11 Symmetry

By studying this lesson you will be able to,

- identify the rotational symmetry of a plane figure,
- find the order of rotational symmetry of a plane figure that has rotational symmetry, and
- find the relationship between the number of axes of bilateral symmetry and the order of rotational symmetry of a plane figure which is bilaterally symmetrical.

11.1 Bilateral symmetry

You learnt in Grade 7 that if it is possible to fold a plane figure through a line on it to get two identical parts which coincide with each other, then it is called a **bilaterally symmetrical plane figure**. You also learnt that such a line of a bilaterally symmetrical figure is known as an **axis of symmetry** of the figure.

The two parts on either side of an axis of symmetry of a bilaterally symmetrical figure are equal in shape and area.

If by folding a plane figure along a line, it is divided into two parts which are equal in area and shape, but the two parts do not coincide with each other, then that line is not an axis of symmetry of the plane figure.

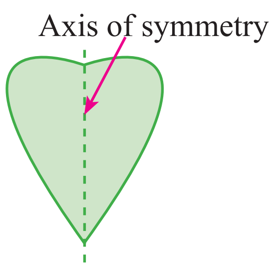


Figure 1

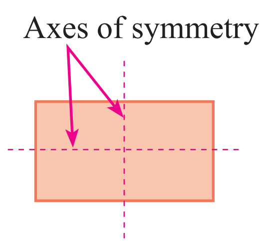


Figure 2

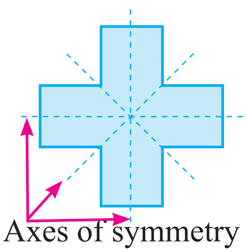


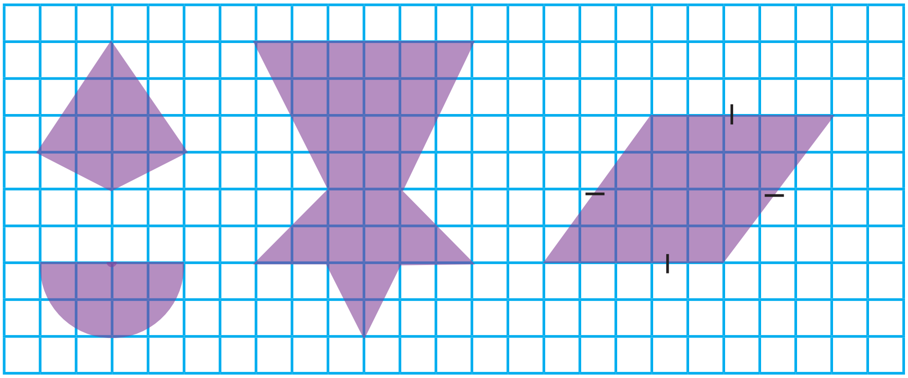
Figure 3

The axes of symmetry are indicated by dotted lines in each figure shown above.

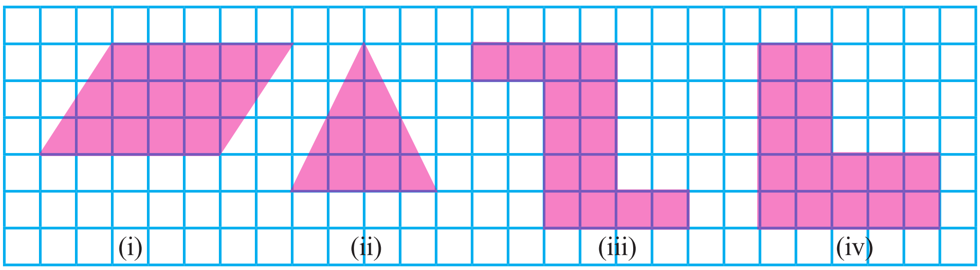
Do the review exercise to recall the facts you learnt in Grade 7 about bilateral symmetry.

Review Exercise

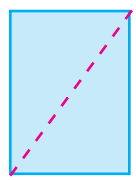
(1) Copy the following plane figures in your exercise book and draw the axes of symmetry of each figure.



(2) Select the figures that are bilaterally symmetrical from the following figures and write down their numbers.



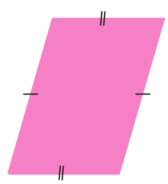
(3) The dotted line shown in the figure divides the given rectangle into two equal parts. Samith says that the dotted line represents an axis of symmetry of the rectangle. Explain why his statement is not true.



(4) (i) Copy the given parallelogram onto a tracing paper and cut it out.

(ii) Can the figure that was cut out be folded along a line so that the two parts on either side coincide?

(iii) Accordingly, show that a parallelogram need not have bilateral symmetry.



11.2 Rotational symmetry

When a plane figure is rotated about a point on it through one complete rotation in the plane of the figure, it coincides with the original position at least once.

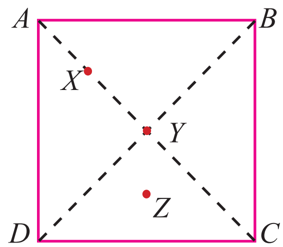
There are some figures, which when rotated about a point on it through one complete rotation, coincide several times with the original position.

The number of times a figure coincides with the original position varies, depending on the point about which it is rotated.

Do the following activity to establish this property further.

Activity 1

Step 1 - Draw a square *ABCD* and mark the points *X*, *Y* and *Z* as in the given figure.



Step 2 - Trace the figure *ABCD* onto a transparent sheet like an oil paper or a plastic paper and mark the points *X*, *Y* and *Z* on it.

Step 3 - Overlap the two figures so that they coincide, and keep them in place with a pin fixed at point *X*.

Step 4 - Rotate the figure on the transparent sheet about the pin point (point *X*) and observe whether the two figures coincide with each other. Find the number of times the two figures coincide with each other when the transparent sheet is rotated once about *X*.

Step 5 - As above, rotate the figure on the transparent sheet once about the points *Y* and *Z* too, and find the number of times the two figures coincide with each other.

Step 6 - Draw the following table in your exercise book and complete it.

Point	*X*	*Y*	*Z*
Number of times the figures coincide			

While doing the above activity you would have observed that when the figure on the transparent sheet was rotated once about the points X and Z, the two figures coincided with each other only once at the completion of one rotation, and that when the figure was rotated about the point Y, it coincided four times with the original figure during one complete rotation.

When a plane figure is rotated about a fixed point in it through one complete rotation (i.e., 360°), if it coincides with the original position before completing one rotation, then it is said to have **rotational symmetry**. The point about which the figure is rotated is called the **centre of rotation**.

When a plane figure that has rotational symmetry is rotated once about a point which is not the centre of rotation, then it coincides with the original figure only when it completes one complete rotation.

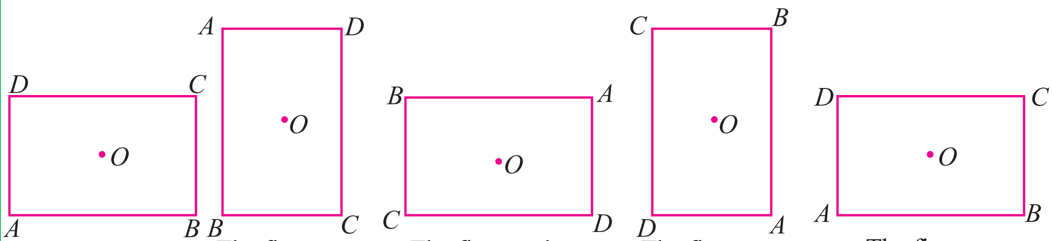
The number of times a figure that has rotational symmetry coincides with itself when it is rotated once about the centre of rotation is called its **order of rotational symmetry**.

From the above activity, it is clear that a square has rotational symmetry, that its centre of rotation is the point at which its axes of symmetry intersect, and that its order of rotational symmetry is 4.

Activity 2

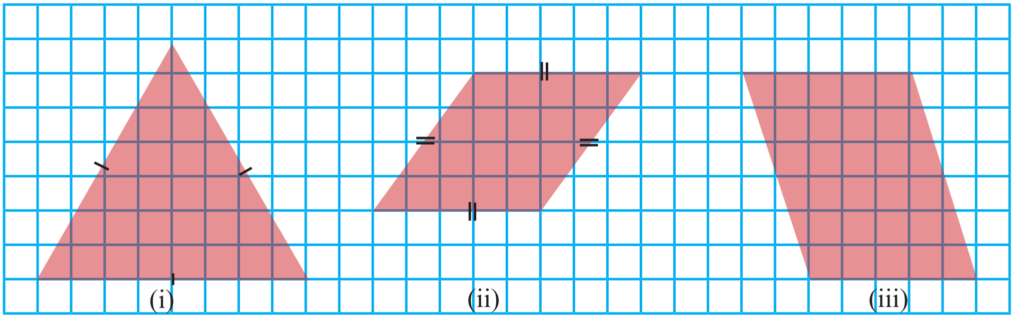
Step 1 - Draw a rectangle in your exercise book and name it $ABCD$.

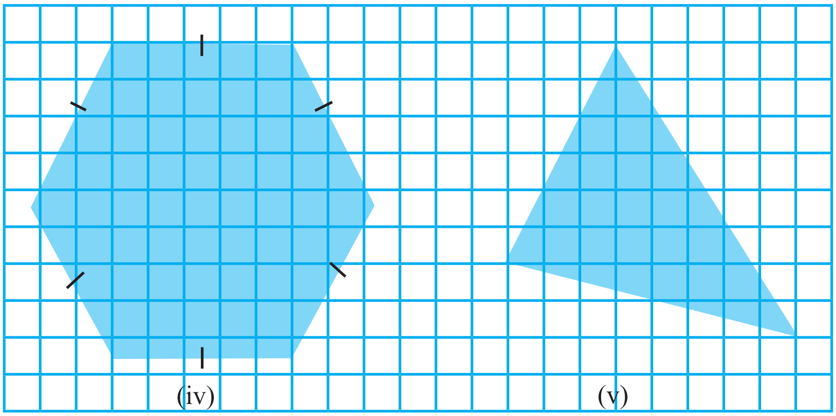
Step 2 - Copy the rectangle $ABCD$ onto a plastic paper and place it on the original figure so that they coincide with each other. By using a pin, rotate the plastic paper about O as in activity 1 and find out whether the rectangle has rotational symmetry. If it does, find its order of rotational symmetry.



The figure when rotated through 90° | The figure when rotated through 180° | The figure when rotated through 270° | The figure when rotated through 360°

Step 3 - Draw the following figures also in your exercise book, and check whether they have rotational symmetry.





Step 4 - Copy the following table and complete it.

If a given figure has rotational symmetry, then write its order of symmetry.

Plane figure	Number of axes of symmetry	Order of rotational symmetry
Rectangle Equilateral triangle Rhombus Parallelogram Regular hexagon Scalene triangle		

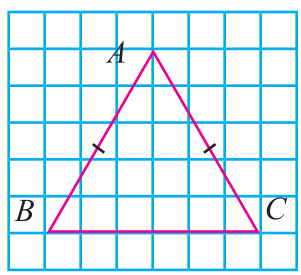
Examine the table given below.

Plane figure	Number of axes of symmetry	Order of rotational symmetry	Rotational symmetry Yes/No
Equilateral triangle	3	3	Yes
Parallelogram	0	2	Yes
Rhombus	2	2	Yes
Rectangle	2	2	Yes
Square	4	4	Yes
Regular pentagon	5	5	Yes
Regular hexagon	6	6	Yes
Regular octagon	8	8	Yes

The following facts are clear according to the above table.

- If a **geometrical plane figure** which has rotational symmetry is also **bilaterally symmetrical**, then its order of rotational symmetry is equal to the number of axes of bilateral symmetry.
- A figure which is not bilaterally symmetrical can have rotational symmetry (parallelogram).
- The center of rotational symmetry of a bilaterally symmetrical plane figure which has rotational symmetry, is the point of intersection of its axes of bilateral symmetry.
- If the order of rotational symmetry of a plane figure is 2 or more, then that figure is said to have rotational symmetry.
- The order of rotational symmetry of a figure that has rotational symmetry is greater than 1.

Exercise 11.1



(1) (i) Draw the isosceles triangle *ABC* in your exercise book as in the figure, and draw its axis of symmetry.

(ii) Copy the triangle *ABC* onto a plastic paper and find out whether an isosceles triangle has rotational symmetry.

(iii) Does every plane figure which is bilaterally symmetrical have rotational symmetry?

(2) (i) Draw a plane figure which has two or more axes of symmetry.

(ii) Find out whether the figure you have drawn has rotational symmetry.

(iii) If it has rotational symmetry, mark the centre of rotation as *P* and write down the order of rotational symmetry of the figure.

(3) Write down the following statements in your exercise book and mark the true statements with "✓" and the false statements with "✗".

(i) Every bilaterally symmetrical plane figure has rotational symmetry.

(ii) Every plane figure which has rotational symmetry is also bilaterally symmetrical.

(iii) If a bilaterally symmetrical plane figure has rotational symmetry, then the number of axes of symmetry is equal to its order of rotational symmetry.

(iv) The point of intersection of the axes of symmetry of a bilaterally symmetrical plane figure which has more than one axis of symmetry is its centre of rotational symmetry.

(v) A scalene triangle does not have rotational symmetry and is also not bilaterally symmetrical.

Summary

- When a plane figure is rotated about a special point in it through an angle of 360°, if it coincides with the original position before completing a full rotation, then it is said to have rotational symmetry.
- The center of rotational symmetry of a bilaterally symmetrical plane figure which has rotational symmetry, is the point of intersection of its axes of bilateral symmetry.
- The order of rotational symmetry of a figure that has rotational symmetry is greater than 1.
- When a plane figure is rotated about its centre of rotation, the number of times it coincides with the original position during a complete rotation is called its order of rotational symmetry.