7

Factors

By studying this lesson you will be able to,

- find the highest common factor of the terms of a set which consists of up to three algebraic terms,
- write an algebraic expression as a product of two factors, where one factor is the highest common factor of the terms of the algebraic expression, and
- establish that an algebraic expression written in terms of its factors is the given algebraic expression, by multiplying the factors.

7.1 The highest common factor (HCF) of several numbers

$6 = 2 \times 3$

You have learnt previously that 2 and 3 are factors of 6.

When a number is written as a product of two whole numbers, those numbers are called **factors** of the original number.

The HCF of two or more numbers is the largest of all the common factors of the given numbers. That is, the largest number by which all the given numbers are divisible is their HCF.

Now let us find the HCF of 6 and 10.

$6 = 1 \times 6$ $\qquad$ $10 = 1 \times 10$
$6 = 2 \times 3$ $\qquad$ $10 = 2 \times 5$

$\therefore$ the factors of 6 are 1, 2, 3 and 6.
The factors of 10 are 1, 2, 5 and 10.

1 and 2 are the common factors of 6 and 10. Since 2 is the larger factor, the HCF of 6 and 10 is 2.

You learnt in Grade 7 how to find the HCF of several numbers by writing each as a product of prime numbers. Let us recall what you learnt through an example.

Let us find the HCF of 6, 12 and 18.
Let us write each number as a product of prime factors.

2 | 6
3 | 3
1

2 | 12
2 | 6
3 | 3
1

2 | 18
3 | 9
3 | 3
1

$6 = 2 \times 3$
$12 = 2 \times 2 \times 3$
$18 = 2 \times 3 \times 3$

We obtain the HCF of 6, 12 and 18 by taking the product of the prime factors which are common to these three numbers.

The HCF of 6, 12 and 18 $= 2 \times 3 = 6$

Note

To find the prime factors of a whole number,

- it is sequentially divided by the prime numbers by which it is divisible, starting from the smallest such prime number, till the answer 1 is obtained.

Review Exercise

Find the HCF of each set of numbers given below.

(i) 12, 18 (ii) 30, 24 (iii) 45, 60
(iv) 6, 12, 18 (v) 15, 30, 75 (vi) 36, 24, 60
(vii) 6, 9, 12 (viii) 15, 30, 45 (ix) 11, 13, 5

7.2 The highest common factor of several algebraic terms

Now let us see what is meant by the HCF of several algebraic terms and how to find it.

Let us find the HCF of the algebraic terms $4x$, $8xy$ and $6xyz$.

Let us write each term as a product of its factors.

$4x = 2 \times 2 \times x$
$8xy = 2 \times 2 \times 2 \times x \times y$
$6xyz = 2 \times 3 \times x \times y \times z$

Here, the coefficient of each algebraic term is written as a product of its prime factors and the unknowns are separated and written as a product.

The common factors of all three algebraic terms, $4x$, $8xy$ and $6xyz$ are 2 and x.

The HCF of the algebraic terms, $4x$, $8xy$ and $6xyz$ is the product of the factors which are common to all three terms.

$\therefore$ The HCF of $4x$, $8xy$ and $6xyz = 2 \times x$
$= 2x$

Example 1

Find the HCF of the algebraic terms in each part given below.

(i) $2pq$, $4pqr$ (ii) $7mn$, $14mnp$, $28mnq$

(i) $2pq = 2 \times p \times q$
$4pqr = 2 \times 2 \times p \times q \times r$

The HCF of $2pq$ and $4pqr$ $= 2 \times p \times q$
$= 2pq$

(ii) $7mn = 7 \times m \times n$
$14mnp = 2 \times 7 \times m \times n \times p$
$28mnq = 2 \times 2 \times 7 \times m \times n \times q$

The HCF of $7mn$, $14mnp$ and $28mnq$ $= 7 \times m \times n$
$= 7mn$

Exercise 7.1

Find the HCF of the algebraic terms in each part given below.

(i) xy, $3xy$, $4x$ (ii) $4c$, $8a$, $4b$
(iii) $2x$, $8x$, $4xy$ (iv) $4p$, $8pq$, $12pq$
(v) $8pqr$, $16qr$, $7mqr$ (vi) $4x$, $6xy$, $8qrx$
(vii) $4x$, $6abx$, $10abxy$ (viii) $6mn$, $12mny$, $15my$

7.3 Writing an algebraic expression as a product of its factors

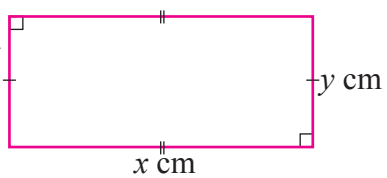
Since 2 and 3 are the prime factors of 6, 6 can be written as a product of its prime factors as $6 = 2 \times 3$.

Now let us consider how to write an algebraic expression as a product of its factors.

Let us find the perimeter of the rectangle in the figure.

Method I

Let us add the lengths of all four sides of the rectangle.
Perimeter of the rectangle $= x + y + x + y$
$= 2x + 2y$



Method II

Let us find the perimeter by multiplying the sum of the length and breadth of the rectangle by two.

Perimeter of the rectangle $= (x + y) \times 2$
$= 2(x + y)$

Since the perimeter of the same rectangle is found by both methods, the two expressions obtained for the perimeter are equal.

$$\therefore\ 2x + 2y = 2(x + y)$$

Writing the algebraic expression $2x + 2y$ as $2(x + y)$, is called **writing the algebraic expression $2x + 2y$ as a product of factors**.

That is, 2 and $(x + y)$ are two factors of the expression $2x + 2y$.

➢ Now let us write the algebraic expression $12x + 18y$ as a product of two factors.

$12x + 18y$ can be expressed as a product of two factors in several ways.

(i) $12x + 18y = 2 \times 6x + 2 \times 9y$
$= 2(6x + 9y)$
In this instance, 2 is taken as a common factor of the two terms.

(ii) $12x + 18y = 3 \times 4x + 3 \times 6y$
$= 3(4x + 6y)$
In this instance, 3 is taken as a common factor of the two terms.

(iii) $12x + 18y = 6 \times 2x + 6 \times 3y$
$= 6(2x + 3y)$
In this instance, 6 is taken as a common factor of the two terms.

Since there is no common factor in $2x$ and $3y$, which are the terms of the expression within brackets, 6 is the HCF of the terms $12x$ and $18y$.

When writing such an algebraic expression as a product of factors, the convention is to write the first factor as a number which is the HCF of the terms of the given expression, and the other factor as an algebraic expression, where the HCF of its terms is 1.

Accordingly, when writing an algebraic expression as a product of factors,

- first find the highest common factor of the terms of the algebraic expression,
- take this HCF as one factor and the expression which is obtained by dividing each term of the algebraic expression by this HCF as the other factor, and
- write the algebraic expression as a product of these two factors.

Example 1

Write the expression $36a + 60b$ as a product of factors.

$36a = 2 \times 2 \times 3 \times 3 \times a$

$60b = 2 \times 2 \times 3 \times 5 \times b$

The HCF of the terms $36a$ and $60b = 2 \times 2 \times 3$

$= 12$

$\therefore \quad 36a + 60b = 12 \times 3a + 12 \times 5b$

$= 12\ (3a + 5b)$

$36a \div 12 = 3a$

$60b \div 12 = 5b$

Example 2

Write the expression $12x + 20y + 16z$ as a product of factors.

$12x = 2 \times 2 \times 3 \times x$

$20y = 2 \times 2 \times 5 \times y$

$16z = 2 \times 2 \times 2 \times 2 \times z$

The HCF of $12x$, $20y$ and $16z = 2 \times 2$

$= 4$

$\therefore \quad 12x + 20y + 16z = 4 \times 3x + 4 \times 5y + 4 \times 4z$

$= 4\ (3x + 5y + 4z)$

$12x \div 4 = 3x$

$20y \div 4 = 5y$

$16z \div 4 = 4z$

Exercise 7.2

(1) Fill in the blanks.

(i) $3x + 12 = 3 \times \square + 3 \times \square = 3\ (\square + \square)$

(ii) $15x + 20y = 5 \times \square + 5 \times \square = 5\ (\square + \square)$

(iii) $12a + \square = 6 \times \square + 6 \times \square = 6\ (\square + 3)$

(iv) $12x + 8y + 20z = 4 \times \square + 4 \times \square + 4 \times \square = 4\ (\square + \square + \square)$

(v) $30x + 24y + 18 = \square\ (5x + \square + \square)$

(2) Write each of the algebraic expressions given below as a product of two factors such that one factor is the HCF of the terms of the expression.

(a) (i) $2x + 6y$ (ii) $8x + 12y$ (iii) $15a + 18b$

(iv) $9x + 27y$ (v) $4p + 20q$ (vi) $12p + 30q$

(vii) $20a - 30b$ (viii) $36a - 54b$ (ix) $60p - 90q$

(b) (i) $5x - 10y + 25$ (ii) $3a + 15b - 12$ (iii) $18 - 12m + 6n$

(iv) $10a - 20b - 15$ (v) $9c - 18a + 9$ (vi) $12d + 6 + 18c$

(vii) $3x + 6y - 3$ (viii) $10m + 4n - 2$ (ix) $12a - 8b + 4$

(x) $9 + 3b + 6c$ (xi) $3a^2 - 6ab + 9b^2$ (xii) $4a^2 - 16ab - 12c$

7.4 Writing an algebraic expression as a product of factors where one factor is a negative number

Since $(-12) = (-6) \times 2$, we have that (-6) is a factor of (-12).

Since $(-12) = 6 \times (-2)$, (-2) is also a factor of (-12).

Since $12 = (-6) \times (-2)$, both (-6) and (-2) are factors of 12.

Example 1

(i) Write (-15) as a product of two factors, such that (-3) is a factor.

$(-15) = (-3) \times 5$

(ii) Write 10 as a product of two factors such that (-2) is a factor.

$10 = (-2) \times (-5)$

Accordingly, (-2) and (-5) are two factors of 10.

Now let us consider an instance where one factor of the algebraic expression is a negative number.

Let us consider the algebraic expression $-2x + 6y$. Here, 2 is a common factor of $-2x$ and $6y$.

Therefore, $-2x + 6y = 2(-x + 3y)$

Since $-2x = (-2) \times x$ and $6y = (-2) \times (-3) \times y$,

(-2) is also a common factor of $-2x$ and $6y$.

$\therefore -2x + 6y = (-2) \times x + (-2) \times (-3)\, y$

$= (-2)(x + (-3)\, y)$

$= -2\,(x - 3y)$

$\therefore$ the algebraic expression $-2x + 6y$ can also be written as a product of two factors as $-2\,(x - 3y)$.

Example 2

Write down each of the algebraic expressions given below as a product of two factors such that one factor is a negative number.

(i) $-4x - 16y$ (ii) $-8m + 24n - 16$

(i) $-4x - 16y = (-4) \times x + (-16)y$

$= (-4) \times x + (-4) \times (+4)\, y$

$= (-4)(x + (+4)\, y)$

$= -4(x + 4y)$

(ii) $-8m + 24n - 16 = -8 \times 1m + (-8) \times (-3)\, n + (-8) \times (+2)$

$= -8(m - 3n + 2)$

Note

When one factor is a negative number, the sign of each term of the other factor is opposite to that of the corresponding term in the original algebraic expression.

Exercise 7.3

(1) (i) Write (–20) as a product of two factors such that (–4) is one of the factors.

(ii) Write 12 as a product of two factors such that (–4) is one of the factors.

(2) Write each algebraic expression given below as a product of two factors such that one factor is a negative number.

(i) $12x - 4y$ (ii) $-12x + 4y$ (iii) $-12x - 4y$

(iv) $-3a + 15b - 6c$ (v) $-12a + 18b - 24c$ (vi) $-8p + 40q - 24$

7.5 More on writing an algebraic expression as a product of two factors

Let us consider the algebraic expression $pq + pr$.

$pq = p \times q$

$pr = p \times r$

Since p is a factor of each term of this expression, p is a common factor of the two terms.

$\therefore\ pq + pr = p \times q + p \times r$

$= p(q + r)$

Accordingly, when writing an algebraic expression as a product of factors,

- first find the HCF of the terms of the algebraic expression,
- take the HCF as one factor and the expression which is obtained by dividing each term of the algebraic expression by the HCF as the other factor, and
- write the algebraic expression as a product of these two factors.

Example 1

Write the expression $18x + 24xy + 12xz$ as a product of two factors.

The HCF of the terms $18x$, $24xy$ and $12xz$ is $6x$

$\therefore\ 18x + 24xy + 12xz = 6x \times 3 + 6x \times 4y + 6x \times 2z$

$= 6x\,(3 + 4y + 2z)$

Note

- Let us simplify $6 \div 9$.

You have learnt that $6 \div 9 = \frac{6 \div 3}{9 \div 3} = \frac{2}{3}$.

Moreover, this can also be simplified as $\frac{6}{9} = \frac{{}^{1}\cancel{3} \times 2}{{}_{1}\cancel{3} \times 3} = \frac{2}{3}$.

- Let us simplify $3xy \div 5y$.

$3xy \div 5y = \frac{3xy}{5y} = \frac{3 \times x \times y}{5 \times y}$

Since y represents a number, it can be simplified as above.

$\frac{3 \times x \times \cancel{y}^{1}}{5 \times \cancel{y}_{1}} = \frac{3 \times x}{5} = \frac{3x}{5}$

Example 2

Write the expression $15pq + 45qr + 60q$ as a product of factors.

$15pq = 3 \times 5 \times p \times q$

$45qr = 3 \times 3 \times 5 \times q \times r$

$60q = 2 \times 2 \times 3 \times 5 \times q$

The HCF of $15pq$, $45qr$ and $60q$ $= 3 \times 5 \times q$

$= 15q$

$\therefore 15pq + 45qr + 60q = 15q\ (p + 3r + 4)$

$15pq \div 15q = p$
$45qr \div 15q = 3r$
$60q \div 15q = 4$

Example 3

Write the expression $3a + 6ab + 12ac$ as a product of factors.

Here $3a = 3 \times a$

$6ab = 3 \times 2 \times a \times b$

$12ac = 2 \times 2 \times 3 \times a \times c$

HCF of $3a$, $6ab$ and $12ac = 3 \times a$

When the HCF $3a$ is separated out as a common factor and written we obtain,

$3a + 6ab + 12ac = 3a\ (1 + 2b + 4c)$.

Note that when the expression within brackets is multiplied by $3a$, the original expression, $3a + 6ab + 12ac$ is obtained.

$3a\ (1 + 2b + 4c) = 3a + 6ab + 12ac$

$\therefore$ $3a + 6ab + 12ac$ is the product of the two factors $3a$ and $(1 + 2b + 4c)$.

Exercise 7.4

(1) Write each algebraic expression given below as a product of two factors.

(i) $ab + ac$ (ii) $p + pq$ (iii) $xyz + xpq$

(iv) $3x + 6xy$ (v) $15pq - 20pr$ (vi) $4p - 16pq + 12pr$

(vii) $2a - 8ab - 8ac$ (viii) $5x - 10xy - 5xz$ (ix) $3ab - 9abc$

(2) Write each of the following algebraic expressions as a product of two factors Establish the accuracy of your answer by simplifying the product.

(i) $xyz - 2xyp$ (ii) $12x - 20xy$ (iii) $ab + ac - ad$

(iv) $p + pq + pqr$ (v) $xp - xy - x$ (vi) $6ab - 8ab^2 + 12ac$

(3) Join each algebraic expression in group *A* with the algebraic expression in group *B* which it is equal to.

A	*B*
(i) $2\ (x + 2y + 5)$	$10a - 2ac + 4ab$
(ii) $4\ (2a + b + 3c)$	$15xyz - 25xy + 20xz$
(iii) $5\ (2a - 1 + 3b)$	$4p^2r + 2qr + 2pqr$
(iv) $4\ (3x - 2y + 5z)$	$12x - 8y + 20z$
(v) $4p\ (a + b + 1)$	$2x + 4y + 10$
(vi) $2a\ (5 - c + 2b)$	$12x - 6xy + 9xz$
(vii) $x\ (2 - 3y + 3y^2)$	$8a + 4ab - 4ac$
(viii) $4a\ (2 + b - c)$	$4ap + 4bp + 4p$
(ix) $5x\ (3yz - 5y + 4z)$	$10a - 5 + 15b$
(x) $3x\ (4 - 2y + 3z)$	$8a + 4b + 12c$
(xi) $2r\ (2p^2 + q + pq)$	$2x - 3xy + 3xy^2$

(4) Complete the table given below.

Original expression	After factoring the expression
.....	$4\ (3a + 2b + 3a^2)$
$9a + 27ac^2 + 18ab$	.....
.....	$3a\ (2p + 3r + 6)$
.....	$2a\ (a + 3b + 2ac)$
$8xy + 24xp + 40xq$	.....
.....	$2\ (3ab + 4bc - 5ac)$
.....	$3x\ (2pq + 3x + 6p)$
.....	$6\ (2xy^2 + 3xy + 4z)$
$3ab - 6ab + 12ac$	.....
$8xy - 12px - 20axy$	.....

(5) Fill in the blanks in the table.

Algebraic expression	One factor of the algebraic expression	As a product of two factors
$-4x + 12$	4	.....
$-4x + 12$	-4	.....
$-6x + 8y$	2	.....
$-6x + 8xy$	$-2x$	.....
$-2a + 4b - 6c$	2	.....
$-2a + 4b - 6c$	-2	.....
$-3ab - 9b$	$-3b$	.....
$2xy - 8xyz$	$2xy$	.....
$5xy + 10xy + 10py$	.....	.....

Summary

- When writing an algebraic expression as a product of factors,
- first find the HCF of the terms of the algebraic expression,
- take the HCF as one factor and the expression which is obtained by dividing each term of the algebraic expression by the HCF as the other factor, and
- write the algebraic expression as a product of these two factors.