took to finish the book.



(4) When kamala was hospitalized due to an illness, the doctor instructed her to drink $\frac{1}{10}$ l of liquid once every $\frac{1}{2}$ hour. Calculate the amount of liquid that kamala drinks during $3\frac{1}{2}$ hours in millilitres.



13.5 Multiplying a mixed number by a mixed number

When multiplying a mixed number by a mixed number, first write each mixed number as an improper fraction.

Let us simplify $1\frac{1}{2} \times 1\frac{2}{5}$.

$$1\frac{1}{2} \times 1\frac{2}{5} = \frac{3}{2} \times \frac{7}{5} \text{ (first the mixed numbers need to be written as improper fractions)}$$

$$= \frac{3 \times 7}{2 \times 5}$$

$$= \frac{21}{10} = 2\frac{1}{10}$$



$5(x-y)$

$\sqrt{64}$



$\frac{7}{10}$

$(-1)^1$



Example 1

Simplify $1\frac{3}{5} \times 2\frac{3}{4}$.

$$\begin{aligned}
 1\frac{3}{5} \times 2\frac{3}{4} &= \frac{28}{5} \times \frac{11}{4} \\
 &= \frac{2 \times 11}{5 \times 1} \\
 &= \frac{22}{5} = 4\frac{2}{5}
 \end{aligned}$$

Example 2

Simplify $1\frac{1}{4} \times 3\frac{1}{2} \times \frac{1}{4}$.

$$\begin{aligned}
 1\frac{1}{4} \times 3\frac{1}{2} \times \frac{1}{4} &= \frac{5}{4} \times \frac{7}{2} \times \frac{1}{4} \\
 &= \frac{35}{32} \\
 &= 1\frac{3}{32}
 \end{aligned}$$

Exercise 13.4

(1) Simplify the following.

(i) $2\frac{1}{2} \times 1\frac{3}{5}$

(ii) $1\frac{1}{2} \times 4\frac{1}{3}$

(iii) $3\frac{3}{4} \times 1\frac{1}{5}$

(iv) $1\frac{2}{3} \times 3\frac{3}{4}$

(v) $6\frac{1}{4} \times 2\frac{2}{5}$

(vi) $10\frac{2}{3} \times 2\frac{1}{4}$

(vii) $1\frac{3}{7} \times 1\frac{1}{100}$

(viii) $5\frac{1}{4} \times 2\frac{2}{7}$

(ix) $3\frac{1}{2} \times 4\frac{4}{5} \times \frac{5}{14}$

(x) $3\frac{3}{10} \times 2\frac{1}{3} \times 4\frac{2}{7}$

Summary



When a fraction is multiplied by a whole number, the numerator of the resultant fraction is the product of the whole number and the numerator of the given fraction, and its denominator is the same as that of the given fraction



When a fraction is multiplied by a fraction, the numerator of the resultant fraction is the product of the numerators of the given fractions and its denominator is the product of the denominators of the given fractions.