4 Directed Numbers

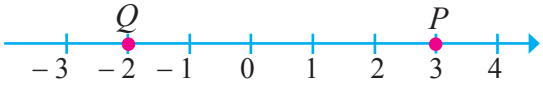
By studying this lesson you will be able to,

- subtract a directed number from another directed number, and
- multiply directed numbers and divide a directed number by a directed number.

4.1 Directed numbers

Let us recall what you learnt in Grade 7 about directed numbers.

Consider the following number line on which the points P and Q are marked.



- On this number line, the point P represents the directed number $(+3)$ and the point Q represents the directed number (-2).
- $(+3)$ is most often written as 3.
- (-2) and $(+3)$, are located on the number line in opposite directions to each other from zero.
- + (positive) sign is used to denote the direction in which the directed number $(+3)$ is located with respect to zero on the number line.
- – (negative) sign is used to denote the opposite direction, in which the directed number (-2) is located.

The **magnitude** of a number represented by a point on the number line is the distance on the number line from zero to that point.

Furthermore, a number gets its sign as + or – according to the position of that number, whether it is to the right or to the left of the point which represents 0.

- Since the distance from zero to point P is 3 units, the magnitude of the directed number $(+3)$ is 3. The magnitude of the directed number (-2) is 2.

In a directed number, the numerical value shows its magnitude and the + or – sign its direction.

$(+3)$, (-7), $(+2.5)$, (-3.4), $\left(+3\frac{1}{2}\right)$, $\left(-5\frac{1}{4}\right)$ are some examples of directed numbers.

Note

- It is important to note that, while using the symbol + or – to denote the direction of a number, the symbol + is also used to denote the addition of two directed numbers and the symbol – is used to denote subtraction of a directed number from another directed number.
- We have to understand that the symbols + and – are used in two different senses here.
- To differentiate this clearly, we write directed numbers within brackets.

Adding directed numbers

Since the sign of a directed number is important, we should pay special attention to the sign when performing mathematical operations.

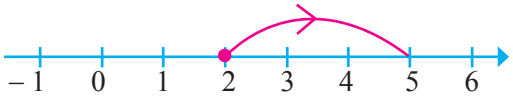
You learnt in Grade 7 how the addition of directed numbers can be explained easily by using the number line.

The addition of directed numbers can be explained easily by using the number line in the following manner too.

➤ Let us find the value of (+2) + (+3) by using the number line.

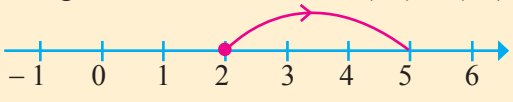
- Mark the directed number (+2) on the number line.

- From this point, move 3 units, which is the magnitude of (+3), to the right, which is the direction of (+3) along the number line.



- The directed number (+5) represented by the ending point is the sum of the above two directed numbers.

That is, the directed number which is obtained when you move 3 units to the right along the number line from (+2) is (+5).



∴ (+2) + (+3) = (+5)

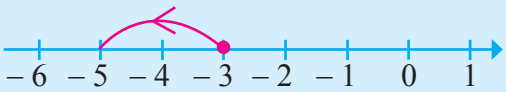
The steps we followed are given below in order.

When adding a directed number to another directed number do the following.

- Mark the point which represents the first directed number on the number line.
- From that point, move a distance **equal to the magnitude of the second directed number** towards the **direction of the second directed number**.
- The directed number which is represented by the ending point is the answer.

Example 1

Find the value of $(-3) + (-2)$ by using the number line.



From (-3), when you move two units to the left, which is the direction of (-2), the directed number you end at is (-5).

$\therefore\ (-3) + (-2) = (-5)$

• Adding directed numbers without using the number line

What you learnt in Grade 7 about the addition of directed numbers without using the number line is presented here.

Let us find the value without using the number line.

When adding two directed numbers of the same sign, first add the two numbers without considering their signs. Include the same sign in the answer.

(i) $(+3) + (+2) = (+5)$

(ii) $(-4) + (-6) = (-10)$

When adding two directed numbers of different signs (positive and negative), first find the difference of their numerical values, without considering their signs. Include the sign of the directed number having the larger magnitude in the answer.

(iii) Let us find the value of $(+8) + (-3)$.

$8 - 3 = 5$

$\therefore\ (+8) + (-3) = (+5)$

(iv) Let us find the value of $(+4.2) + (-6.3)$.

$6.3 - 4.2 = 2.1$

$\therefore\ (+4.2) + (-6.3) = (-2.1)$

Do the following review exercise to recall what you learnt in Grade 7 about directed numbers.

Review Exercise

(1) Find the value of each of the following by using the number line.

(i) (+2) + (+6) (ii) (+8) + (−5) (iii) (−2) + (+3)
(iv) (−3) + (−4) (v) (+4) + (−6)

(2) Find the value of each of the following.

(i) (+2) + (+3) (ii) (−4) + (−2) (iii) (−3) + (+5)
(iv) (+4) + (−10) (v) (−7) + (+7) (vi) (+2) + (+5) + (+3)
(vii) (−3) + (−1) + (−4) (viii) (+2) + (+4) + (−9) (ix) $\left(+\frac{5}{7}\right) + \left(-\frac{2}{7}\right)$
(x) (+3.4) + (−5.2) (xi) (−8.11) + (+8.11)

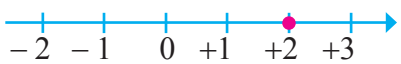
4.2 Subtracting a directed number from another directed number

Now let us consider how to subtract a directed number from another directed number by using the number line. Let us first find out what is meant by the direction opposite to that of a given directed number.

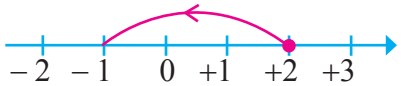
- The magnitude of (+3) is 3 and its direction is toward the right.
  The **direction opposite** to that of (+3) is towards the left.
- The magnitude of (−3) is 3 and its direction is towards the left.
  The **direction opposite** to that of (−3) is towards the right.

➤ Let us find the value of (+2) – (+3) by using the number line.

- First mark the directed number (+2) on the number line.



- From this point, move 3 units which is the magnitude of (+3), towards the left, which is the direction opposite to that of (+3).



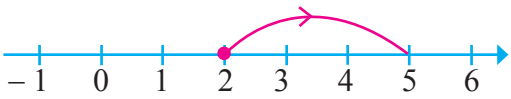
- The answer is the directed number represented by the ending point.

The answer is obtained from the point which is 3 units to the left of (+2).
∴ (+2) − (+3) = (−1)

When subtracting a directed number from a directed number, do the following.

- Mark the point which represents the first directed number on the number line.
- From this point, move a distance equal to the magnitude of the second directed number in the direction opposite to that of the second directed number.
- The directed number which is represented by the ending point is the answer.

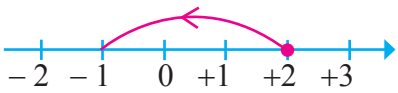
Finding the value of (+2) + (+3).



The directed number which is represented by the ending point, when you move 3 units along the number line **in the direction of (+3)** from (+2) is obtained as the answer.

∴ (+2) + (+3) = (+5)

Finding the value of (+2) – (+3).

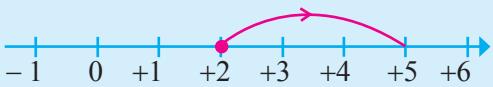


The directed number which is represented by the ending point, when you move 3 units along the number line **in the direction opposite to that of (+3)** from (+2) is obtained as the answer.

∴ (+2) – (+3) = (–1)

Example 1

Find the value of (+2) – (–3) by using the number line.



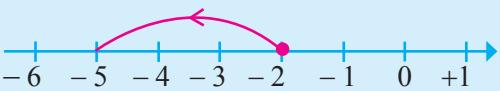
The magnitude of (–3) is 3 and the direction opposite to that of (–3) is towards the right.

The answer is the directed number which is represented by the point located 3 units to the right of (+2).

∴ (+2) – (–3) = (+5)

Example 2

Find the value of (–2) – (+3) by using the number line.



The magnitude of (+3) is 3 and the direction opposite to that of (+3) is towards the left.

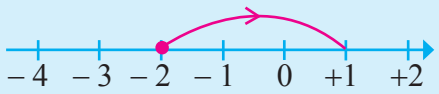
The answer is the directed number which is represented by the point located 3 units to the left of (–2).

∴ (–2) – (+3) = (–5)

Example 3

Find the value of (–2) – (–3) by using the number line.

The magnitude of (–3) is 3 and the direction opposite to that of (–3) is towards the right.



The answer is the directed number which is represented by the point located 3 units to the right of (–2).

$\therefore$ (–2) – (–3) = (+1)

Exercise 4.1

(1) Find the value by using the number line.

(i) (+4) – (+2) (ii) (+1) – (–2) (iii) (–2) – (+3)

(iv) (–1) – (–3) (v) (–6) – (–5) (vi) (+2) – (–2)

• More on subtracting a directed number from a directed number

By solving the equation $a + 1 = 0$, let us find the value of a which satisfies this equation.

The value of a cannot be 0 or a positive whole number.

Let us subtract 1 from both sides of the equation $a + 1 = 0$.

$a + 1 - 1 = 0 - 1$

$a = -1$

By taking the value of a in this equation to be (–1),

we obtain the relationship (–1) + 1 = 0.

This can also be written as 1 + (–1) = 0.

(–1) is called the **additive inverse** of (+1).

Furthermore, the additive inverse of (–1) is (+1).

- Likewise, every positive number has a corresponding additive inverse which is a negative number of equal magnitude.
- Similarly, every negative number has an additive inverse which is a positive number of equal magnitude.

The number	The additive inverse of the number
(+5)	(−5)
(−5)	(+5)
(+2)	(−2)
(−2)	(+2)
(+ 3.5)	(−3.5)
$\left(-\frac{2}{3}\right)$	$\left(+\frac{2}{3}\right)$

Now let us consider subtracting a directed number from another directed number without using the number line.

$5 - 2 = 3$.

Let us consider subtracting 2 from 5 by 5 and 2 as directed numbers.

Let us write the additive inverse of 2 as a directed number and add it to 5.

The additive inverse of (+ 2) is (−2).

$(+5) + (-2) = 3$

Subtracting a number from another number is the same as adding the additive inverse of the second number to the first number.

Hence, $5 - 2 = (+5) - (+2)$
$= (+5) + (-2)$
$= (+3)$

Example 4

Find the value of (+2) − (−4).
The additive inverse of (−4) is (+4).
$\therefore (+2) - (-4) = (+2) + (+4)$
$= (+6)$

Example 5

Find the value of (−5) − (+2).
The additive inverse of (+2) is (−2).
$\therefore (-5) - (+2) = (-5) + (-2)$
$= (-7)$

Example 6

Find the value of (−7) − (−3).
The additive inverse of (−3) is (+3).
$\therefore (-7) - (-3) = (-7) + (+3)$
$= (-4)$

Example 7

Find the value of (−12) − (−15) − (+5).
$(-12) - (-15) - (+5) = (-12) + (+15) + (-5)$
$= (+3) + (-5)$
$= (-2)$

Example 8

Find the value of $\left(+\frac{3}{5}\right) - \left(+\frac{1}{5}\right)$.

$$\left(+\frac{3}{5}\right) - \left(+\frac{1}{5}\right) = \left(+\frac{3}{5}\right) + \left(-\frac{1}{5}\right)$$
$$= \left(+\frac{2}{5}\right)$$

Example 9

Find the value of $\left(-5\frac{1}{2}\right) - (+2)$.

$$\left(-5\frac{1}{2}\right) - (+2) = \left(-5\frac{1}{2}\right) + (-2)$$
$$= \left(-7\frac{1}{2}\right)$$

Example 10

Find the value of $(-3.2) - (+1.4)$.

$$(-3.2) - (+1.4) = (-3.2) + (-1.4)$$
$$= (-4.6)$$

Example 11

Find the value of $(-8.4) - (-2.1)$.

$$(-8.4) - (-2.1) = (-8.4) + (+2.1)$$
$$= (-6.3)$$

Exercise 4.2

(1) Fill in each cage with the suitable directed number.

(i) $(-5) - (+3) = (-5) + \square$
$= \square$

(ii) $(-3) - (-4) = (-3) + \square$
$= \square$

(iii) $(+7) - (-1) = (+7) + \square$
$= \square$

(iv) $(+7) - (-2) = (+7) + \square$
$= \square$

(2) Find the value of each of the following.

(a) (i) $(+4) - (+1)$ (ii) $(-8) - (-2)$ (iii) $(-3) - (-7)$
(iv) $(+9) - (-6)$ (v) $(-5) - (-5)$ (vi) $0 - (+3)$
(vii) $(-11) - (+4)$ (viii) $(+2) + (-1) - (-4)$ (ix) $(-5) - (+2) - (-6)$
(x) $(+4) - (+2) - (+8)$

(b) (i) $\left(+4\frac{1}{2}\right) - (-2)$ (ii) $\left(-6\frac{1}{4}\right) - \left(-\frac{1}{4}\right)$ (iii) $(+15.7) - (-2.3)$
(iv) $(-2) - (+3.5) - (-4.1)$ (v) $\left(+3\frac{1}{2}\right) - (-2) - \left(-\frac{1}{3}\right)$

4.3 Multiplying directed numbers

Now let us consider the multiplication of two directed numbers.

➤ Let us find the value of $(+6) \times (+2)$.

- Obtain the product of the magnitudes of the two directed numbers without considering their signs.
  $6 \times 2 = 12$
- The two directed numbers are of the same sign. Therefore the answer is positive. $\therefore (+6) \times (+2) = (+12)$

➤ Let us find the value of $(-6) \times (+2)$.

- Obtain the product of the magnitudes of the two directed numbers without considering their signs.
  $6 \times 2 = 12$
- The two directed numbers are of opposite signs. Therefore, the answer is negative.
  $\therefore (-6) \times (+2) = (-12)$

When multiplying two directed numbers,

- Find the product of the magnitudes of the two directed numbers without considering their signs.
- If the two directed numbers are of the same sign, include the positive sign in the answer.
- If the two directed numbers are of opposite signs, include the negative sign in the answer.

Example 1

Simplify $(-6) \times (-2)$.

$6 \times 2 = 12$

The two directed numbers are of the same sign. Therefore the answer is positive.

$\therefore (-6) \times (-2) = (+12)$

Example 2

Simplify $(+6) \times (-2)$.

$6 \times 2 = 12$

The two directed numbers are of opposite signs. Therefore the answer is negative.

$\therefore (+6) \times (-2) = (-12)$

Example 3

Simplify the following .

(i) $(+2) \times (+5)$ (ii) $(-2) \times (+3)$ (iii) $(+5) \times (-3)$ (iv) $(-4) \times (-3) \times (+2)$

(i) $(+2) \times (+5) = (+10)$

(ii) $(-2) \times (+3) = (-6)$

(iii) $(+5) \times (-3) = (-15)$

(iv) $(-4) \times (-3) \times (+2) = (+12) \times (+2) = (+24)$

Example 4

Simplify $(+2.5) \times (-5)$.

$2.5 \times 5 = 12.5$

$\therefore (+2.5) \times (-5) = (-12.5)$

Example 5

Simplify $(-3.4) \times (-12)$.

$3.4 \times 12 = 40.8$

$\therefore (-3.4) \times (-12) = (+40.8)$

Exercise 4.3

(1) Find the value.

(i) $(+5) \times (+4)$ (ii) $(-5) \times (+4)$ (iii) $(-10) \times (-5)$

(iv) $(+7) \times (-3)$ (v) $(-1) \times (-4)$ (vi) $(+11) \times 0$

(vii) $(-6) \times (+4)$ (viii) $(+12) \times (-3)$ (ix) $(-2) \times (+2) \times (-5)$

(x) $(-3) \times (-1) \times (+2) \times (-5)$ (xi) $(+2.5) \times (+2)$ (xii) $(+4.1) \times (-23)$

4.4 Dividing a directed number by a directed number

➤ Let us find the value of $(+6) \div (+2)$.

- Let us divide the two directed numbers by considering their magnitudes only, without considering their signs.

  $6 \div 2 = 3$

- The two directed numbers are of the same sign. Therefore the answer is positive.

  $\therefore (+6) \div (+2) = (+3)$

➤ Let us find the value of (–6) ÷ (+2).

- Let us divide the two directed numbers by considering their magnitudes only, without considering their signs.
  6 ÷ 2 = 3
- The two directed numbers are of opposite signs. Therefore, the answer is negative.
  ∴ (–6) ÷ (+2) = (–3)

When dividing a directed number by another directed number,

- Divide by considering their magnitudes, without considering their signs.
- Include the positive sign in the answer, if the two directed numbers are of the same sign.
- Include the negative sign in the answer, if the two directed numbers are of opposite signs.

Example 1

Simplify (–6) ÷ (–2).

6 ÷ 2 = 3

The two directed numbers are of the same sign. Therefore, the answer is positive.

(–6) ÷ (–2) = (+3)

Example 2

Simplify (+6) ÷ (–2).

6 ÷ 2 = 3

The two directed numbers are of opposite signs. Therefore, the answer is negative.

∴ (+6) ÷ (–2) = (–3)

Example 3

Simplify the following.

(i) (+15) ÷ (+5) (ii) (–9) ÷ (+3) (iii) (+15) ÷ (–3) (iv) (–9) ÷ (–3)

(i) (+15) ÷ (+5) = (+3)

(ii) (–9) ÷ (+3) = (–3)

(iii) (+15) ÷ (–3) = (–5)

(iv) (–9) ÷ (–3) = (+3)

Exercise 4.4

(1) Find the value of each of the following.

(i) $(+10) \div (+2)$ (ii) $(-12) \div (-4)$ (iii) $(+15) \div (-3)$

(iv) $(-21) \div (+7)$ (v) $(-5) \div (+5)$ (vi) $\frac{(-20)}{(-4)}$

(vii) $\frac{(+2) \times (+8)}{(-4)}$ (viii) $\frac{(-36)}{(-6) \times (-2)}$ (ix) $\frac{(+5) \times (-4)}{(-2) \times (-2)}$

(x) $\frac{(-9) \times (-8)}{(-4) \times (+3)}$

(2) Fill in each cage with the suitable directed number.

(i) $(-20) \div \square = (-10)$ (ii) $(+18) \div \square = (-6)$ (iii) $\square \div (-2) = (+5)$

(iv) $(+4) \div \square = (-4)$ (v) $\frac{(+3) \times \square}{(-2)} = (+6)$ (vi) $\frac{\square \times (+7)}{(+2) \times \square} = \frac{(-28)}{\square} = (+7)$

Summary

- Subtracting a number from another number is the same as adding the additive inverse of the second number to the first number.
- A positive number is obtained, when two directed numbers of the same sign are multiplied or divided.
- A negative number is obtained when two directed numbers of opposite signs are multiplied or divided.