12 Triangles and Quadrilaterals

By studying this lesson, you will be able to,

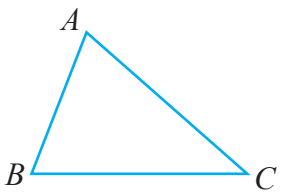
- obtain a value for the sum of the interior angles of a triangle and of a quadrilateral,
- show that the sum of the exterior angles of a triangle and of a quadrilateral is 360°, and
- perform calculations associated with angles of triangles and quadrilaterals.

12.1 Triangles

You have learnt that a polygon formed with three straight line segments is called a **triangle.**

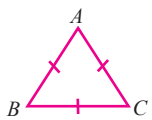
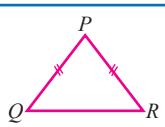
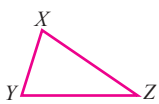
A triangle has three sides and three angles. They are called the elements of the triangle.

The three sides of the triangle ABC are AB, BC and CA. The three angles of the triangle ABC are $A\hat{B}C$, $B\hat{C}A$ and $C\hat{A}B$.

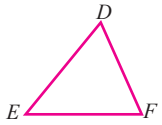
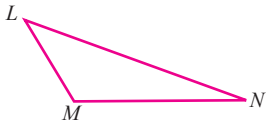
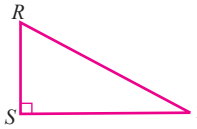


You have learnt in Grade 7 how to classify a triangle according to the lengths of its sides and the magnitudes of its angles.

• Classification of triangles according to the lengths of the sides

Triangle	Figure	Note
Equilateral triangle		The lengths of all three sides are equal
Isosceles triangle		The lengths of two sides are equal
Scalene triangle		All three sides are unequal in length

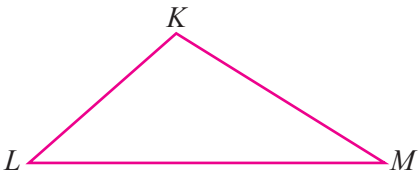
• Classification of triangles according to the angles

Triangle	Figure	Note
Acute triangle		The magnitude of each angle is less than 90^{o}.
Obtuse triangle		The magnitude of one angle is greater than 90^{o}.
Right triangle		The magnitude of one angle is 90^{o}.

Do the following review exercise to recall the facts you learnt in Grade 7 on triangles and angles.

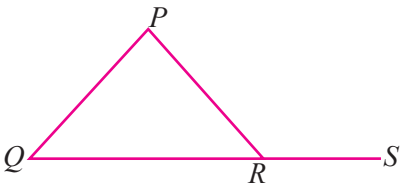
Review Exercise

(1) Name the three sides and the three angles of the triangle shown in the figure.

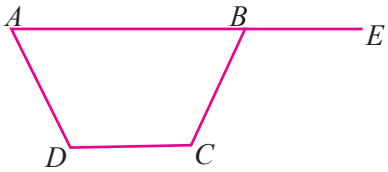


(2) (i) Draw an obtuse triangle and name it ABC.
(ii) Measure $A\hat{B}C$, $B\hat{A}C$ and $A\hat{C}B$ and write down their magnitudes.

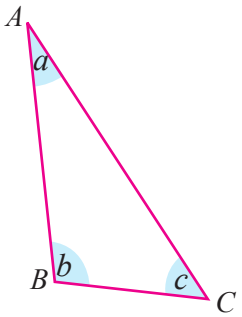
(3) (i) Draw a triangle PQR as in the figure and produce QR to S.
(ii) Measure $P\hat{R}Q$ and $P\hat{R}S$ and write down their magnitudes.



(4) (i) Draw a quadrilateral $ABCD$ and produce AB to E.
(ii) Measure $E\hat{B}C$ and $A\hat{B}C$ and write down their magnitudes.



12.2 The sum of the interior angles of a triangle

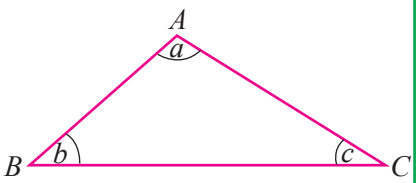


The angles located within the triangle ABC are named a, b, and c. Since they are located within the triangle, they are called the **interior angles of the triangle** ABC**.**

Engage in the following activity in order to find the sum of the interior angles of a triangle.

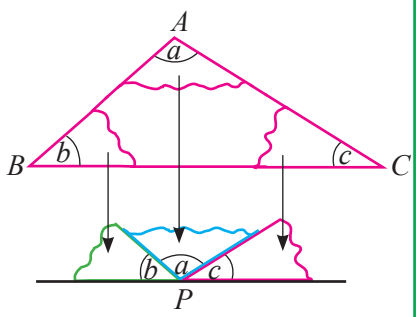
Activity 1

Step 1 - Draw any triangle on a piece of white paper and name its vertices as A,B and C and its interior angles as a,b and c respectively, as shown in the figure.



Step 2 - Cut and separate out the three angles a, b and c as shown in the figure.

Step3 - In your exercise book, paste the three angles a,b and c that were cut out, as shown in the figure, without overlapping them and such that the point P on the line is the common vertex.



Step 4 - Establish the fact that the three pasted angles are located on a straight line, by keeping a ruler. Write down the value of $a + b + c$.

➢ Draw another triangle in your exercise book, measure the three interior angles and find their sum.

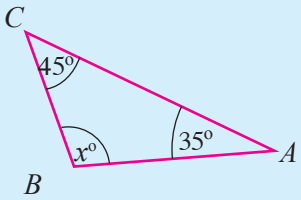
It must be clear to you from the above activity that the sum of the three interior angles of a triangle can be presented as the sum of three angles located on a straight line, completely covering one side of it.

Since the sum of the angles at a point on a straight line is 180°, it can be concluded that the sum of the three interior angles of a triangle is 180°.

The sum of the interior angles of a triangle is 180°.

Example 1

Find the magnitude of $A\hat{B}C$ in the figure.



Since the sum of the interior angles of a triangle is 180°,

$45 + 35 + x = 180$

$80 + x = 180$

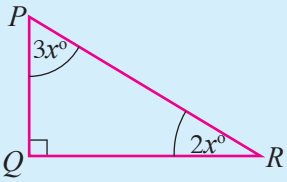
$x + 80 - 80 = 180 - 80$

$x = 100$

$A\hat{B}C = 100°$

Example 2

Find the magnitude of $Q\hat{P}R$ in the figure.



$3x + 2x + 90 = 180$

$5x + 90 = 180$

$5x + 90 - 90 = 180 - 90$

$5x = 90$

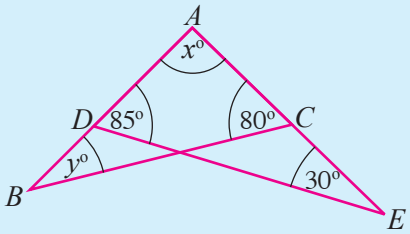
$\frac{5x}{5} = \frac{90}{5}$

$x = 18$

$\therefore Q\hat{P}R = 3 \times 18° = 54°$

Example 3

Find the values of x and y according to the information marked in the figure.



Since the sum of the interior angles of the triangle ADE is 180°,

$85 + 30 + x = 180$

$115 + x = 180$

$x + 115 - 115 = 180 - 115$

$x = 65$

Since the sum of the interior angles of the triangle ABC is 180°,

$x + 80 + y = 180$

$65 + 80 + y = 180$ (substituting $x° = 65°$)

$y + 145 = 180$

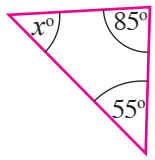
$y + 145 - 145 = 180 - 145$

$y = 35$

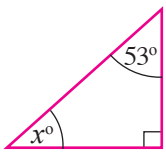
Exercise 12.1

(1) Find the magnitude of the angle marked as x in each figure.

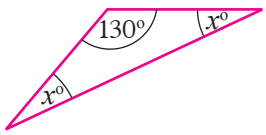
(i)



(ii)



(iii)



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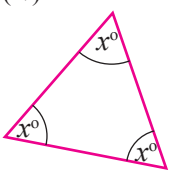
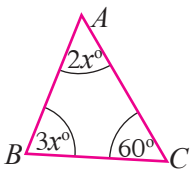


(2) Find the magnitude of each of the angles in each triangle.

(i)



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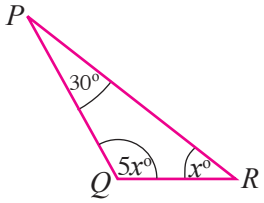
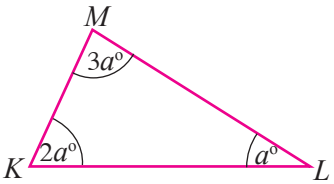
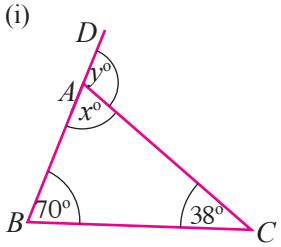


(iii)



(3) Find the magnitude of each of the angles denoted by an English letter in each figure.

(i)



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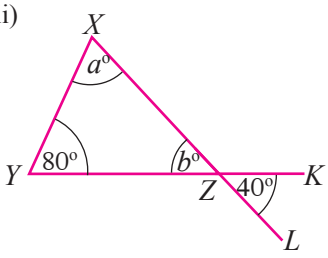
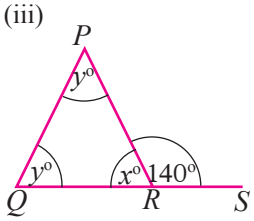
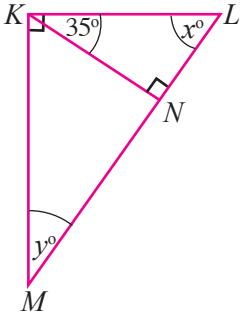


(iii)



(iv)



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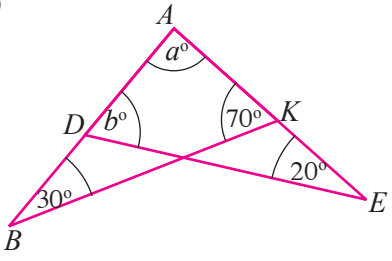
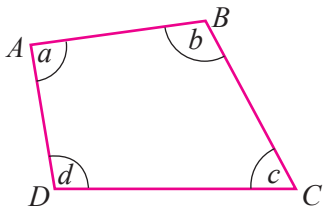


12.3 The sum of the interior angles of a quadrilateral

You learnt in Grade 6 that a closed rectilinear plane figure which consists of 4 sides is called a **quadrilateral**. A quadrilateral has 4 sides and 4 angles.

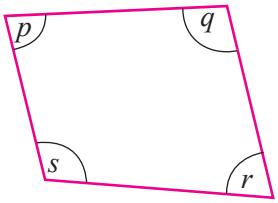


The sides of the quadrilateral *ABCD* are *AB, BC, CD* and *DA*. Its sides can also be named *BA, CB, DC* and *AD*.

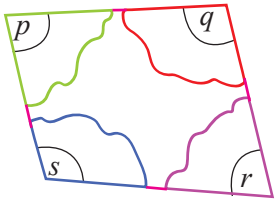
The interior angles of the quadrilateral *ABCD* in the figure are marked as *a*, *b*, *c* and *d*.

Do the activity given below in order to find the sum of the interior angles of a quadrilateral.

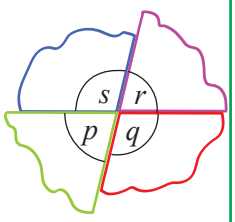
Activity 2



Step 1 - Draw any quadrilateral on a piece of coloured paper and name its interior angles as *p*, *q*, *r* and *s*.



Step 2 - Cut and separate out the angles *p*, *q*, *r*, *s* as shown in the figure.



Step 3 - In your exercise book, paste the angles that were cut out, around a point without overlapping them, such that the vertices of all the angles coincide.

Step 4 - Write down a value for $p + q + r + s$ by considering the sum of the angles around a point.

Step 5 - Draw another quadrilateral in your exercise book, measure its interior angles and obtain a value for their sum.

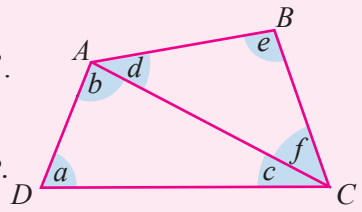
In the above activity, you would have obtained that $p + q + r + s = 360^{\circ}$.

Since the sum of the angles located around a point is 360^o , it can be concluded that the sum of the four interior angles of a quadrilateral is 360^o .

The sum of the interior angles of a quadrilateral is 360^o.

Note:

The quadrilateral $ABCD$ is shown in the figure. By joining the vertices A and C, the triangles ABC and ADC are created.



The sum of the three angles of the triangle ADC is 180°.

Accordingly, $a + b + c = 180^o$.

The sum of the three angles of the triangle ABC is 180^o.

Accordingly, $d + e + f = 180^o$.

$\therefore$ The sum of the interior angles of the quadrilateral = The sum of the interior angles of the triangle ADC + The sum of the interior angles of the triangle ABC

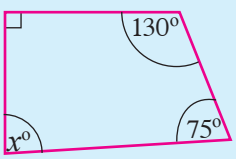
$= (a + b + c) + (d + e + f)$

$= 180^o + 180^o = 360^o$

Accordingly, the sum of the interior angles of a quadrilateral is 360^o.

Example 1

Find the value of x in the figure.



Since the sum of the interior angles of a quadrilateral is 360^o,

$x + 90 + 130 + 75 = 360$

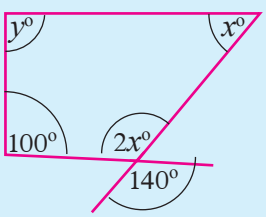
$x + 295 = 360$

$x + 295 - 295 = 360 - 295$

$x = 65$

Example 2

Find the values of x and y in the figure.



Since vertically opposite angles are equal,

$2x = 140$

$x = 70$

Since the sum of the interior angles of a quadrilateral is 360°,

$y + 100 + 2x + x = 360$

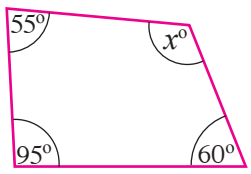
$y + 100 + 140 + 70 = 360$

$y + 310 - 310 = 360 - 310 = 50$

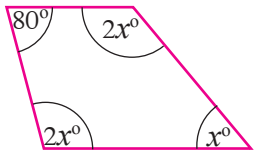
Exercise 12.2

(1) Find the value of x in each figure given below.

(i)



(ii)



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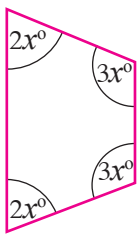

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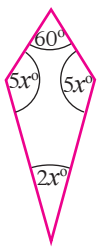
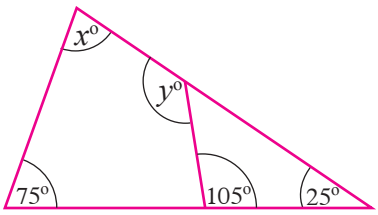
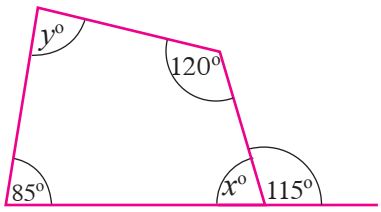


(2) Find the values of x and y in each figure given below.

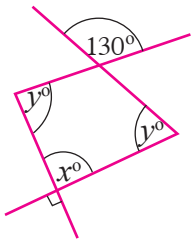
(i)



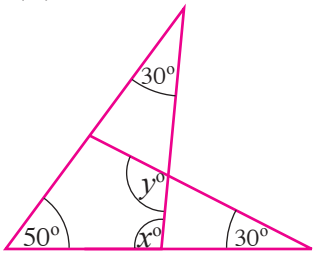
(ii)



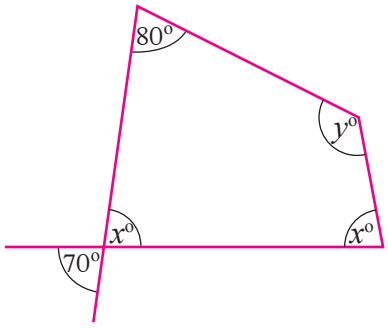
(iii)



(iv)



(v)

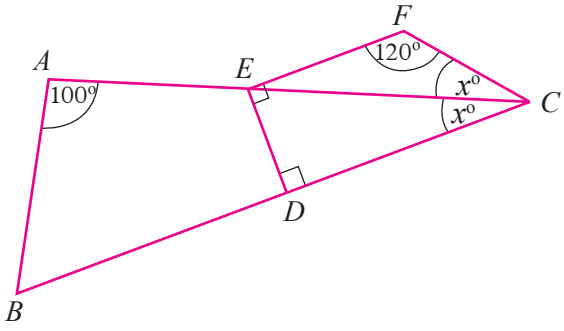


(3) Find the magnitude of each of the following angles, based on the information marked in the figure.

(i) $D\hat{C}F$

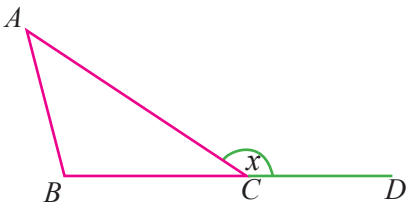
(ii) $A\hat{B}C$

(iii) $A\hat{E}D$



12.4 Exterior angles of a triangle

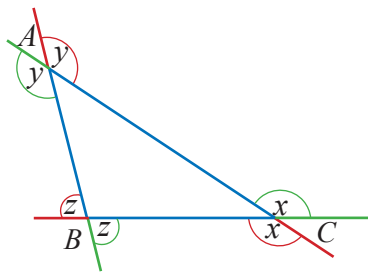
In the triangle ABC, the side BC is produced to D. The angle ACD with arms AC and CD, coloured in green, which is then formed, is **an exterior angle of the triangle ABC**.



As shown in the figure, more exterior angles can be created by producing the other sides of the triangle ABC.

Although there are two exterior angles formed at every vertex of a triangle, they are equal in magnitude since they are vertically opposite angles.

When one exterior angle at each vertex is considered, then the sum of these angles is said to be the **sum of the exterior angles of the triangle**.

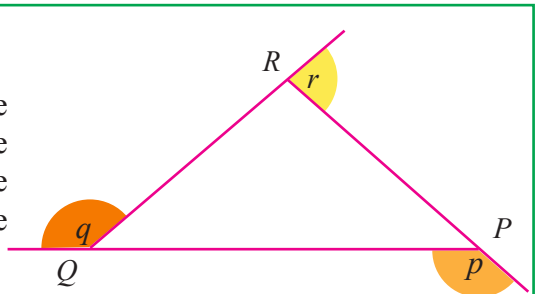


• The sum of the exterior angles of a triangle

Let us engage in activity 3 in order to obtain a value for the sum of the exterior angles of a triangle.

Activity 3

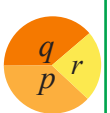
Step 1 - Draw any triangle on a piece of paper and draw three exterior angles at its three vertices as shown in the figure.



Step 2 - As shown in the figure, cut and separate out laminas of the three exterior angles using a blade.

Step 3 - In your exercise book, paste the three exterior angles (the three laminas) that were cut out, around a point without overlapping them, such that the vertices of the three exterior angles coincide.

Step 4 - Find the sum $p + q + r$ of the exterior angles of the triangle, by using the knowledge on the sum of the angles around a point.



➢ Draw another triangle in your exercise book, produce the sides to form exterior angles at the three vertices, and by measuring them, obtain the sum of the exterior angles of the triangle.

It is clear from the above activity, that the three exterior angles of a triangle can be positioned as three angles around a point.

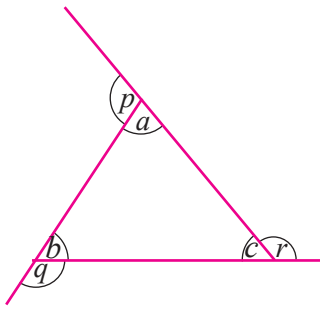
Since the sum of the angles around a point is 360˚, the sum of the exterior angles of a triangle is also 360˚.

This result is obtained by measuring the angles as well.
According to the given figure,

$$(a+p)+(b+q)+(c+r) = 180^o+180^o+180^o$$
$$= 540^o$$
$$\therefore (a+b+c)+(p+q+r) = 540^o$$

Since, $a+b+c = 180^o$,

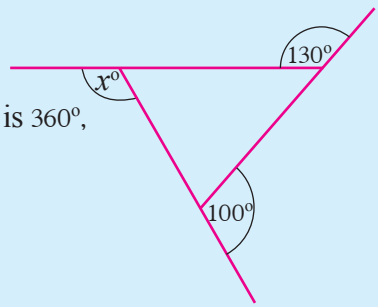
$$180^o+(p+q+r) = 540^o$$
$$\therefore p+q+r = 540^o-180^o$$
$$= 360^o$$



The sum of the exterior angles of a triangle is 360˚.

Example 1

Find the value of x in the figure.

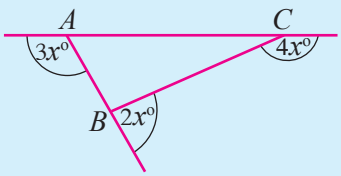


Since the sum of the exterior angles of a triangle is 360°,

$$130+100+x = 360$$
$$230+x = 360$$
$$x+230-230 = 360-230$$
$$x = 130$$

Example 2

Find the magnitudes of the three exterior and the three interior angles of the triangle ABC.



$3x + 2x + 4x = 360$

$9x = 360$

$\frac{9x}{9} = \frac{360}{9}$

$\therefore x = 40$

$\therefore$ The exterior angle at vertex $A = 3x^o = 3 \times 40^o = 120^o$

The exterior angle at vertex $B = 2x^o = 2 \times 40^o = 80^o$

The exterior angle at vertex $C = 4x^o = 4 \times 40^o = 160^o$

Since the sum of the angles on a straight line is 180°,

the interior angle at vertex $A = 180^o - 120^o = 60^o$

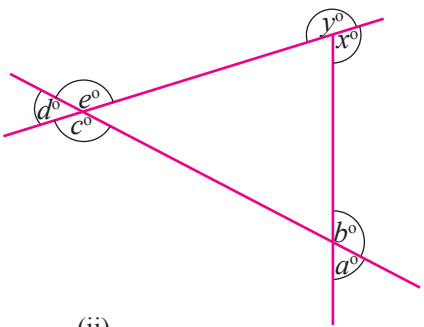
the interior angle at vertex $B = 180^o - 80^o = 100^o$

the interior angle at vertex $C = 180^o - 160^o = 20^o$

Exercise 12.3

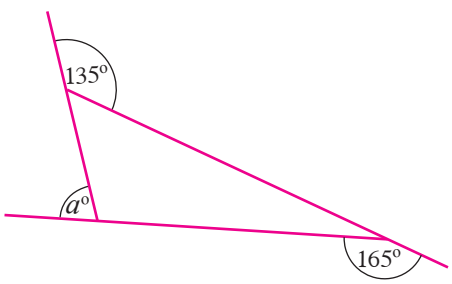
(1) (i) Select and write the exterior angles from among the angles *a, b, c, d, e, x* and *y* shown in the figure.

(ii) Explain why the other angles are not exterior angles.

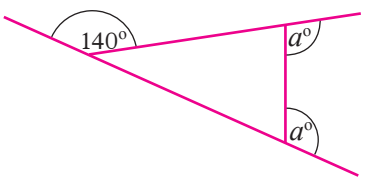


(2) Find the value of each of the angles denoted by an English letter in each figure given below.

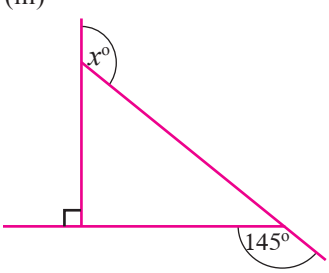
(i)



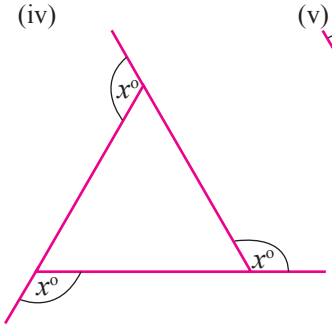
(ii)



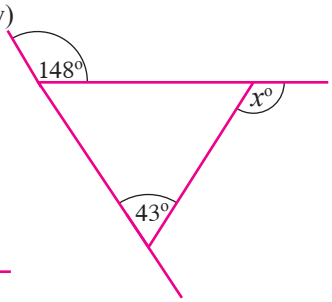
(iii)



(iv)



(v)

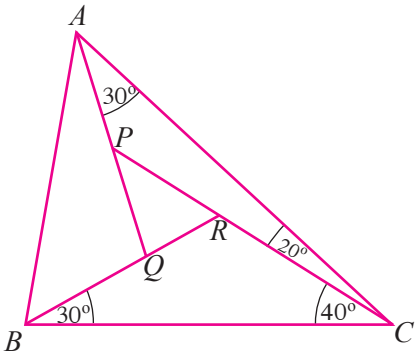


(3) According to the information marked in the figure,

(i) find $B\hat{R}C$.

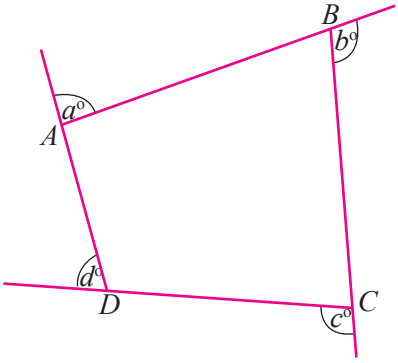
(ii) find $A\hat{P}C$.

(iii) find $B\hat{Q}A$.



12.5 Exterior angles of a quadrilateral

The exterior angles created by producing the sides of the quadrilateral $ABCD$ are marked in the figure as a, b, c and d.



A quadrilateral has four vertices. Hence, there are four exterior angles.

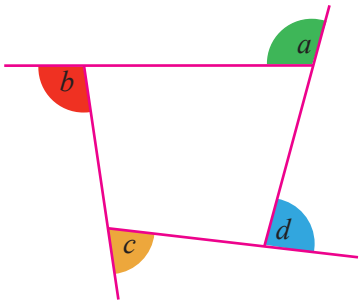
Although there are two exterior angles formed at every vertex of a quadrilateral, they are equal in magnitude since they are vertically opposite angles.

When one exterior angle at each vertex is considered, then the sum of these angles is said to be the **sum of the exterior angles of the quadrilateral**.

Let us engage in the activity given below in order to find the sum of the exterior angles of a quadrilateral.

Activity 4

Step 1 - Draw any quadrilateral, and draw four exterior angles at its four vertices.



Step 2 - As shown in the figure, cut and separate out laminas of the exterior angles with a blade.

Step 3 - Obtain a value for $a + b + c + d$, by pasting the four exterior angles that were cut out, around a point without overlapping them, such that their vertices coincide.

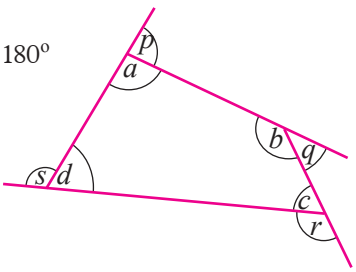
➢ Draw another quadrilateral in your exercise book and obtain a value for the sum of its exterior angles by measuring them.

It is clear from the above activity that the sum of the exterior angles of a quadrilateral is 360˚.

The sum of the exterior angles of a quadrilateral is 360˚.

According to the given figure,

$$a + p + b + q + c + r + d + s = 180^o + 180^o + 180^o + 180^o$$
$$(a + b + c + d) + (p + q + r + s) = 720^o$$
Since $a + b + c + d = 360^o$,
$$360^o + p + q + r + s = 720^o$$
$$= 720^o - 360^o$$
$$= 360^o$$

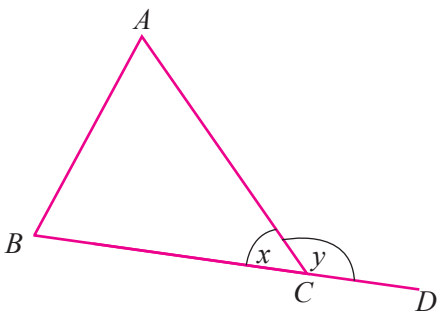


• The sum of an exterior angle and an interior angle at one vertex of a triangle and of a quadrilateral

The interior and exterior angles of a triangle at one vertex are shown in the figure as x and y.

These two angles are located on the straight line BD, at the point C.

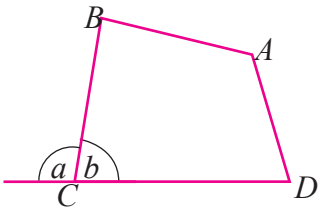
Since the sum of the angles at a point on a straight line is 180˚, $x + y = 180^o$.



At a vertex of a triangle, interior angle + exterior angle = 180º.

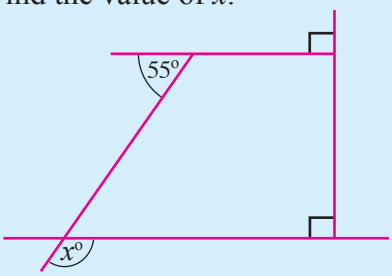
As for a triangle, the sum of the interior angle and the exterior angle at each vertex of a quadrilateral is 180^o.

$\therefore \quad a + b = 180^o$



Example 1

Find the value of x.



$x + 55 + 90 + 90 = 360$

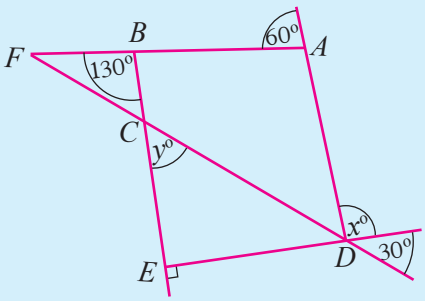
$x + 235 = 360$

$x = 360 - 235$

$x = 125$

Example 2

Find the values of x and y according to the information marked in the figure.



Since the sum of the exterior angles of the quadrilateral $ABED$ is 360^o,

$60 + 130 + 90 + x = 360$

$x + 280 = 360$

$x + 280 - 280 = 360 - 280$

$x = 80$

By taking the sum of the exterior angles of the quadrilateral $ABCD$,

$60 + 130 + y + (30 + x) = 360$

$190 + y + 30 + 80 = 360$

$y + 300 = 360$

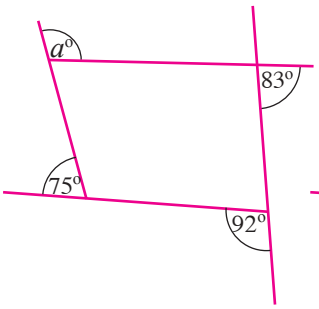
$y = 360 - 300$

$y = 60$

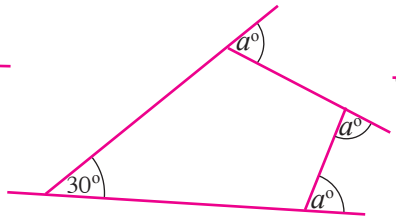
Exercise 12.4

(1) Find the value of a, marked in each figure.

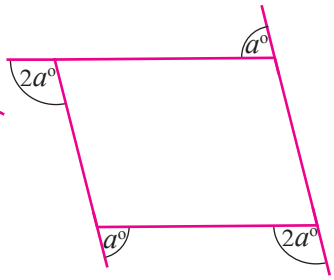
(i)



(ii)

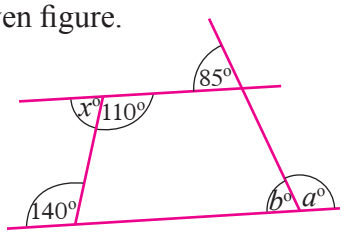


(iii)



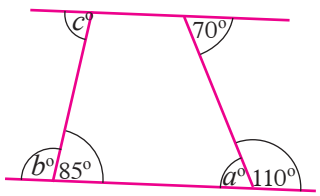
(2) Find the value of each of the angles based on the given figure.

(i) What is the value of x?
(ii) What is the value of a?
(iii) What is the value of b?

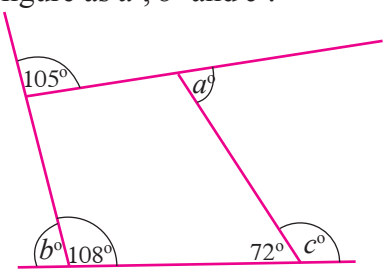


(3) Find the magnitudes of the angles marked in the figure as a^o, b^o and c^o.

(i)

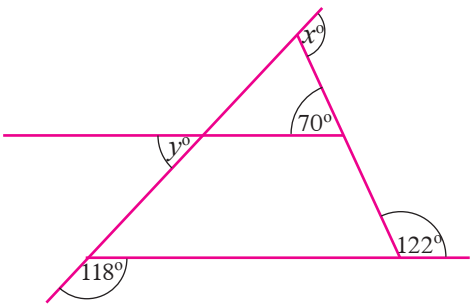


(ii)

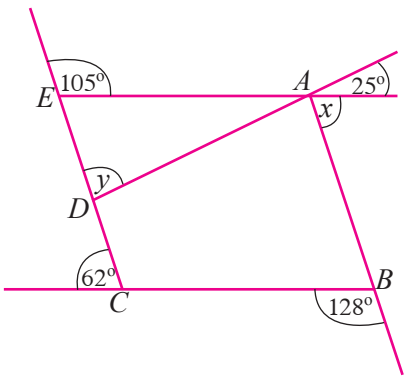


(4) Find the values of x and y in each figure.

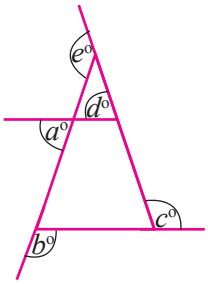
(i)



(ii)



(5)



(i) What is the value of $a + b + c + d$?

(ii) What is the value of $b + c + e$?

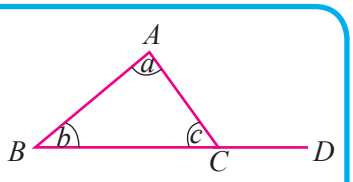
(iii) According to the answers of (i) and (ii), show that $e = a + d$.

Summary

- The sum of the interior angles of a triangle is 180°.
- The sum of the interior angles of a quadrilateral is 360°.
- The sum of the exterior angles of a triangle is 360°.
- The sum of the exterior angles of a quadrilateral is 360°.

Think

(1) Show that $A\hat{C}D = a + b$.



(2) (i) $ABCD$ is not a polygon. Explain the reason.

(ii) $a + b = c + d$. Explain the reason.

(iii) Show that the value of $a + b + c + d$ is less than 360°.