1 Number Patterns

By studying this lesson you will be able to,

- identify the *n*th term of a given number pattern, and
- find any term of a number pattern when the *n*th term is given.

1.1 Number patterns and terms of a number pattern

Let us write the odd numbers from 3 to 11 in ascending order.

3, 5, 7, 9, 11

This is the number pattern of the odd numbers from 3 to 11 written in ascending order.

- When numbers are written in a certain order according to a specific method or rule, starting from a certain number, it is called a **number pattern**.
- Every number in a given number pattern is called a **term of the number pattern**.
- The first number of a number pattern is called the first term and the following numbers in order are called the second term, third term, fourth term, etc.
- Commas are used to separate the terms of a number pattern.

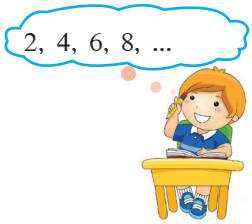
Let us consider again the number pattern 3, 5, 7, 9, 11 of the odd numbers from 3 to 11 written in ascending order.

The first term of the above pattern is 3 and the fourth term is 9. The last or the 5th term is 11. There are only five terms in this number pattern. Therefore the number of terms is finite.

Such number patterns, where the number of terms is finite, are called **finite number patterns**.

Let us write the even numbers starting from 2 in ascending order.

2, 4, 6, 8, …



You learnt in Grade 6 that this is the number pattern of the even numbers starting from 2 written in ascending order.

Since the exact number of terms in this number pattern cannot be specified, that is, since it is infinite, we cannot write down all its terms. Therefore, the first few terms are written such that the pattern can be identified, and as above, three dots are used to denote the rest of the terms.

Such number patterns where the number of terms is not finite, are called **infinite number patterns**.

Example 1

Write the terms of each of the following number patterns.

(i) The number pattern of the prime numbers between 1 and 17, written in ascending order.

(ii) The number pattern of the odd numbers starting from 1, written in ascending order.

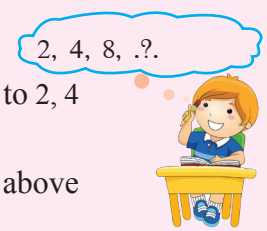
(iii) The number pattern starting from 1 and followed by the terms 2 and 1 written alternatively.

(i) 2, 3, 5, 7, 11, 13 (ii) 1, 3, 5, 7, 9, …

(iii) 1, 2, 1, 2, 1, 2, …

Note

Consider the number pattern 2, 4, 8,…



A number pattern with the first, second and third terms equal to 2, 4 and 8 respectively is given above.

We can easily write two different number patterns with the above first three terms.

(i) 2, 4, 8, 16, 32, 64, …
Here a term is multiplied by 2 to get the next term.

(ii) 2, 4, 8, 10, 20, 22, 44, …
Here, the second term is obtained by adding two to the first term, the third term is obtained by multiplying the second term by two, the fourth term is obtained by adding two to the third term, etc.

An important fact that can be learnt from this is that there can be more than one number pattern having the same first few terms.

Exercise 1.1

(1) Fill in the blanks.

(i) In the number pattern 1, 3, 5, 7, 9, …

the first term = ...
the second term = ...
the fourth term = ...

(ii) In the number pattern 4, 8, 12, 16, 20, …

the first term = ...
the second term = ...
the third term = ...

(2) Write the terms of each of the following number patterns.

(i) The number pattern of the even numbers between 1 and 9 written in ascending order.

(ii) The number pattern of the multiples of 6 from 6 to 36 written in ascending order.

(iii) The number pattern of the even numbers greater than 7 written in ascending order.

(iv) The number pattern of the prime numbers starting from 2 written in ascending order.

(3) Copy the below given statements in your exercise book and mark the correct statements with a ✓ and the incorrect statements with a ✗ .

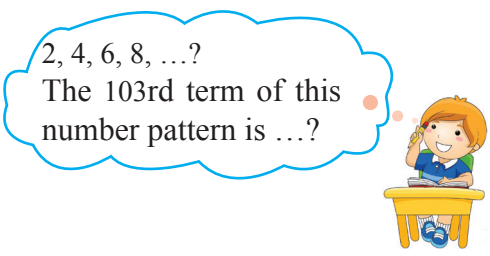
(i) The terms of a number pattern have to be in ascending order.

(ii) The terms of a number pattern have to be different from each other.

(iii) If the 10th terms of two number patterns are different, then the two number patterns are different to each other.

1.2 The general term of a number pattern

Let us consider how we can easily find any term of a number pattern.



When the nth term of a number pattern is written as an algebraic expression in n, it is called the **general term of the number pattern**.

Using the general term, the value of any term in the number pattern can be found.

• The general term of the number pattern of the multiples of a number

➤ Let us consider the number pattern of the multiples of 2, starting from 2 and written in ascending order.

This number pattern is 2, 4, 6, 8, …

Although the terms from the fifth term onwards are not written, we know that the fifth term is 10, the sixth term is 12 and the seventh term is 14.

Let us find the nth term of this number pattern.
The table given below, shows how the value of each term is obtained.

Term	Value of the term	How the value of the term is obtained
First term	2	2×1
Second term	4	2×2
Third term	6	2×3
Fourth term	8	2×4
⋮	⋮	⋮
Tenth term	?	2×10
⋮	⋮	⋮
nth term	?	$2 \times n$
⋮	⋮	⋮

According to the 3rd column of the table, the nth term of the above number pattern is $2 \times n$; that is $2n$.

The nth term of this number pattern is $2n$. This is called the **general term** of this number pattern. By substituting suitable values for n in $2n$, we can obtain the values of the relevant terms.

The value of n in the general term of a number pattern should always be a positive integer.

The above number pattern is the same as **the even numbers starting from 2 and written in ascending order**.

- The general term of the number pattern of the even numbers starting from 2 and written in ascending order is $2n$.
- The general term of the number pattern of the multiples of 2 starting from 2 and written in ascending order is $2n$.

Example 1

In the number pattern of the multiples of 2 starting from 2 and written in ascending order,

(i) find the 11th term.
(ii) find the 103rd term.
(iii) find which term is 728.

(i) General term = $2n$
Since $n = 11$, the 11th term = $2 \times 11 = 22$

(ii) The 103rd term = 2×103
$= 206$

(iii) Since 728 is a multiple of 2, it should be a term of this number pattern. To identify which term it is, the general term should be equated to 728 and the value of n satisfying this equation should be found.

$$2n = 728$$
$$\frac{2n}{2} = \frac{728}{2}$$
$$n = 364$$

Accordingly, 728 is the 364th term of this number pattern.

➤ Let us consider the number pattern of the multiples of 3 starting from 3 and written in ascending order.

This number pattern is 3, 6, 9, 12, …

How the values of the terms of this number pattern are obtained is shown in the following table.

Term	Value of the term	How the value of the term is obtained
First term	3	3×1
Second term	6	3×2
Third term	9	3×3
Fourth term	12	3×4
⋮	⋮	⋮
Tenth term	?	3×10
⋮	⋮	⋮
nth term	?	$3 \times n$
⋮	⋮	⋮

According to the third column of this table, the nth term in this number pattern is $3 \times n$; that is $3n$.

The general term of the number pattern of the multiples of 3 starting from 3 and written in ascending order is $3n$.

Accordingly,

- the general term of the number pattern of the multiples of 4 starting from 4 and written in ascending order is $4n$.
- the general term of the number pattern of the multiples of 7 starting from 7 and written in ascending order is $7n$.

Example 2

The general term of the number pattern of the multiples of 3 starting from 3 and written in ascending order is $3n$.
(i) Find the 13th term of this number pattern.
(ii) Find which term 87 is of this number pattern.

(i) The general term of the number pattern of the multiples of 3 starting from 3 and written in increasing order is $3n$

The 13th term of this number pattern $= 3 \times 13 = 39$

(ii) $3n = 87$

Let us find the value of n that satisfies this equation.

$$\frac{3n}{3} = \frac{87}{3}$$
$$n = 29$$

$\therefore$ 87 is the 29th term of this number pattern.

Example 3

In the number pattern of the multiples of 4 starting from 4 and written in ascending order, with general term $4n$,
(i) what is the 10th term?
(ii) what is the 11th term?
(iii) which term is 100?
(iv) is 43 a term of this number pattern? What are the reasons for your answer?

(i) The general term of the number pattern of the multiples of $4 = 4n$

10th term $= 4 \times 10$
$= 40$

(ii) The general term of the number pattern of the multiples of $4 = 4n$

11th term $= 4 \times 11$
$= 44$

(iii) Since the general term of the number pattern of the multiples of 4 is $4n$,

$$4n = 100$$
$$\frac{4n}{4} = \frac{100}{4}$$
$$n = 25$$

$\therefore$ 100 is the 25th term.

(iv) When $4n = 43$

$$\frac{4n}{4} = \frac{43}{4}$$
$$n = 10\frac{3}{4}$$ (This is not a positive integer)

$\therefore$ 43 cannot be a term of this number pattern.

43 is not a multiple of 4. Therefore, it can be said that 43 is not a term of this number pattern.

Exercise 1.2

(1) Copy the table given below and complete it.

Number pattern	First term	General term
5, 10, 15, 20, ...		
10, 20, 30, 40, ...		
8, 16, 24, 32, ...		
7, 14, 21, 28, ...		
12, 24, 36, 48, ...		
1, 2, 3, 4, ...		

(2) Write the number pattern of the multiples of 5 between 3 and 33 written in ascending order.

(3) In the number pattern 11, 22, 33, 44, … of the multiples of 11 starting from 11 and written in ascending order,

(i) what is the general term?

(ii) what is the 9th term?

(iii) which term is 121?

(4) In the number pattern 9, 18, 27, 36, … of the multiples of 9 starting from 9 and written in ascending order,

(i) what is the general term ?

(ii) what is the 11th term?

(iii) which term is 270?

(5) In the number pattern with general term $100n$,
(i) what is the 11th term?
(ii) which term is 500?

(6) What is the smallest multiple of 3 larger than 100? Which term is it in the number pattern of the multiples of 3 starting from 3?

(7) What is the nth term (general term) of the pattern of the even numbers greater than 1 but less than 200 written in ascending order? The smallest value of n is 1. What is its largest value?

(8) It has been estimated that in a country having a population of 2 million people, the population will increase by 2 million people every 25 years. Estimate the population of the country in 200 years.

• The general term of the pattern of the odd numbers

You have learnt earlier that odd numbers are numbers which have a remainder of 1 when divided by 2.

1, 3, 5, 7, ... is the pattern of the odd numbers starting from 1 written in ascending order.

Since we obtain a remainder of 1 when an odd number is divided by 2, we should obtain an odd number when 1 is subtracted from any multiple of 2.

Accordingly, let us identify how the pattern of the odd numbers is developed by considering the following table.

Term	Multiples of 2	Multiples of 2 - 1	Odd number
First term	$2 = 2 \times 1$	$(2 \times 1) - 1$	$2 - 1 = 1$
Second term	$4 = 2 \times 2$	$(2 \times 2) - 1$	$4 - 1 = 3$
Third term	$6 = 2 \times 3$	$(2 \times 3) - 1$	$6 - 1 = 5$
⋮	⋮	⋮	⋮
Tenth term	$20 = 2 \times 10$	$(2 \times 10) - 1$	$20 - 1 = 19$
⋮	⋮	⋮	⋮
nth term	$2n = 2 \times n$	$(2 \times n) - 1$	$2n - 1$
⋮	⋮	⋮	⋮

The general term of the pattern of the odd numbers starting from 1 and written in ascending order can be expressed in terms of the general term of the pattern of even numbers starting from 2, written in ascending order.

The general term of the pattern of the odd numbers starting from 1 and written in ascending order is $2n - 1$.

Example 4

In the pattern of the odd numbers 1, 3, 5, 7, … starting from 1,

(i) what is the general term?

(ii) what is the 72nd term?

(iii) which term is 51?

(i) Since this is the pattern of the odd numbers starting from 1, the general term is $2n - 1$.

(ii) When $n = 72$, the seventy second term $= 2 \times 72 - 1$

$= 144 - 1$

$= 143$

(iii) Let us take that 51 is the nth term of this number pattern.

Then, $2n - 1 = 51$

$2n - 1 + 1 = 51 + 1$

$2n = 52$

$\frac{2n}{2} = \frac{52}{2}$

$n = 26$

51 is the 26th term of this number pattern.

Exercise 1.3

(1) In the pattern of the odd numbers starting from 1 and written in ascending order,

(i) what is the 12th term?

(ii) what is the 15th term?

(iii) which term is 89?

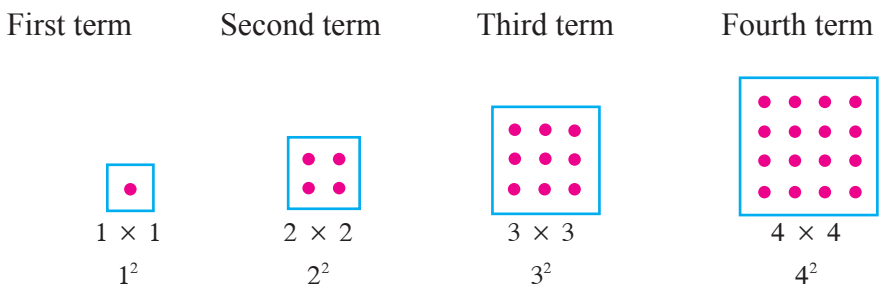
(iv) which term is the greatest odd number less than 100?

(2) Find the value of the sum of the 34th term of the pattern of the even numbers starting from 2 and the 34th term of the pattern of the odd numbers starting from 1.

• General term of the pattern of the square numbers

You have learnt in Grade 6 that 1, 4, 9, 16, … are the square numbers written in ascending order.

This pattern represented by square shaped figures consisting arrangements of dots is given below.



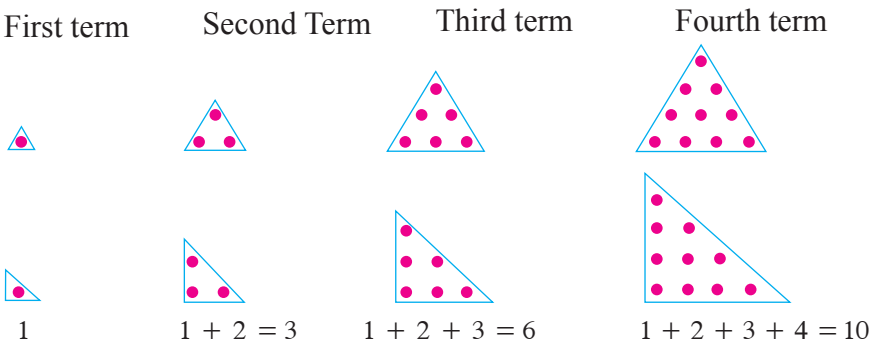
The pattern of the square numbers is developed as follows.

First term	$= 1 \times 1$	$= 1^2$	$= 1$
Second term	$= 2 \times 2$	$= 2^2$	$= 4$
Third term	$= 3 \times 3$	$= 3^2$	$= 9$
⋮	⋮	⋮	⋮
Tenth term	$= 10 \times 10$	$= 10^2$	$= 100$
⋮	⋮	⋮	⋮
nth term	$= n \times n$	$= n^2$	

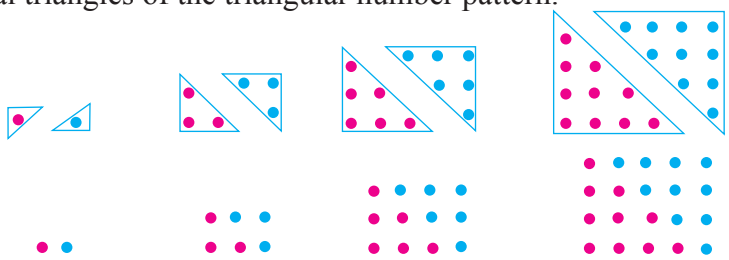
∴ The general term of the pattern of the square numbers starting from 1 and written in ascending order is n^2.

• The general term of the pattern of the triangular numbers

You have learnt in Grade 6 that 1, 3, 6, 10, 15, … are the triangular numbers written in ascending order. They can be represented by dots in both the ways given below.



Rectangular shaped arrangements of dots that have twice the number of dots as each corresponding figure in the triangular number pattern can be obtained by joining together two equal triangles of the triangular number pattern.



Number of rows	1	2	3	4
Number of columns	2	3	4	5
Total number of dots	1×2	2×3	3×4	4×5
Triangular number	$\frac{1 \times 2}{2} = 1$	$\frac{2 \times 3}{2} = 3$	$\frac{3 \times 4}{2} = 6$	$\frac{4 \times 5}{2} = 10$

Therefore, the pattern of triangular numbers is as follows.

$$\text{First term} = \frac{1 \times 2}{2} = 1$$

$$\text{Second term} = \frac{2 \times 3}{2} = 3$$

$$\text{Third term} = \frac{3 \times 4}{2} = 6$$

$$\text{Fourth term} = \frac{4 \times 5}{2} = 10$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$\text{Tenth term} = \frac{10 \times 11}{2} = 55$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$n\text{th term} = \frac{n \times (n + 1)}{2} = \frac{n\,(n + 1)}{2}$$

The general term of the triangular number pattern starting from 1 and written in ascending order is $\frac{n\,(n + 1)}{2}$.

Exercise 1.4

(1) What is the 10th term of the square number pattern starting from 1 and written in ascending order?

(2) What is the 10th term of the triangular number pattern starting from 1 and written in ascending order?

(3) A certain number greater than 1 and less than 50, which is a term of the square number pattern starting from 1 and written in ascending order, is also a term of the triangular number pattern starting from 1 and written in ascending order.

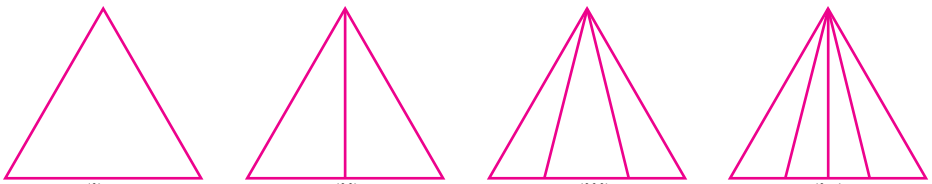
(i) What is this term?

(ii) Which square number is it?

(iii) Which triangular number is it?

(4) "The sum of the 14th and 15th terms of the triangular number pattern starting from 1 is a square number". Show that this statement is true and find which term it is of the square number pattern.

(5) Write the total number of triangles in each figure in order and see whether you can identify the pattern.



(i) (ii) (iii) (iv)

The pattern of the total number of triangles in the figures in the given order, is identical to the pattern of triangular numbers starting from 1 and written in ascending order. Find the total number of triangles in the 8th figure that is drawn according to this pattern.

(6) Sayuni buys a till and starts saving money by putting one rupee into it on the first day. On the second day she puts 2 rupees, on the third day 3 rupees and so forth. How much money is in the till by the end of the 10th day?

Miscellaneous Exercise

(1) In the pattern of the odd numbers starting from 1, commencing from the first term, if the first two terms, then the first three terms, then the first four terms are added and continued accordingly, a special type of numbers is obtained.

(i) What is the special name given to these numbers?

(ii) Find the number that is obtained if 15 of these terms are added in order starting from the first term.

(2) Milk tins brought to a shop to be sold were arranged on a rack in the following manner.

- 10 tins on the lowest shelf and every other shelf having one tin less than the number on the shelf below it.1 tin on the topmost shelf.

(i) Find the number of milk tins that were brought to the shop.

(ii) All the milk tins on the four topmost shelves were sold within two weeks. Find the number of milk tins that were sold.

(3) What is the sum of the integers from 1 to 30?

What is the difference between a set of numbers and a number pattern?

The pattern of the even numbers between 1 and 9 written in ascending order is 2, 4, 6, 8.

If these four numbers are written in descending order as 8, 6, 4, 2, we obtain a different number pattern.
A is the set of even numbers between 1 and 9.

We can write the set of even numbers between 1 and 9 as follows.
$A = \{2, 4, 6, 8\} = \{6, 4, 8, 2\} = \{8, 6, 4, 2\}$

Whatever order the numbers 2, 4, 6, 8 are written within brackets, we obtain the same set. Elements of a set are not named as the first element, the second element, etc.

$\therefore$ Although $\{2, 4, 6, 8\}$ and $\{8, 6, 4, 2\}$ are the same set, the number pattern 2, 4, 6, 8 is not equal to the number pattern 8, 6, 4, 2.

Summary

- The expression in n obtained for the nth term of a number pattern is called its general term.
- The value of n in the general term of a number pattern should always be a positive integer.
- The general term of the number pattern of the even numbers starting from 2 and written in ascending order is $2n$.
- The general term of the number pattern of the odd numbers starting from 1 and written in ascending order is $2n - 1$.
- The general term of the pattern of the square numbers starting from 1 and written in ascending order is n^2.
- The general term of the triangular number pattern starting from 1 and written in ascending order is $\frac{n\,(n\,+\,1)}{2}$.

Think

(1) Can you construct three different number patterns with 1, 2, 4 as the first three terms?
If you can, then write the next two terms of each of those number patterns.