## Square Root

By studying this lesson, you will be able to,

- identify what a perfect square is,
- write the square of each of the whole numbers from 1 to 20 ,
- find the square root of the perfect squares from 1 to 1000 by observation and by considering their prime factors.


### 8.1 Square of a positive integer

A few numbers which can be represented by a square shaped arrangement of dots are given below.

| $\square$ | \%09 | 008 <br> 000 <br> 000 | [ |
| :---: | :---: | :---: | :---: |
| Rows 1 | Rows 2 | Rows 3 | Rows 4 |
| Columns 1 | Columns | Columns 3 | Columns 4 |

You have learnt earlier that the numbers $1,4,9,16, \ldots$ which can be represented as above are called square numbers.
We get the square numbers $1,4,9,16, \ldots$ by multiplying each positive integer by itself. We can write these square numbers using indices as $1^{2}, 2^{2}, 3^{2}, 4^{2}, \ldots$

These are read as the "one squared", "two squared" etc.

| Representation of the square number | Number of rows/ columns | How the square is obtained | Square of the number, using indices |  |
| :---: | :---: | :---: | :---: | :---: |
| 0 | Rows 1, <br> Columns 1 | $1 \times 1$ | $1^{2}$ | 1 |
| 80 | Rows 2, <br> Columns 2 | $2 \times 2$ | $2^{2}$ | 4 |
| $\begin{aligned} & \hline 000 \\ & 000 \\ & 000 \\ & \hline \end{aligned}$ | Rows 3, <br> Columns 3 | $3 \times 3$ | $3^{2}$ | 9 |
| 0000 0000 0000 0000 | Rows 4, <br> Columns 4 | $4 \times 4$ | $4^{2}$ | 16 |

The number we obtain by multiplying a number by itself, is called the square of that number. The square of a positive integer is called a perfect square.
$1,4,9,16, \ldots$ are the squares of the numbers $1,2,3,4, \ldots$ Therefore they are perfect squares.

## Example 1

A square tile is of side length 8 cm . Show that the numerical value of its surface area is a perfect square.

The length of a side of the square tile $=8 \mathrm{~cm}$

$$
\begin{aligned}
\text { Its surface area } & =8 \mathrm{~cm} \times 8 \mathrm{~cm} \\
& =64 \mathrm{~cm}^{2}
\end{aligned}
$$

The numerical value of the area of the square tile $=64=8 \times 8$
64, is the square of 8 , so the numerical value of the surface area of the tile is a perfect square.

## Exercise 8.1

(1) Represent the square of 5 by an arrangement of dots and write down its value.
(2) Complete the table given below and answer the questions accordingly.

| Number | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Square of the <br> number |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

By adding some pairs of perfect squares in the second row, we can obtain another perfect square. Observe the table and write four such relationships.
(3) (i) Write down the perfect square between 10 and 20, and the reason for it.
(ii) Write down the perfect square between 60 and 70 , and the reason for it.
(iii) Write down the perfect square between 80 and 90 , and the reason for it.
(iv) How many perfect squares are there between 110 and 160 ?
(4) Complete the table given below.

| Odd numbers added <br> consecutively | Sum | The perfect square <br> in index form |
| :--- | :---: | :---: |
| 1 | 4 |  |
| $1+3$ |  | $2^{2}$ |
| $1+3+5$ |  |  |
| $1+3+5+7$ |  |  |
| $1+3+5+7+9$ |  |  |

Using the above table, write the special feature of the numbers that are obtained when consecutive odd integers starting from 1 are added together.

### 8.2 The digit in the units place of a perfect square

The table below shows the squares of the numbers from 1 to 15 .

| Number | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Perfect Square | 1 | 4 | 9 | 16 | 25 | 36 | 49 | 64 | 81 | 100 | 121 | 144 | 169 | 196 | 225 |
| The digit in the units <br> place of the perfect <br> square | 1 | 4 | 9 | 6 | 5 | 6 | 9 | 4 | 1 | 0 | 1 | 4 | 9 | 6 | 5 |

- The digit in the units place of the square of a positive integer is the digit in the units place of the square of the digit at the right end (units place) of that positive integer.
- The digit in the units place of a square number is one of the numbers in the 3 rd row of the above table.
- It is clear from the 3 rd row of the table, that the digit in the units place of a perfect square is one of the digits $0,1,4,5,6$ or 9 .
- None of the digits $2,3,7$ and 8 is ever the digit in the units place of a perfect square.


## Example 1

Is 272 a perfect square?
If the digit in the units place of a whole number is $2,3,7$ or 8 , then that number is not a perfect square. In 272, the digit in the units place is 2 . Therefore, it is not a perfect square.
(1) By considering the digit in the units place of each of the numbers given below, show that they are not perfect squares.
(i) 832
(ii) 957
(iii) 513
(2) Give an example of a perfect square which has 9 in its units place.
(3) "If the digit in the units place of a whole number is one of $0,1,4,5,6$ or 9 , then it is a perfect square". Show with an example that this statement is not always true.
(4) Observe the digit in the units place of each number given below and write the digit in the units place of their respective squares.
(i) 34
(ii) 68
(iii) 45

### 8.3 The square root of a perfect square

$16=4 \times 4=4^{2}$. Since 16 is the square of 4 , the square root of 16 is 4 .
$49=7^{2}$, so the square root of 49 is 7 .
$81=9^{2}$, so the square root of 81 is 9 .
To indicate the square root of a number, we use the symbol " $\sqrt{ }$ ".
Accordingly; the square root of $16=\sqrt{16}=\sqrt{4^{2}}=4$,
the square root of $25=\sqrt{25}=\sqrt{5^{2}}=5$,
the square root of $100=\sqrt{100}=\sqrt{10^{2}}=10$,
the square root of $4=\sqrt{4} \quad=2 \quad\left(\right.$ because $\left.2^{2}=4\right)$
the square root of $1=\sqrt{1}=1 \quad\left(\right.$ because $\left.1^{2}=1\right)$
If $c=a^{2}$ where $a$ is a positive number, then $\sqrt{c}=a$. That is, $a$ is the square root of $c$.
If a number is the square of a positive number, then the second number is the square root of the first.

The square roots of perfect squares such as $36,49,64$ can be expressed quickly from memory. However it is not easy to do the same for every perfect square.

We have to use different methods to find them.
Let us see how we can find the square root,

- by using prime factors, and
- by observation.
- Finding the square root of a perfect square using prime factors

Let us find the value of $\sqrt{36}$ using prime factors.
Let us first write 36 as a product of its prime factors.

$$
\begin{aligned}
36 & =2 \times 2 \times 3 \times 3 \\
36 & =(2 \times 3) \times(2 \times 3) \\
& =(2 \times 3)^{2} \\
\therefore \sqrt{36} & =2 \times 3 \\
& =6
\end{aligned}
$$

## Example 1

Find the value of $\sqrt{576}$ using prime factors.

$$
\begin{aligned}
576 & =2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \\
& =(2 \times 2 \times 2 \times 3) \times(2 \times 2 \times 2 \times 3) \\
& =(2 \times 2 \times 2 \times 3)^{2} \quad \text { or } 576=24^{2} \\
\therefore \sqrt{576} & =2 \times 2 \times 2 \times 3 \text { or } \sqrt{576}=24 \\
& =24
\end{aligned}
$$

## Exercise 8.3

(1) Find the value of each of the following.
(i) $\sqrt{(2 \times 5)^{2}}$
(ii) $\sqrt{(2 \times 3 \times 5)^{2}}$
(iii) $\sqrt{(3 \times 5) \times(3 \times 5)}$
(iv) $\sqrt{3 \times 3 \times 7 \times 7}$
(v) $\sqrt{2 \times 2 \times 3 \times 3 \times 3 \times 3}$
(2) Find the square root using prime factors.
(i) 144
(ii) 400
(iii) 900
(iv) 324
(v) 625
(vi) 484
(3) What is the side length of a square shaped parking lot of area $256 \mathrm{~m}^{2}$ ?
(4) The area of a square land is $169 \mathrm{~m}^{2}$. Find the length of a side of the land.


- Finding the square root of a perfect square by observation
$>$ Finding the digit in the units place of the square root of a perfect square


## Activity 1

(1) Complete the table given below by considering the perfect squares you have identified so far and their square roots.

| (i) | Perfect squares with the digit 1 in the units place | 1 | 81 | 121 | 361 | 441 |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- |
|  | Square roots of these perfect squares | (ii) | Perfect squares with the digit 4 in the units place |  |  |  |
|  | Square roots of these perfect squares |  |  |  |  |  | (iii) | Perfect squares with the digit 5 in the units place |  |
| :--- | :--- |
|  | Square roots of these perfect squares |

(2) Complete the table given below using the information in (i) to (vi) in the above table.

| Digit in the units place of the perfect <br> square | Digit in the units place of the <br> square root |
| :---: | :---: |
| 1 |  |
| 4 |  |
| 5 |  |
| 6 |  |
| 9 |  |
| 0 |  |

According to the above activity, the digit in the units place of the square root of a perfect square, which depends on the digit in the units place of the perfect square, is as follows.

| Digit in the units place of the perfect square | Digit in the units place of the <br> square root |
| :---: | :--- |
| 1 | 1 or 9 |
| 4 | 2 or 8 |
| 5 | 5 |
| 6 | 4 or 6 |
| 9 | 3 or 7 |
| 0 | 0 |

$>$ Finding the digit in the tens place of the square root of a perfect square between 101 and 1000

Since $40 \times 40=1600$, the square root of a number between 101 and 1000 will be less than 40 . Therefore, the square root of such a number will only have digits in the units place and the tens place.

The digit in the tens place of the square root of a perfect square is as follows.

- If the digit in the hundreds place of a number is a perfect square, then the square root of that digit is the digit in the tens place of the answer (the square root of the perfect square).
- If the digit in the hundreds place of a number is not a perfect square, then the square root of the perfect square which is closest and less than the digit in the hundreds place, is the digit in the tens place of the answer.


## Example 1

Find $\sqrt{961}$.

- Since the digit in the units place is 1 , the digit in the units place of the square root must be 1 or 9 .
- The digit in the hundreds place of the given number is 9 , so the digit in the tens place of the square root is $\sqrt{9}$, which is 3 .
$\therefore \sqrt{961}$ is either 31 or 39 .

| 31 | 39 |
| ---: | ---: |
| $\times 31$ | $\times 39$ |
| 31 | 351 |
| 93 | $\underline{117}$ |
| $\underline{961}$ | $\underline{1521}$ |

Since $31^{2}=961$,
$\therefore \sqrt{961}=31$

## Example 2

Find the square root of 625 .


- Since the digit in the units place of 625 is 5 , the digit in the units place of its square root is 5 .
- Since the digit in the hundreds place of 625 is 6 , the digit in the tens place of the square root, is the square root of the perfect square closest to 6 and less than it.
- The perfect square closest to 6 and less than it is 4 . Its square root is 2 .
$\therefore \sqrt{625}$ is 25 .


## Example 3

Find $\sqrt{784}$.

## Method I



- Since the digit in the units place of 784 is 4 , the digit in the units place of its square root is 2 or 8 .
- Since the digit in the hundreds place of 784 is 7 , the digit in the tens place of the square root, is the square root of the perfect square closest to 7 and less than it.
- The perfect square closest to 7 and less than it is 4 . Its square root is 2 .
$\therefore \sqrt{784}$ is either 22 or 28 .

$$
\therefore \sqrt{784}=28
$$

| 22 | 28 |
| ---: | ---: |
| $\times 22$ | $\times 28$ |
| 44 | 224 |
| 44 |  |
| $\underline{484}$ | $\underline{56} 7$ |

## Method II

The squares of the multiples of 10 which are less than 1000 are 100, 400 and 900.784 lies between 400 and 900 .

When written in order, we get;
$400<784<900$.
$\therefore \sqrt{400}<\sqrt{784}<\sqrt{900}$ (the square roots of these numbers)
That is, $20<\sqrt{784}<30$
Therefore, $\sqrt{784}$ lies between 20 and 30 .

The digit in the units place of 784 is 4 . So the digit in the units place of the square root is either 2 or 8 . Therefore, $\sqrt{784}$ is either 22 or 28 .

784 is closer to 900 than to 400 .
As shown in the right $28^{2}=784$

$$
28
$$

$\therefore \sqrt{784}$ is 28 .

$$
\frac{\times 28}{224}
$$

Let us verify this.

56
$\underline{\underline{784}}$

## Example 4

Show that 836 is not a perfect square.


- If 836 is a perfect square, then the digit in the units place of the square root should be 4 or 6 .
- The digit in the hundreds place of 836 is 8 . Since the closest perfect square less than 8 is 4 , the digit in the tens place of the square root is $\sqrt{4}$, which is 2 .
Therefore, if 836 is a perfect square, then its square root must be 24 or 26 .
But $24 \times 24=576$ and $26 \times 26=676$. Therefore, 836 is not a perfect square.


## Exercise 8.4

(1) Complete the table given below.

| Perfect square | Square root of the <br> perfect square |
| :---: | :---: |
| 9 | $\sqrt{9}=\sqrt{3}^{2}=3$ |
| 36 |  |
| 64 |  |
| 121 |  |
| 400 |  |
| 900 |  |

(2) Check whether the given numbers are perfect squares, and find the square root of each number which is a perfect square.
(i) 169
(ii) 972
(iii) 441
(iv) 716
(v) 361
(vi) 484
(vii) 1522
(viii) 529
(ix) 372
(x) 624
(3) The value of $\sqrt{324}$ is a whole number between 15 and 20. Find $\sqrt{324}$ by observing the last digit.
(4) 625 is a perfect square. Its square root is a whole number between 20 and 30 . Find $\sqrt{625}$.
(5) Find the square root of each of the numbers given below by observation.
(i) 256
(ii) 441
(iii) 729
(iv) 361
(v) 841

## Miscellaneous Exercise

(1) In the given figure, $A B C D$ is a square of side length $8 \mathrm{~cm}, B T R S$ is a square of side length 6 cm and $A T P Q$ is a square of side length 10 cm .
(i) Find the area of the square $A B C D$.
(ii) Find the area of the square $B T R S$.
(iii) Find the area of the square $A T P Q$.
(iv) Find a special relationship between the areas of the three squares.
(2) The value of $\sqrt{500}$ cannot be found by using prime
 factors. Explain the reason for it.
(3) Show that $8^{2}-5^{2}=(8+5)(8-5)$ and show the same relationship for another pair of perfect squares.

## Summary

We obtain a perfect square when we multiply a positive integer by itself.
© If a number is a square of a positive integer, then the square root of that number is that positive integer of which is the square.
[1 The symbol " $\sqrt{ }$ " is used to denote the square root of a positive number.
$\square$ The square root of a perfect square can be found by observing the last digit of that number.
[1] The square root of a perfect square can also be found using prime factors.

