## Indices

By studying this lesson you will be able to,

- express a power of a product as a product of powers,
- express a product of powers as a power of a product, and
- find the value of a power of a negative integer by expansion.


### 10.1 Indices

Let us recall what was learnt in Grade 7 about indices.
You learnt in Grade 7 that $2^{3}$ and $x^{4}$ are respectively a power of 2 and a power of $x$. In $2^{3}$, the base is 2 and the index is 3 .

These can be expanded and written as products as $2^{3}=2 \times 2 \times 2$ and $x^{4}=x \times x \times x \times x$.

Accordingly, $3 x^{2} y^{3}=3 \times x \times x \times y \times y \times y$ and $3 a b=3 \times a \times b$.
Since $6=2 \times 3$, we say that 6 is the product of 2 and 3 .
Similarly, since $3 a b=3 \times a \times b$, we say that $3 a b$ is the product of $3, a$ and $b$.

Do the following review exercise to recall the facts that have been learnt so far regarding indices.

## Review Exercise

(1) Complete the following table.

| Number | Index Notation | Base | Index |
| :---: | :---: | :---: | :---: |
| 8 | $2^{3}$ | ......... | ......... |
| 9 | ......... | ......... | ......... |
| 16 | $\ldots$ | 2 | $\ldots$ |
| ........ | ........ | 4 | 2 |
| 1000 | ......... | 10 | ......... |

(2) Expand and write each of the following expressions as a product.
(i) $3 x^{2}$
(ii) $2 p^{2} q$
(iii) $4^{2} x^{3}$
(iv) $5^{2} x^{2} y^{2}$
(3) Write down each of the following numbers as a product of powers of which the bases are prime numbers.
(i) 20
(ii) 48
(iii) 100
(iv) 144
(4) Write 64 in index notation (i) with base 2
(ii) with base 4
(iii) with base 8 .

### 10.2 Expressing a power of a product as a product of powers

$2 \times 3$ is the product of 2 and 3 . $(2 \times 3)^{2}$ is a power of the product $2 \times 3$.
Let us write $(2 \times 3)^{2}$ as a product of powers of 2 and 3 .

$$
\begin{aligned}
(2 \times 3)^{2} & =(2 \times 3) \times(2 \times 3) \\
& =2 \times 3 \times 2 \times 3 \\
& =2 \times 2 \times 3 \times 3 \\
& =2^{2} \times 3^{2}
\end{aligned}
$$

$$
\therefore(2 \times 3)^{2}=2^{2} \times 3^{2}
$$

Now let us write $(2 \times 3)^{3}$ as a product of powers of 2 and 3 .

$$
\begin{aligned}
(2 \times 3)^{3} & =(2 \times 3) \times(2 \times 3) \times(2 \times 3) \\
& =2 \times 3 \times 2 \times 3 \times 2 \times 3 \\
& =2 \times 2 \times 2 \times 3 \times 3 \times 3 \\
& =2^{3} \times 3^{3}
\end{aligned}
$$

$$
\therefore(2 \times 3)^{3}=2^{3} \times 3^{3}
$$

Accordingly, the power of a product can be written in this manner as a product of powers of the factors of the given product.

Now let us consider a power of a product that contains unknown terms.

$$
\begin{aligned}
(a b)^{3} & =a b \times a b \times a b \\
& =a \times b \times a \times b \times a \times b \\
& =a \times a \times a \times b \times b \times b \\
& =a^{3} \times b^{3}=a^{3} b^{3} \\
(a b)^{3} & =a^{3} b^{3}
\end{aligned}
$$

Let us similarly express $(a b c)^{3}$ as a product of powers of $a, b$ and $c$.

$$
\begin{aligned}
(a b c)^{3} & =(a b c) \times(a b c) \times(a b c) \\
& =(a \times b \times c) \times(a \times b \times c) \times(a \times b \times c) \\
& =(a \times a \times a) \times(b \times b \times b) \times(c \times c \times c) \\
& =a^{3} \times b^{3} \times c^{3}=a^{3} b^{3} c^{3} \\
\therefore(a b c)^{3} & =a^{3} b^{3} c^{3}
\end{aligned}
$$

Accordingly, a power of a product can be written as a product of powers of the factors of the given product.

- Let us express $4 a^{2}$, as a power of a product.

$$
\begin{aligned}
4 a^{2}=4 \times a^{2} & =2^{2} \times a^{2} \\
& =(2 \times a)^{2} \\
& =(2 a)^{2}
\end{aligned}
$$

This can be established further through the following examples.

## Example 1

Express each of the following powers of products as a product of powers of the factors of the given product.
(i) $(2 x)^{3}$
(ii) $(3 a b)^{2}$
(i) $(2 x)^{3}=2^{3} \times x^{3}$

$$
=2^{3} x^{3}
$$

(ii) $(3 a b)^{3}$

$$
\begin{aligned}
(3 a b)^{3} & =3^{3} \times a^{3} \times b^{3} \\
& =3^{3} a^{3} b^{3}
\end{aligned}
$$

## Example 2

Express $36 x^{2}$, as a power of a product.
Since $36=6^{2}$,

$$
\begin{aligned}
36 x^{2}=6^{2} \times x^{2} & =(6 \times x)^{2} \\
& =(6 x)^{2}
\end{aligned}
$$

## Example 3

Express $a^{3} b^{3}$ as a power of a product.

$$
\begin{aligned}
a^{3} b^{3} & =a^{3} \times b^{3} \\
& =(a \times b)^{3} \\
& =(a b)^{3}
\end{aligned}
$$

## Exercise 10.1

(1) Express each of the following powers of products as a product of powers of the factors of the given product.
(a) (i) $(2 \times 5)^{2}$
(ii) $(3 \times 5)^{3}$
(iii) $(11 \times 3 \times 2)^{3}$
(iv) $(a \times b)^{2}$
(v) $(x \times y)^{5}$
(vi) $(4 \times x \times y)^{3}$
(b) $\left(\right.$ i) $(5 a)^{2}$
(ii) $(6 p)^{2}$
(iii) $(4 y)^{3}$
(iv) $(3 a)^{3}$
(v) $(2 y)^{4}$
(vi) $(2 a b)^{2}$
(2) Find the value of each of the following powers of products. Write each power of a product as a product of powers of the factors of the given product and obtain the value again by simplifying the answer.
(i) $(2 \times 5)^{3}$
(ii) $(2 \times 3)^{3}$
(iii) $(11 \times 2)^{3}$
(iv) $(3 \times 7)^{2}$
(v) $(5 \times 7)^{3}$
(vi) $(13 \times 2 \times 3)^{2}$
(3) Express each of the following products of powers as a power of a product.
(i) $5^{2} \times 2^{2}$
(ii) $5^{2} \times 11^{2}$
(iii) $3^{3} \times 4^{3} \times 2^{3}$
(iv) $x^{2} \times y^{2}$
(v) $p^{3} \times q^{3}$
(vi) $a^{5} \times b^{5} \times x^{5}$
(vii) $100 m^{2}$
(viii) $225 t^{2}$
(ix) $8 y^{3}$
(4) Show that $1000 x^{3}=(10 x)^{3}$.

### 10.3 The power of a negative integer

$-1,-2,-3$ are negative integers. Do the following activity to find the value of a power of these negative integers.

## Activity 1

Complete the following table by using the knowledge on multiplying integers.

| Integer | Its second power | Its third power | Its fourth power |
| :---: | :---: | :---: | :---: |
| $\begin{gathered} 2 \\ -1 \\ -2 \\ -3 \end{gathered}$ | $\begin{aligned} 2^{2} & =2 \times 2=4 \\ (-1)^{2} & =(-1) \times(-1)=1 \end{aligned}$ | $2^{3}=2 \times 2 \times 2=8$ | $2^{4}=2 \times 2 \times 2 \times 2=16$ $\qquad$ $\qquad$ $\qquad$ |

[^0]
## Example 1

Find the value of $(-2)^{4}$.

$$
\begin{aligned}
(-2)^{4} & =2^{4} \\
& =16
\end{aligned}
$$

## Example 2

Find the value of $(-5)^{3}$.

$$
\begin{aligned}
(-5)^{3} & =-(5)^{3} \\
& =-125
\end{aligned}
$$

## Exercise 10.2

(1) Find the value.
(a) (i) $(-1)^{1}$
(ii) $(-1)^{2}$
(iii) $(-1)^{3}$
(iv) $(-1)^{4}$
(v) $1^{1}$
(vi) $1^{1003}$
(vii) $1^{2018}$
(viii) $1^{0}$
(b)
(i) $(-4)^{2}$
(ii) $(-4)^{3}$
(iii) $(-4)^{4}$
(iv) $(-5)^{1}$
(v) $(-5)^{2}$
(vi) $(-5)^{3}$
(vii) $(-10)^{1}$
(viii) $(-10)^{2}$
(2) Show that $(-1)^{8}>(-1)^{9}$.

## Miscellaneous Exercise

(1) Express each of the following products of powers as a power of a product.
(i) $(2 x)^{2} \times y^{2}$
(ii) $(3 a)^{2} \times b^{2}$
(iii) $p^{3} \times(2 q)^{3}$
(iv) $(2 x)^{3} \times(3 y)^{3}$
(v) $(5 a)^{3} \times(2 b)^{3}$
(vi) $a^{3} \times(2 b)^{3} \times c^{3}$
(2) Show that $(3 a)^{2} \times(2 x)^{2}=36 a^{2} x^{2}$.
(3) Arrange in ascending order of the values.
(i) $2^{3},(-10)^{1},(-1)^{10}, 3^{2}$
(ii) $(-2)^{4},(-2)^{5},(-1)^{4},(-1)^{5}$
(4) If $a$ is a negative integer, show that $a^{2}>a^{3}$.

## Summary

[1] If $a, b, c$ and $n$ are positive integers, then $(a b)^{n}=a^{n} \times b^{n}=a^{n} b^{n}$ and $(a b c)^{n}=a^{n} \times b^{n} \times c^{n}=a^{n} b^{n} c^{n}$.
The value of any power of a positive integer is positive.
(1) The value of an odd power of a negative integer is negative.

The value of an even power of a negative integer is positive.

## REVISION EXERCISE - FIRST TERM

(1) (i) Find the value of $\sqrt{361}$.
(ii) Evaluate $5 \mathrm{t} 75 \mathrm{~kg} \times 12$.
(iii) Write down the value of $(-11)^{11}$.
(iv) What is the complement of the angle $28^{\circ}$ ?
(v) What is the supplement of the angle $28^{\circ}$ ?
(vi) (a) Find the value of $x$.
(b) Find the value of $a$.

(vii) Write down the number of faces, edges and vertices of a dodecahedron.
(viii) Fill in the blanks.

$$
12 x-36 y+4=4(\square x-\square y+\square)
$$

(2) (a) Find the value of each of the following.
(i) $(-5)+(-3)$
(ii) $(-7)+4$
(iii) $13+(-5)$
(iv) $(-5)-(-2)$
(v) $(-7)-(-10)$
(vi) $0-(-5)$
(b) Find the value of each of the following.
(i) $(-12) \times(-3)$
(ii) $(+8) \times(-5)$
(iii) $(+12) \div(-3)$
(iv) $(-12) \div(-3)$
(v) $(-12) \times 0$
(vi) $0 \div(-100)$
(c) Fill in the cages and re-write the following.
(i) $24 \div \square=(-4)$
(ii) $(-16) \div \square=(-4)$
(iii) $32 \div \square=(-4)$
(iv) $(-10)+\square=-6$
(v) $(-5)+\square=(-6)$
(vi) $(-2) \times(-4)=$
(3) The general term of the triangular number pattern starting from 1 is $\frac{n(n+1)}{2}$.
(i) Write the first term of the triangular number pattern.
(ii) Write the 19th and 20th terms of this pattern.
(iii) It is given that $10 \times 11=110$. Find which term of the triangular number pattern is 55 ?
(iv) It is given that $18 \times 19=342$. Find which term of the triangular number pattern is 171?
(v) Show that the sum of the 19th and 20th terms of the triangular number pattern is equal to the 20th term of the square number pattern which starts from 1.
(4) (i) Find the perimeter of the square design.
(ii) Find the perimeter of the isosceles triangular design.

(iii) The two designs are pasted together as shown to form a composite figure. Find the perimeter of the composite figure.

(5)


The straight lines $A B$ and $C D$ are drawn such that they intersect each other perpendicularly at $P$. The given figure is obtained by joining $A C, C B$ and $D B$ and then producing them.
(i) Write three pairs of complementary angles.
(ii) Write three pairs of supplementary angles.
(iii) Write 4 pairs of vertically opposite angles.
(iv) What is the magnitude of $F \hat{B} G$ ?
(v) $C \hat{B} D$ and $D \hat{B} G$ are a pair of supplementary angles. Write down the magnitude of $D \hat{B} G$.
(vi) Name an angle which is a supplement of $C \hat{B} P$.
(vii) Write down the value of the angle you named.
(viii) Find the magnitude of $C \hat{B} F$.
(ix) Find the sum of the angles around the point $B$ and establish the fact that the sum of the angles around a point is $360^{\circ}$.
(6) (i) The perimeter of a rectangle is $16 x+10$ units. Its length is $5 x+3$ units. Write an algebraic expression for its breadth.

(ii) A cuboid of length, breadth and height equal to $2 n, n$ and $n-1$ units respectively is shown in the figure. Show that the sum of the lengths of all its edges is $4(4 n-1)$ units.

(7) Simplify the following.
(i) $5(c-2)+12$
(ii) $7(d-9)-d$
(iii) $4(f+5)+2 f-3$
(iv) $-2 g(h+4)-3 g(h-2)$
(v) $4 h(i+2)-7(i-1)$
(8)

(9)
(i) Express $4 y^{2}$ as a power of a product.
(ii) Write $(8 a b)^{2}$ as a product of powers and simplify it.
(iii) Simplify $(2 p)^{3} \times(3 p)^{3}$.
(iv) Show that $6^{3}$ is equal to $8 \times 27$.
(v) Show that when $(-3)^{4}$ is simplified, the same value as of $9^{2}$ is obtained.
(vi) Without obtaining the value of $(-15)^{3} \times(-27)^{4}$, explain whether the answer is a positive value or a negative value
(10) A signboard near an old bridge indicates that the maximum load which the bridge can support is 8 t . A lorry of mass 5.5 metric tons is loaded with 80 bags of cement of mass 50 kg each.
(i) Calculate and show that it is dangerous for the lorry loaded
 with the bags of cement to cross the bridge.
(ii) What is the minimum number of bags of cement that should be removed for the lorry to safely cross the bridge?
(11) Simplify the following.
(a)
(i) $(+7)+(-3)$
(ii) $(-5)+(-4)$
(b)
(i) $(+10)-(-3)$
(c)
(ii) $(-7)-(-3)$
(i) $(+4) \times(-3)$
(iii) $(+12)+(-18)$
(iii) $(-7)-(+20)$
(ii) $(-5) \times(-6)$
(iv) $\left(+5 \frac{1}{2}\right)+(-3)$
(iv) $(+17)-(-12)$
(iii) $(-1) \times(+4.8)$
(v) $(+3.7)+(-6.3)$
(v) $(+8.7)-(-2.3)$
(iv) $(-20) \div(+4)$
(v) $(-35) \div(-5)$
(12) Expand the given algebraic expressions and simplify them.
(i) $5(2 x-3)-4 x+7$
(ii) $x(3 y+5)-8 x y+2$
(iii) $-3 a(5-7 b)+5(a-2)$
(13) Simplify the following.
(i) $4 a+7 b-3(a+c)$
(ii) $2(3 x-7)-2 x+5$
(iii) $3 a(a+7)+5 a^{2}-20 a+4$
(14) Find the value of each algebraic expression when $x=-2, y=3$ and $z=-3$.
(i) $3 x+4 y$
(ii) $x^{2} y+5 y^{2}$
(iii) $4(2 x-3 y-4 z)$
(15) Write down the geometrical shape of the faces of each solid given below.
(i) Regular tetrahedron
(ii) Cube
(iii) Regular octahedron
(iv) Regular dodecahedron
(v) Regular icosahedron
(16) Write down the HCF of each of the following groups of terms.
(i) $3 x, 12 x y, 15 y$
(ii) $12 x, 6 x y, 9 x^{2}$
(iii) $3 a^{2} b, 15 a b, 15 y$
(iv) $4 x^{2} y, 6 x y, 8 x y^{2}$
(17) Factorize the following expressions.
(i) $8 x+4 y+12$
(ii) $15 x^{2}+3 x y$
(iii) $6 a^{2} b-15 a b+18 a b c$
(iv) $-4 m n-20 m^{2}+12 m$
(18) (i) Write down the perfect squares that are in the range of values from 1 to 100 .
(ii) The digit in the units place of a perfect square is 6 . Write two digits that could be in the units place of its square root.
(iii) Which digits do not appear in the units place of a perfect square?
(iv) Find $\sqrt{900}$.
(19) Fill in the blanks.
(i) $3 \mathrm{t}=\ldots \ldots \ldots . . \mathrm{kg}$.
(ii) $3500 \mathrm{~kg}=$ $\qquad$ t $\qquad$
$\qquad$
(iii) $4.05 \mathrm{t}=\ldots \ldots \ldots . . . \mathrm{kg}$.
(iv) $12450 \mathrm{~kg}=$ $\qquad$ t.
(v) $10 \mathrm{t} 50 \mathrm{~kg}=\ldots \ldots \ldots \ldots . \mathrm{kg}$.
(20) Evaluate the following.
(i) $3^{2} \times 5$
(ii) $4^{3} \times 2^{2}$
(iii) $2^{3} \times 3^{2}$
(iv) $(-4)^{2} \times 5^{3}$
(v) $(-3)^{3} \times 2^{2}$
(vi) $(-1)^{4} \times 5^{2} \times 4$


[^0]:    - the value of any power of a positive integer is positive.
    - the value of an odd power of a negative integer is negative.
    - the value of an even power of a negative integer is positive.

