



$5(x - y)$

$\sqrt{64}$



$\frac{7}{10}$

$(-1)^n$



# Solids

By studying this lesson you will be able to,

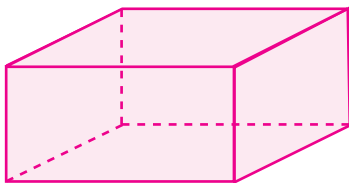
- prepare models of regular octahedrons, regular dodecahedrons and regular icosahedrons,
- verify Euler's relationship for the above solids by considering the number of edges, vertices and faces of these solids, and
- from given, solids identify the platonic solids and describe their characteristics.

## 6.1 Solids

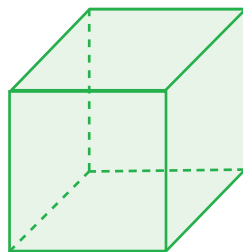
You have learnt that an object which has a specific shape and which occupies a certain amount of space is called a solid object.

You have also learnt that the surfaces of solids objects are plane surfaces or curved surfaces.

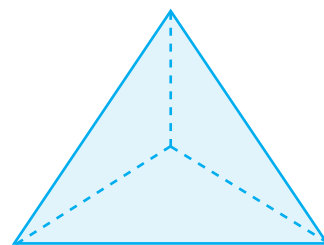
Some solids you have studied in Grades 6 and 7 are illustrated below.



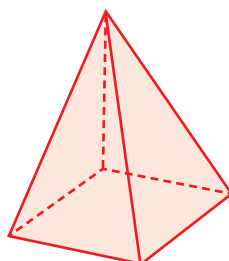
A cuboid



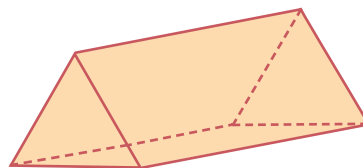
A cube



A regular tetrahedron



A square pyramid



A triangular prism

Do the review exercise to recall what you have learnt about solids in Grades 6 and 7.



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$$(-1)^7$$



## Review Exercise

- (1) Fill in the blanks in the table given below.

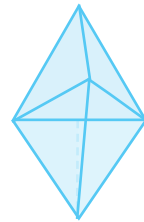
Solid	Number of edges	Number of faces	Number of vertices
Cuboid	12	6	8
Cube			
Regular tetrahedron			
Square pyramid			
Triangular prism			

- (2) Draw nets that can be used to construct the following solids.

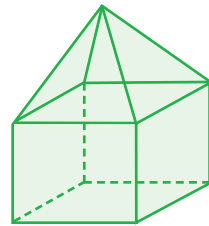
(i) Square pyramid

(ii) Triangular prism

- (3) A figure of a solid constructed by pasting together two triangular faces of two identical regular tetrahedrons, one on the other, is given here. Find the number of edges, faces and vertices of this solid.



- (4) A solid constructed by joining a cube and a square pyramid is shown in the figure. Find the number of edges, faces and vertices of this solid.



## 6.2 Octahedron

Diamonds and certain other gems used in jewellery are cut in the shape of an octahedron.

A solid which has 8 faces is called an **octahedron**.





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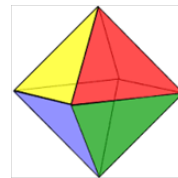


$\frac{7}{10}$

$(-1)^n$



A solid object which has eight identical equilateral triangular shaped faces is called a **regular octahedron**. The figure shows a regular octahedron.

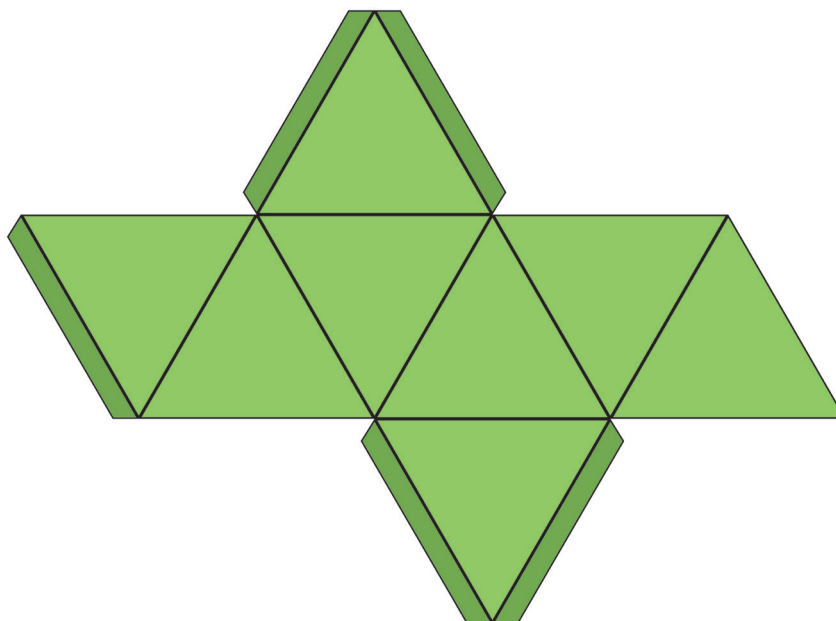


Let us identify the characteristics of a regular octahedron by engaging in the following activity.



### Activity 1

**Step 1 -** Copy the given figure on a thick piece of paper such as a Bristol board, or get a photo copy of the figure and paste it on a thick piece of paper.



**Step 2 -** Cut out the figure drawn or pasted on the Bristol board and prepare a model of a regular octahedron by folding along the edges and pasting along the pasting allowances.

**Step 3 -** By considering the model you prepared, find the number of faces, edges and vertices of a regular octahedron. Examine and identify the special features of the model.



$$5(x-y)$$

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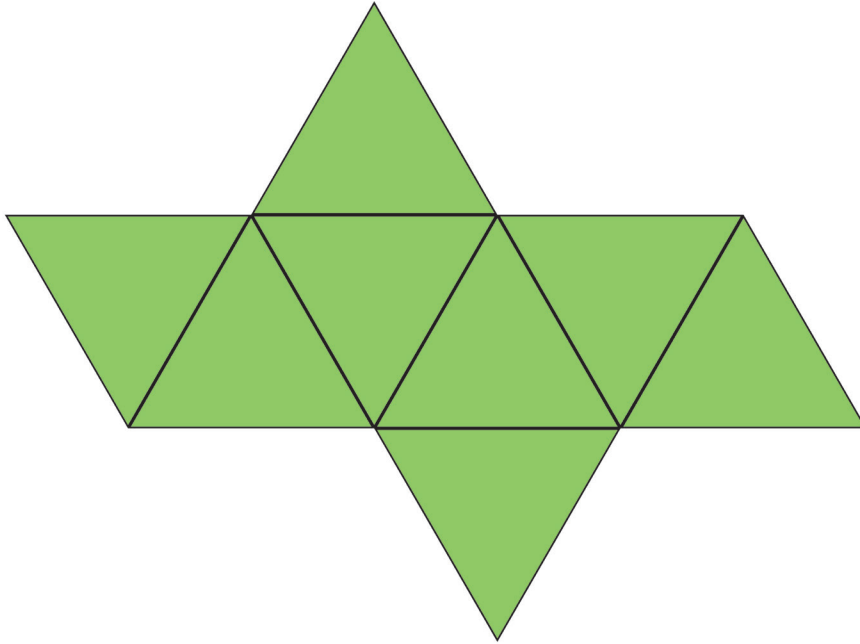
$$\frac{7}{10}$$

$$(-1)^1$$



**Step 4** - Write the special features you identified in your exercise book.

The figure you obtain by removing the pasting allowances of the above figure, which was used to prepare a model of a regular octahedron, is called a **net of the regular octahedron**.



The object you constructed during the above activity is a model of a regular octahedron.

#### Features you can identify in a regular octahedron

- There are 8 faces in a regular octahedron.
- All faces are the shape of identical equilateral triangles.
- There are 6 vertices in a regular octahedron.
- There are 12 edges in a regular octahedron. All are straight edges.



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$1\frac{7}{10}$

$(-1)^1$

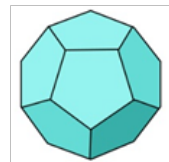


## 6.3 Dodecahedron

Models of this shape are used for decorations and ornaments.



A solid object which has 12 regular pentagonal faces is called a **regular dodecahedron**. The figure shows a regular dodecahedron.

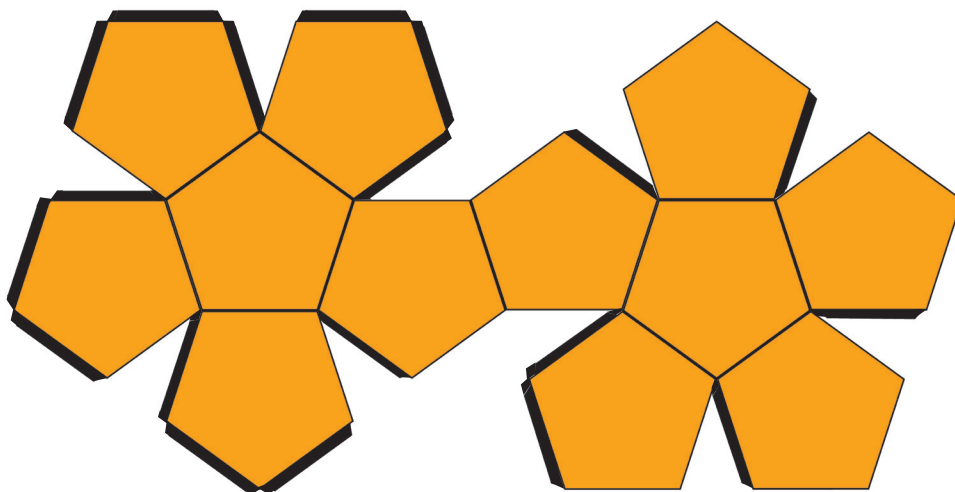


Let us identify the characteristics of a regular dodecahedron by engaging in the following activity.



### Activity 2

**Step 1** - Copy the given figure on a thick piece of paper such as a Bristol board or get a photo copy of the figure and paste it on a thick piece of paper.



**Step 2** - Cut out the figure drawn or pasted on the Bristol board and prepare a model of a regular dodecahedron by folding along the edges and pasting along the pasting allowances.



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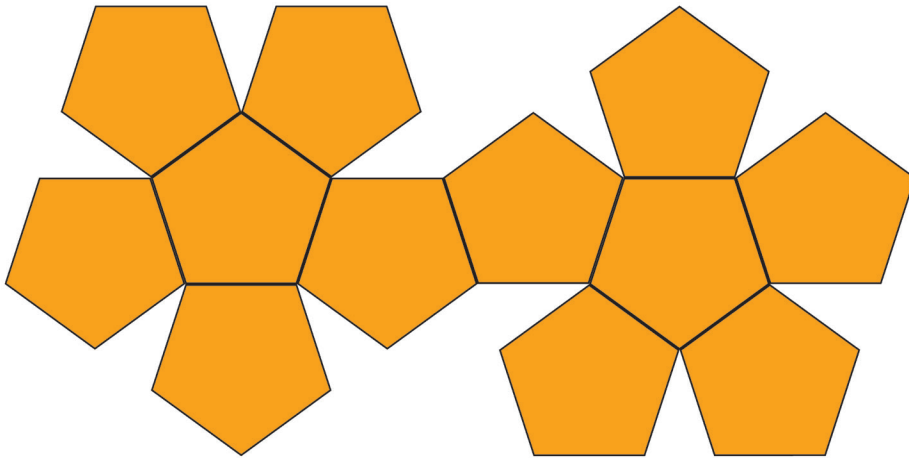
$$(-1)^7$$



**Step 3** - By considering the model you prepared, find the number of faces, edges and vertices of a regular dodecahedron. Examine and identify the special features of the model.

**Step 4** - Write the special features you identified in your exercise book.

The figure you obtain by removing the pasting allowances of the above figure which was used to prepare a model of a regular dodecahedron, is called a **net of the regular dodecahedron**.



The object you constructed during the above activity is a model of a regular dodecahedron.

### Features you can identify in a regular dodecahedron

- There are 12 faces in a regular dodecahedron.
- All the faces of a regular dodecahedron take the shape of identical regular pentagons.
- There are 20 vertices in a regular dodecahedron.
- There are 30 edges in a regular dodecahedron. All are straight edges.



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$(-1)^n$



## 6.4 Icosahedron

A model which can be used in decorations such as Vesak lanterns is given here. It is known as an icosahedron.



A solid which has twenty equilateral triangular faces is called a **regular icosahedron**. The figure shows a regular icosahedron.

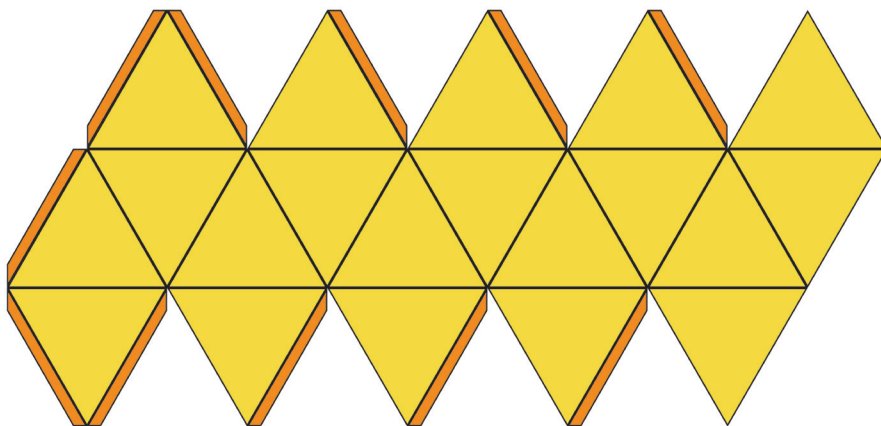


Let us identify the characteristics of a regular icosahedron by engaging in activity 3.



### Activity 3

**Step 1** - Copy the given figure on a thick piece of paper such as a Bristol board, or get a photo copy of the figure and paste it on a thick piece of paper.



**Step 2** - Cut out the figure drawn or pasted on the Bristol board and prepare a model of a regular icosahedron by folding along the edges and pasting along the pasting allowances.

**Step 3** - By considering the model you prepared, find the number of faces, edges and vertices of a regular icosahedron. Examine and identify the special features of the model.

**Step 4** - Write the special features you identified in your exercise book.

The figure you obtain by removing the pasting allowances of the above figure which was used to prepare a model of a regular icosahedron is called a **net of the regular icosahedron**.



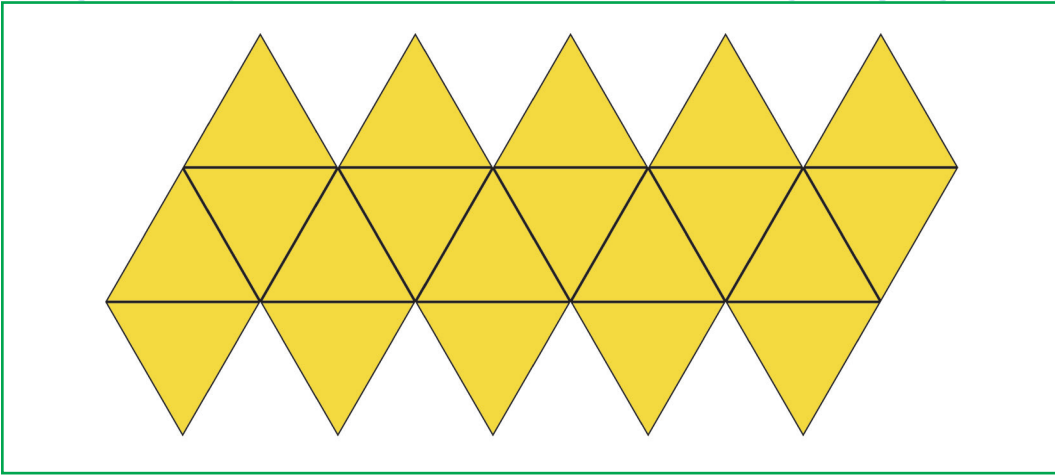
$$5(x-y)$$

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$$(-1)^7$$



The object you constructed during the above activity is a model of a regular icosahedron.

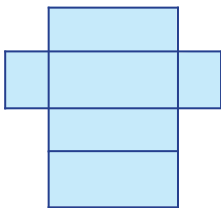
### Features you can identify in a regular icosahedron

- There are 20 faces in a regular icosahedron.
- The faces of a regular icosahedron take the shape of identical equilateral triangles.
- There are 12 vertices in a regular icosahedron.
- There are 30 edges in a regular icosahedron. All are straight edges.

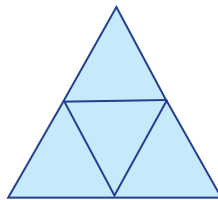
### Exercise 6.1

(1) Name the solid which can be constructed using each net given below.

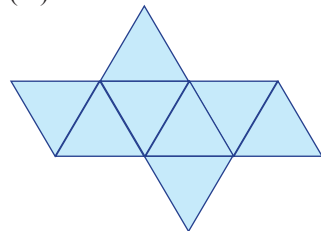
(i)



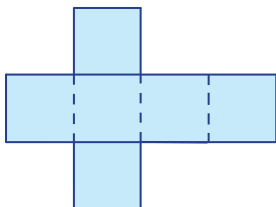
(ii)



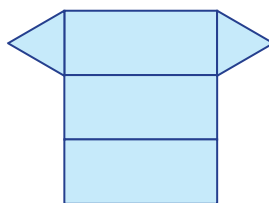
(iii)



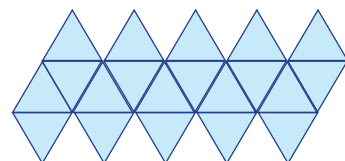
(iv)



(v)



(vi)







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$(-1)^1$



### 6.5 Verification of Euler's relationship for solids

You learnt in Grade 7 about the relationship between the edges, vertices and faces of a solid, which was first presented by the Swiss mathematician Euler. Let us recall what you learnt.

#### Euler's relationship

In a solid with straight edges, the sum of the number of faces and the number of vertices is two more than the number of edges.

This relationship can be expressed as follows.

$$\begin{array}{rcl} \text{Number of Vertices} & + & \text{Number of Faces} & = & \text{Number of Edges} + 2 \\ V & + & F & = & E + 2 \end{array}$$



#### Activity 4

Fill in the blanks in the table given below by observing the solids you constructed in activities 1, 2 and 3.

Solid	Number of vertices ( $V$ )	Number of faces ( $F$ )	Number of edges ( $E$ )	$V + F - E$	Is Euler's relationship satisfied?
Regular Octahedron					
Regular Dodecahedron					
Regular Icosahedron					

#### Exercise 6.2

- Verify Euler's relationship for a regular tetrahedron by considering the number of faces, vertices and edges it has.
- For a square pyramid,
  - write down the number of faces, vertices and edges.
  - show that the above values satisfy Euler's relationship.
- If a certain solid has 9 edges and 6 vertices, and if Euler's relationship is satisfied, find the number of faces it has.



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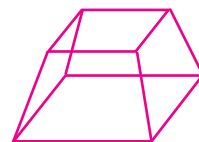
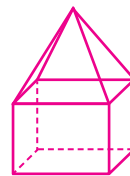


$$\frac{7}{10}$$

$$(-1)^7$$



- (4) A figure of a composite solid is shown here. Determine with reasons whether Euler's relationship is satisfied for this solid.
- (5) A certain solid has 10 edges and 6 faces. Find the number of vertices it has, if Euler's relationship is satisfied.
- (6) The figure given here shows a pyramid of which the upper portion has been cut out and removed. Verify Euler's relationship for this solid.

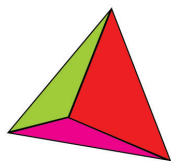


## 6.6 Platonic solids

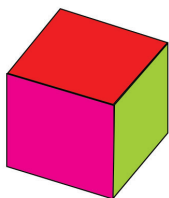
**Platonic solids** are solids having identical regular polygonal faces and with the same number of faces meeting at every vertex.

You have learnt about the five types of solids which are considered as platonic solids. They are the regular tetrahedron, cube, regular octahedron, regular dodecahedron and the regular icosahedron.

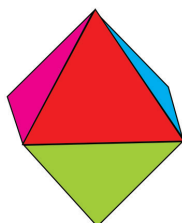
They are called **platonic solids**.



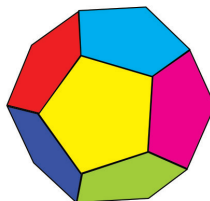
Regular  
Tetrahedron



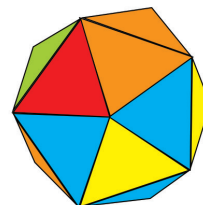
Cube



Regular  
Octahedron



Regular  
Dodecahedron



Regular  
Icosahedron



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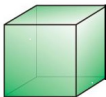
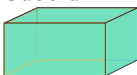


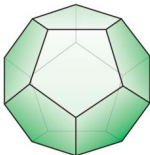


$\frac{7}{10}$

$(-1)^n$

**Exercise 6.3**

(1) Complete the table given below.

Solid	Shapes of the faces of the solid	Are all the faces regular?	Are the number of faces meeting at each vertex equal?	Number of faces meeting at a vertex	Is the solid a platonic solid?
Cube 	Square	Regular	Equal	3	Yes
Cuboid 					
Regular tetrahedron 					
Regular octahedron 					
Regular dodecahedron 					



$$5(x-y)$$


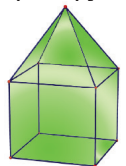
$$\sqrt{64}$$



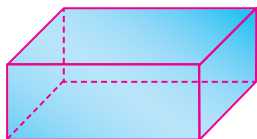
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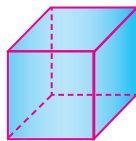


Solid	Shapes of the faces of the solid	Are all the faces regular?	Are the number of faces meeting at each vertex equal?	Number of faces meeting at a vertex	Is the solid a platonic solid?
Regular icosahedron 					
Composite solid consisting of a cuboid and a square pyramid 					

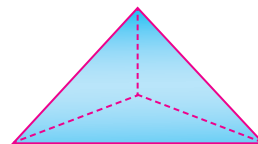
- (2) Construct a regular icosahedron and 20 regular tetrahedrons such that the icosahedron and the tetrahedrons have edges of equal length. Construct a composite solid by pasting a tetrahedron on each face of the icosahedron. For the composite figure, find
- the number of edges.
  - the number of faces.
  - the number of vertices.
- (3) From the following, select the platonic solids and write down the corresponding numbers.



(i)



(ii)



(iii)



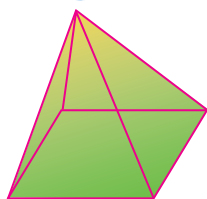
$5(x - y)$

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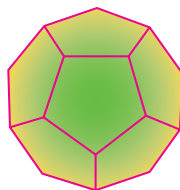


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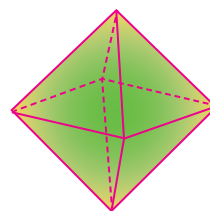
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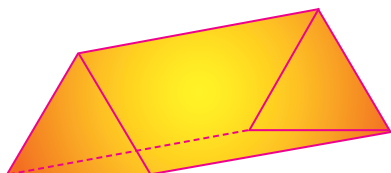
(iv)



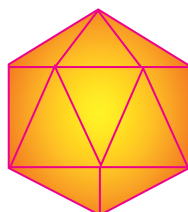
(v)



(vi)






(vii)



(viii)

## Summary

-  The sum of the number of faces and the number of vertices of a solid with straight edges is 2 more than the number of edges.
-  Solids having identical regular polygonal faces and with the same number of faces meeting at every vertex are called platonic solids.
-  The five types, regular tetrahedrons, cubes, regular octahedrons, regular dodecahedrons and regular icosahedrons are the only solids that are platonic solids.

Solid	Shape of a face	Number of faces	Number of edges	Number of vertices
Cube	Square	6	12	8
Cuboid	Rectangle	6	12	8
Regular tetrahedron	Equilateral triangle	4	6	4
Square pyramid	One face is square shaped. The other 4 faces take the shape of identical triangles	5	8	5
Triangular prism	3 rectangular faces and 2 triangular faces	5	9	6
Regular octahedron	Equilateral triangle	8	12	6
Regular dodecahedron	Regular pentagon	12	30	20
Regular icosahedron	Equilateral triangle	20	30	12