3

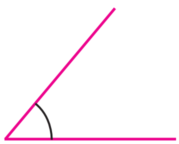
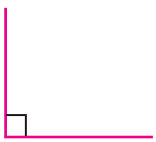
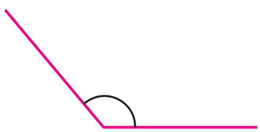
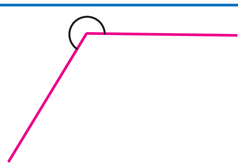
Angles

By studying this lesson you will be able to,

- identify pairs of complementary angles, supplementary angles, adjacent angles and vertically opposite angles,
- identify that the sum of the angles which lie around a point on one side of a straight line is 180˚,
- identify that the sum of the angles around a point on a plane is 360˚,
- identify that the vertically opposite angles created by two intersecting straight lines are equal, and
- calculate the magnitudes of angles associated with straight lines.

3.1 Angles

You have learnt in Grade 7 that the standard unit used to measure angles is **degrees** and that one degree is written as 1˚.

Angle	Figure	Note
Acute Angle		An angle of magnitude less than 90° is called an **acute angle**.
Right Angle		An angle of magnitude 90° is called a **right angle**.
Obtuse Angle		An angle of magnitude greater than 90° but less than 180° (that is, an angle of magnitude between 90° and 180°) is called an **obtuse angle**.
Straight Angle		An angle of magnitude 180° is called a **straight angle**.
Reflex Angle		An angle of magnitude between 180° and 360° is called a **reflex angle**.

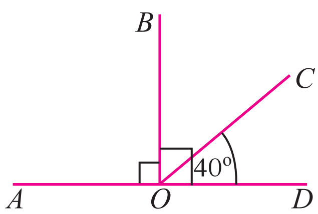
Do the review exercise to recall the above given facts which you learnt in Grade 7 under the lesson on angles.

Review Exercise

(1) Copy the two groups A and B given below and join them appropriately.

A	**B**
135^{o}	Acute angle
90^{o}	Right angle
180^{o}	Obtuse angle
35^{o}	Straight angle
245^{o}	Reflex angle
190^{o}	
280^{o}	

(2) By considering the given figure, find the magnitude of each of the angles given below and write the type of each angle.

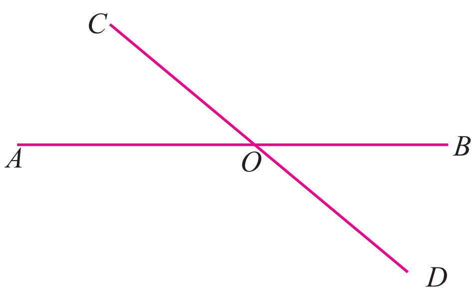


(i) $A\hat{O}B$ (ii) $C\hat{O}D$
(iii) $B\hat{O}D$ (iv) $B\hat{O}C$
(v) $A\hat{O}C$ (vi) $A\hat{O}D$

(3) Draw the following angles using a protractor and name them.

(i) $P\hat{Q}R = 60^{o}$ (ii) $A\hat{B}C = 90^{o}$ (iii) $X\hat{Y}Z = 130^{o}$ (iv) $K\hat{L}M = 48^{o}$

(4) As shown in the figure, draw two straight line segments AB and CD such that they intersect each other at O.



(i) Measure the magnitude of each of the angles $A\hat{O}C$, $C\hat{O}B$, $B\hat{O}D$ and $A\hat{O}D$ and write them down.

(ii) What is the value of $A\hat{O}C + C\hat{O}B$?

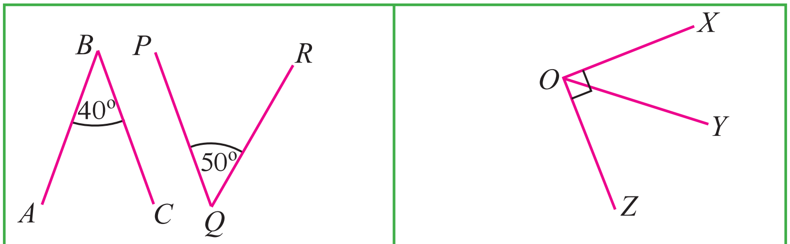
(iii) Are the two angles $A\hat{O}C$ and $B\hat{O}D$ equal to each other?

3.2 Complementary angles and supplementary angles

Let us identify what complementary angles and supplementary angles are.

• Complementary angles

Two pairs of angles are shown in the figure given below. Let us consider the sum of the magnitudes of each pair of angles.



$A\hat{B}C + P\hat{Q}R = 40^{\circ} + 50^{\circ}$
$= 90^{\circ}$

$X\hat{O}Y + Y\hat{O}Z = 90^{\circ}$

The sum of the magnitudes of the two angles of each pair is obtained as 90°.

If the sum of a pair of acute angles is 90°, then that pair of angles is called a pair of **complementary angles**.

According to this explanation, in the figure given above,

the angles $A\hat{B}C$ and $P\hat{Q}R$ are a pair of complementary angles, and
the angles $X\hat{O}Y$ and $Y\hat{O}Z$ are a pair of complementary angles.

The acute angle which needs to be added to a given acute angle for the sum of the two angles to be 90° is called the **complement** of the given angle.

$30^{\circ} + 60^{\circ} = 90^{\circ}$. Hence, the complement of 30° is 60°.

Example 1

Calculate the complement of 38°.

Since $90^{\circ} - 38^{\circ} = 52^{\circ}$, the complement of 38° is 52°.

Example 2

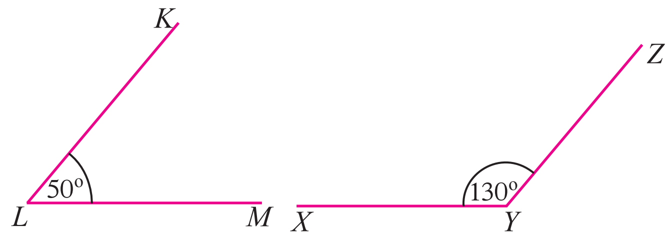
If $A\hat{B}C = 48^{\circ}$, $P\hat{Q}R = 66^{\circ}$, $K\hat{L}M = 42^{\circ}$ and $X\hat{Y}Z = 24^{\circ}$; name the pairs of complementary angles among these angles.

$48^{\circ} + 42^{\circ} = 90^{\circ}$. $\therefore$ $A\hat{B}C$ and $K\hat{L}M$ are a pair of complementary angles.
$66^{\circ} + 24^{\circ} = 90^{\circ}$. $\therefore$ $P\hat{Q}R$ and $X\hat{Y}Z$ are a pair of complementary angles.

• Supplementary angles

Let us consider the sum of the two angles given in the figure.

$K\hat{L}M + X\hat{Y}Z = 50^{\circ} + 130^{\circ}$
$= 180^{\circ}$



If the sum of a pair of angles is 180˚, then that pair of angles is called a pair of **supplementary angles.**

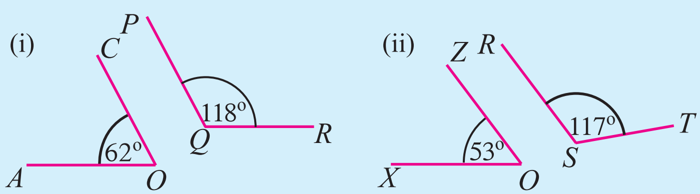
According to this explanation, $K\hat{L}M$ and $X\hat{Y}Z$ are a pair of supplementary angles.

The angle which needs to be added to a given angle of less than 180˚ for the sum to be 180° is called the **supplement** of the given angle.

$60^{\circ} + 120^{\circ} = 180^{\circ}$
$\therefore$ The supplement of 60° is 120°.

Example 3

Explain whether the pairs of angles given in the figure are supplementary angles.



(i) $A\hat{O}C + P\hat{Q}R = 62^{\circ} + 118^{\circ}$
$= 180^{\circ}$
$\therefore$ $A\hat{O}C$ and $P\hat{Q}R$ are a pair of supplementary angles.

(ii) $X\hat{O}Z + R\hat{S}T = 53^{o} + 117^{o}$
$= 170^{o}$
Since the sum of the two angles is not 180°, $X\hat{O}Z$ and $R\hat{S}T$ are not a pair of supplementary angles.

Exercise 3.1

(1) Copy and complete.

(i) The complement of 60^{o} is ....
The supplement of 60^{o} is ....

(ii) The complement of 75^{o} is ....
The supplement of 75^{o} is ....

(iii) The complement of 25^{o} is ....
The supplement of 25^{o} is ....

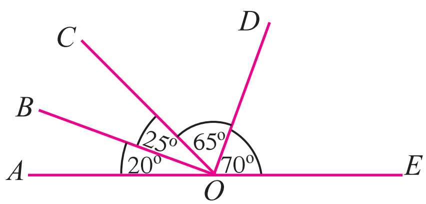
(iv) The complement of 1^{o} is ....
The supplement of 1^{o} is ....

(2) From among the angles $A\hat{B}C = 72^{o}$, $P\hat{Q}R = 15^{o}$, $X\hat{Y}Z = 28^{o}$, $K\hat{L}M = 165^{o}$, $B\hat{O}C = 18^{o}$, $M\hat{N}L = 108^{o}$ and $D\hat{E}F = 75^{o}$, select and write down,

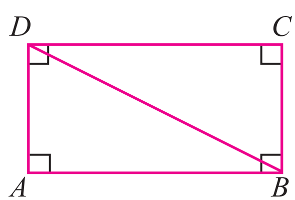
(i) two pairs of complementary angles.
(ii) two pairs of supplementary angles.

(3) According to the figure given here,

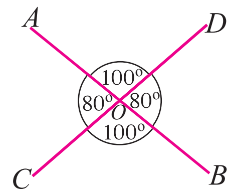
(i) what is the sum of $B\hat{O}C$ and $C\hat{O}D$?
(ii) what is the complement of $B\hat{O}C$?
(iii) what is the magnitude of $A\hat{O}D$?
(iv) what is the sum of $A\hat{O}D$ and $D\hat{O}E$?
(v) what is the supplement of $D\hat{O}E$?
(vi) what is the complement of $D\hat{O}E$?



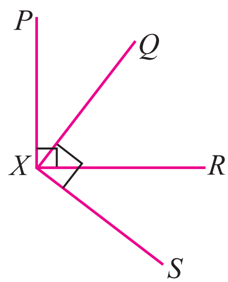
(4) (i) Write two pairs of complementary angles in the given figure.



(ii) The straight line segments AB and CD intersect at O. Write four pairs of supplementary angles in the figure.



(5) Write two pairs of complementary angles according to the information marked in the given figure.



(6) Copy these statements in your exercise book and place a ✓ in front of the correct statements and a ✗ in front of the incorrect statements.

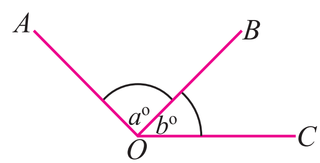
(i) The complement of an acute angle is an acute angle.

(ii) The complement of an acute angle is an obtuse angle.

(iii) The supplement an obtuse angle is an obtuse angle.

(iv) The supplement of an acute angle is an obtuse angle.

3.3 Adjacent angles



Let us consider the arms and the vertex of the two angles $A\hat{O}B$ and $B\hat{O}C$ in the figure.

The arms of $A\hat{O}B$ are AO and BO. The vertex is O.
The arms of $B\hat{O}C$ are BO and CO. The vertex is O.

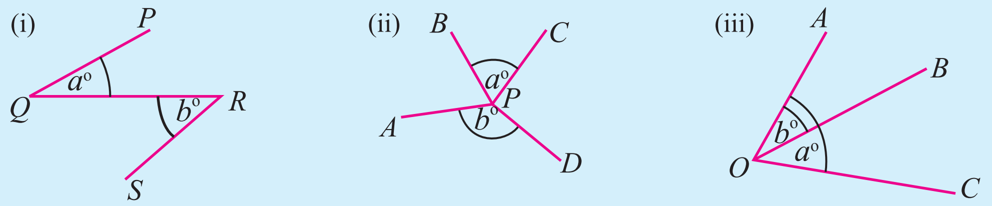
The arm BO belongs to both angles. Hence, BO is a common arm. The vertex of both angles is O. Hence, O is the **common vertex**. Moreover, these two angles are located on either side of the **common arm** OB .

A pair of angles which have a common arm and a common vertex and are located on either side of the common arm is called a pair of **adjacent angles**.

According to this explanation, $A\hat{O}B$ and $B\hat{O}C$ in the figure given above are a pair of adjacent angles.

Example 1

Explain whether the pairs of angles denoted by a and b in the figures given below are pairs of adjacent angles.



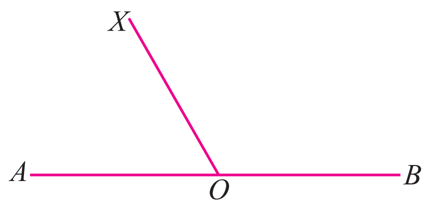
(i) QR is the common arm of both angles. The two angles are located on either side of QR. But there isn't a common vertex. Hence, $P\hat{Q}R$ and $Q\hat{R}S$ are not adjacent angles.

(ii) Both angles have a common vertex. But they do not have a common arm. Therefore, $B\hat{P}C$ and $A\hat{P}D$ are not adjacent angles.

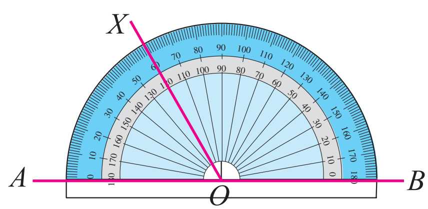
(iii) The angles $A\hat{O}B$ and $A\hat{O}C$ have a common arm and a common vertex. The common arm is AO. However, the two angles are not located on either side of the common arm. Therefore, $A\hat{O}B$ and $A\hat{O}C$ are not adjacent angles.

• Adjacent angles on a straight line

A pair of adjacent angles named $A\hat{O}X$ and $B\hat{O}X$ is created by the straight line XO meeting the straight line AB at O. Let us measure these two angles by using a protractor.



It is clear that in the figure, $A\hat{O}X = 60^o$ and $B\hat{O}X = 120^o$ (You can read the magnitudes of both angles at the same time by placing the base line of the protractor on the line AOB).

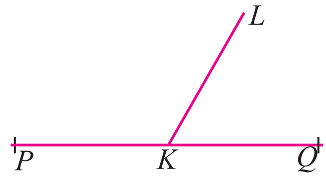


Activity 1

Step 1 - Draw a straight line segment in your exercise book and name it PQ.

Step 2 - Draw the straight line KL with the point K located on PQ.

Step 3 - Measure $P\hat{K}L$ and $Q\hat{K}L$ using the protractor and write down their magnitudes.



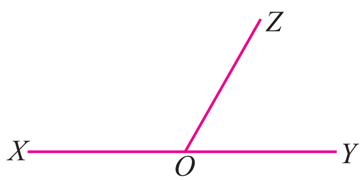
Step 4 - Fill in the blanks below the figure.

$P\hat{K}L + Q\hat{K}L =$ .... + ...

$=$ ....

Step 5 - As above, engage in the activity for another two figures, and investigate the possible conclusion that can be drawn.

The line segment XY is divided into the two line segments OX and OY by the point O located on XY.



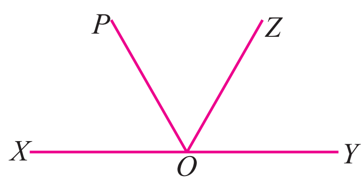
The sum of the two adjacent angles $X\hat{O}Z$ and $Z\hat{O}Y$, where OZ is the common arm, and OX and OY are the other arms, can be shown to be 180° by measuring the two angles separately.

This establishes the fact that a pair of adjacent angles, located on a straight line in this manner is a pair of supplementary angles.

Let us divide the angle $X\hat{O}Z$ into two by the straight line OP in the figure.

Then $X\hat{O}Z = X\hat{O}P + P\hat{O}Z$

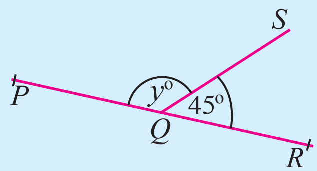
$\therefore\ X\hat{O}P + P\hat{O}Z + Z\hat{O}Y = X\hat{O}Z + Z\hat{O}Y = 180^{\circ}$.



The sum of the angles around a point on a straight line, located on one side of the straight line is 180°.

Example 2

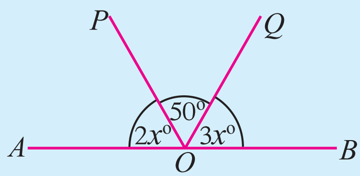
In the given figure, PR is a straight line segment. Find the magnitude of $P\hat{Q}S$.



$$y + 45 = 180$$
$$y + 45 - 45 = 180 - 45$$
$$y = 135$$
$$P\hat{Q}S = 135^\circ$$

Example 3

Find the magnitude of $A\hat{O}P$ according to the information marked in the figure.



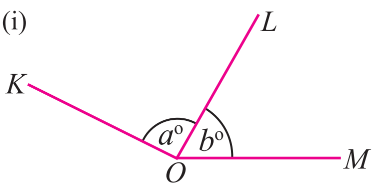
$2x + 50 + 3x = 180$ (the sum of the angles on a straight line is 180°)

$$5x + 50 = 180$$
$$5x + 50 - 50 = 180 - 50$$
$$\frac{5x}{5} = \frac{130}{5}$$
$$x = 26$$
$$\therefore\ A\hat{O}P = 2x^\circ = 2 \times 26^\circ = 52^\circ$$

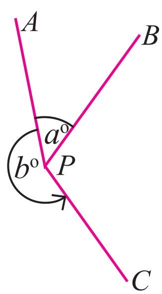
Exercise 3.2

(1) Write whether the pair of angles marked as a and b in each figure is a pair of adjacent angles.

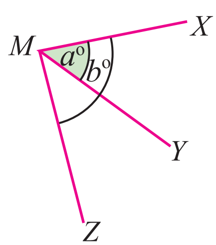
(i)



(ii)

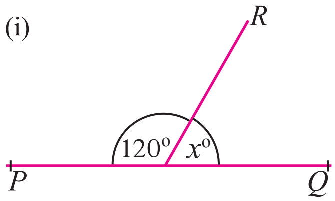


(iii)

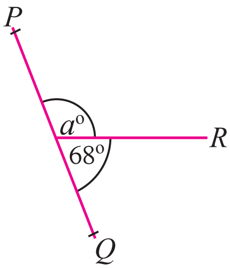


(2) If PQ is a straight line segment in each figure given below, find the magnitude of the angle marked by an English letter.

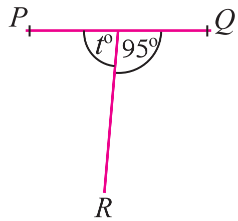
(i)



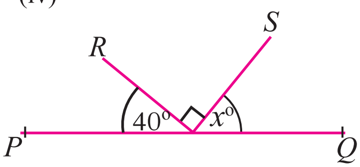
(ii)



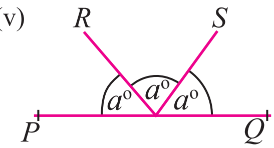
(iii)



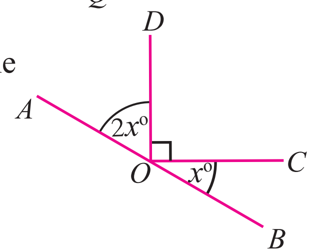
(iv)



(v)



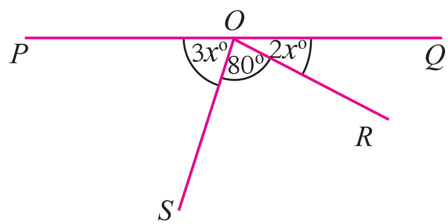
(3) In the figure, if AB is a straight line segment, find the magnitude of $A\hat{O}D$.



(4) PQ is a straight line segment. According to the information marked in the figure,

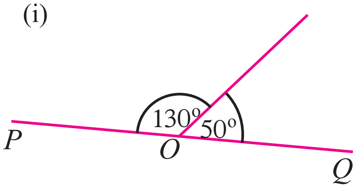
(i) find the magnitude of $P\hat{O}S$.

(ii) find the magnitude of $S\hat{O}Q$.

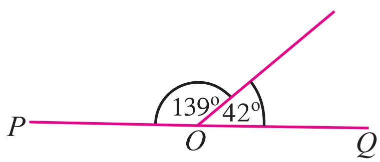


(5) Conclude whether POQ in each of the given figures is a straight line.

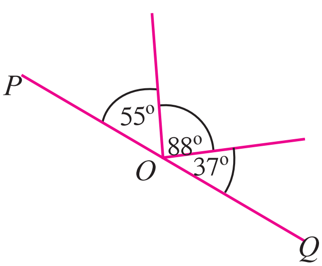
(i)



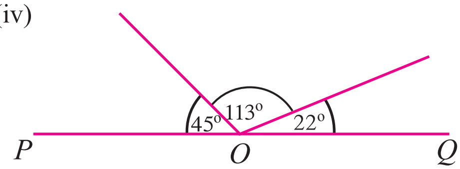
(ii)



(iii)

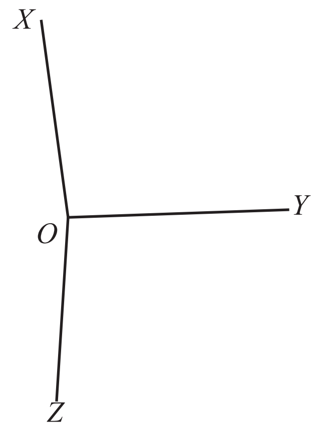


(iv)

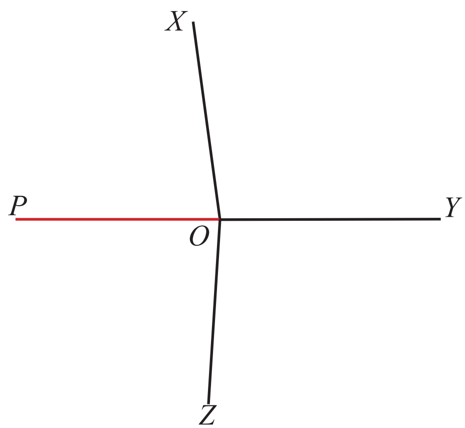


3.4 The sum of the angles around a point on a plane

Consider the angles $X\hat{O}Y$, $Y\hat{O}Z$ and $Z\hat{O}X$ located around the point O in the figure. Let us find the value of $X\hat{O}Y + Y\hat{O}Z + Z\hat{O}X$.



To do this, produce the straight line YO to P.



Method 1

Since POY is a straight line,

$$P\hat{O}X + X\hat{O}Y = 180^\circ$$
$$P\hat{O}Z + Z\hat{O}Y = 180^\circ$$
$$\therefore\ P\hat{O}X + X\hat{O}Y + P\hat{O}Z + Z\hat{O}Y = 180^\circ + 180^\circ$$
$$= 360^\circ$$

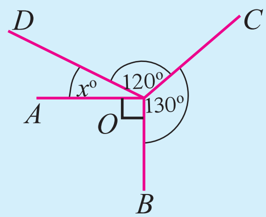
Method 2

$$Z\hat{O}X = Z\hat{O}P + P\hat{O}X$$
$$\therefore\ X\hat{O}Y + Y\hat{O}Z + Z\hat{O}X = X\hat{O}Y + Y\hat{O}Z + Z\hat{O}P + P\hat{O}X$$
$$= \underbrace{X\hat{O}Y + P\hat{O}X}_{\text{Supplementary Angles}} + \underbrace{Y\hat{O}Z + Z\hat{O}P}_{\text{Supplementary Angles}}$$
$$= 180^\circ + 180^\circ = 360^\circ$$

> The sum of the angles located around a point on a plane is 360°.

Example 1

Find the magnitude of the angle marked as $A\hat{O}D$ in the given figure.

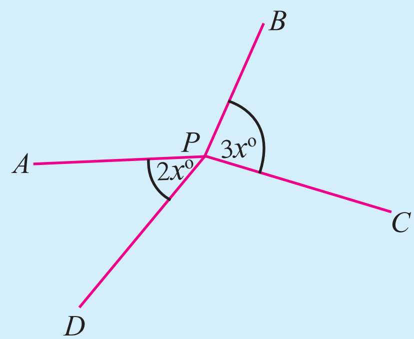


$x + 120 + 130 + 90 = 360$ (the sum of the angles around a point is 360^{o})

$$x + 340 = 360$$
$$x + 340 - 340 = 360 - 340$$
$$x = 20$$
$$\therefore A\hat{O}D = 20^{o}$$

Example 2

If $A\hat{P}B = 150^{o}$ and $D\hat{P}C = 100^{o}$ in the figure, find the magnitude of $B\hat{P}C$.



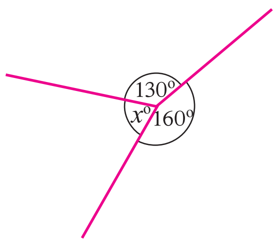
Because the sum of the angles around P is 360^{o},

$$2x + 150 + 3x + 100 = 360$$
$$5x + 250 = 360$$
$$5x + 250 - 250 = 360 - 250 = 110$$
$$\frac{5x}{5} = \frac{110}{5}$$
$$x = 22$$

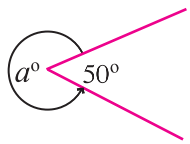
$\therefore B\hat{P}C = 3 \times 22^{o} = 66^{o}$

Exercise 3.3

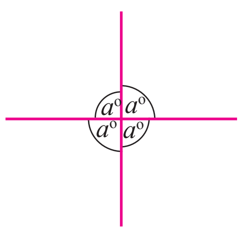
(1) Find the value of x^{o}.



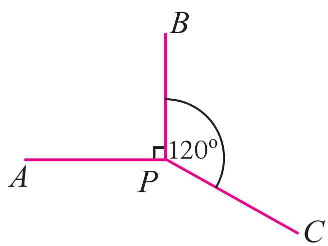
(2) Find the value of a^{o}.



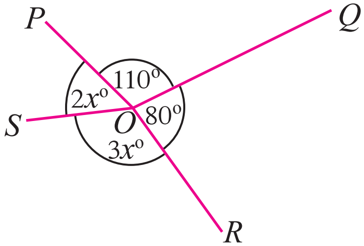
(3) Find the value of a^{o}.



(4) Find the magnitude of $A\hat{P}C$.

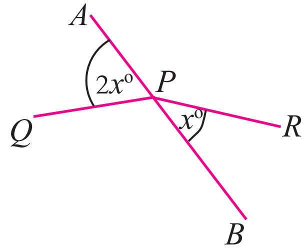


(5) Find the magnitude of $S\hat{O}R$.



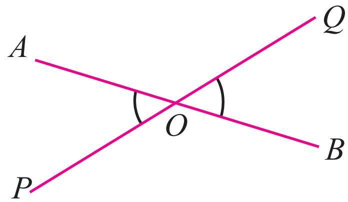
(6) AB is a straight line.

If $A\hat{P}R = 150°$, find the magnitude of $Q\hat{P}B$.



3.5 Vertically opposite angles

The two straight lines AB and PQ shown in the figure intersect at point O. The two angles $A\hat{O}P$ and $B\hat{O}Q$ which are located vertically opposite each other as shown here are called **vertically opposite angles**.

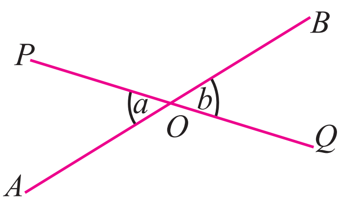


The two angles $A\hat{O}Q$ and $B\hat{O}P$ in the figure are also a pair of vertically opposite angles.

Two pairs of vertically opposite angles are always created by the intersection of two straight lines. Each pair has a common vertex and the two angles are located vertically opposite each other across the common vertex.

Activity 2

Step 1 - In your exercise book, draw two straight lines which intersect each other as shown in the figure and include the information given in the figure.



Step 2 - Copy the figure on a tissue paper and name it also as in the above figure.

Step 3 - Keep the two drawn figures such that they coincide with each other and hold them in place with a pin at point O.

Step 4 - Rotate the tissue paper half a circle around the point O and see whether the two angles a and b coincide with each other.

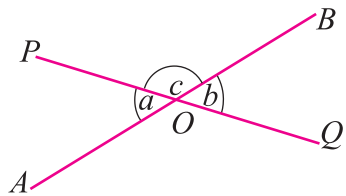
Step 5 - Engage in the activity as above for another two cases and examine whether the vertically opposite angles coincide with each other.

Investigate the conclusion that can be drawn from this activity.

It can be concluded based on the above activity, that vertically opposite angles created by the intersection of two straight lines are equal to each other.

Vertically opposite angles created by the intersection of two straight lines are equal to each other.

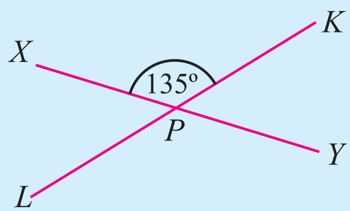
Let us investigate whether this is true by another method.
PQ and *AB* in the figure are straight line segments.



$a + c = 180^\circ$ (*AB* is a straight line)
$b + c = 180^\circ$ (*PQ* is a straight line)
$\therefore\ a + c = b + c$
$a + c - c = b + c - c$ (subtracting c from both sides)
$\therefore\ a = b$
$\therefore$ The vertically opposite angles $A\hat{O}P$ and $B\hat{O}Q$ are equal to each other.

Example 1

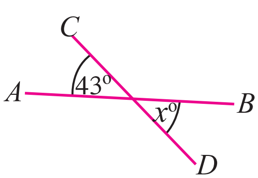
Find the magnitude of each angle around the point *P* in the given figure, where *XY* and *KL* are straight line segments.

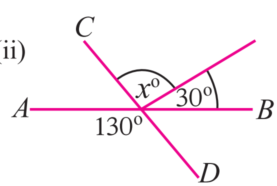


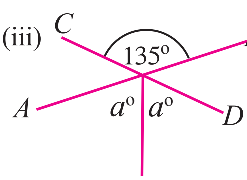
$L\hat{P}Y = X\hat{P}K$ (vertically opposite angles are equal)
$\therefore\ L\hat{P}Y = 135^\circ$
$X\hat{P}L + 135^\circ = 180^\circ$ (the sum of the angles on the straight line *LK* is 180°)
$\therefore\ X\hat{P}L = 180^\circ - 135^\circ$
$= 45^\circ$
$K\hat{P}Y = X\hat{P}L$ (vertically opposite angles are equal)
$\therefore\ K\hat{P}Y = 45^\circ$

Exercise 3.4

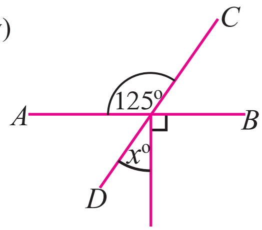
(1) Find the magnitude of each of the angles marked by an English letter in the figures given below (*AB*, *CD* and *EF* are straight lines).

(i)


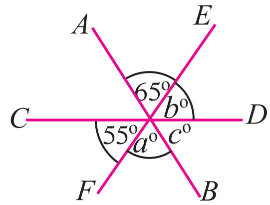
(ii)


(iii)


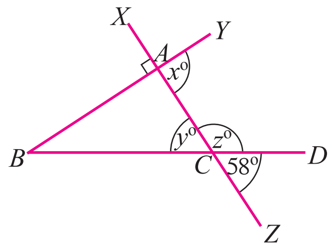
(iv)



(v)



(2) (i) Find the values of the angles denoted by x, y and z in the given figure (BY, BD and XZ are straight lines).



(ii) $A\hat{B}C$ and $A\hat{C}B$ are a pair of complementary angles. What is the magnitude of $A\hat{B}C$?

Summary

- If the sum of a pair of acute angles is 90°, then that pair of angles is called a pair of complementary angles.
- The acute angle which needs to be added to a given acute angle for the sum of the two angles to be 90° is called the complement of the given angle.
- If the sum of a pair of angles is 180°, then that pair of angles is called a pair of supplementary angles.
- The angle which needs to be added to a given angle of less than 180° for the sum to be 180° is called the supplement of the given angle.
- A pair of angles which have a common arm and a common vertex and are located on either side of the common arm is called a pair of adjacent angles.
- The sum of the angles located around a point on one side of a straight line is 180°.
- The sum of the angles located around a point on a plane is 360°.
- Vertically opposite angles created by the intersection of two straight lines are equal to each other.