FWC Nov. 2016 | Combined Maths I | Gr : 13 (2017)

1. when $n=1$ $f(1)=1\in\mathbb{Z}^+$ (5)

$f(p)=\frac{p^3}{3}+\frac{2p}{3}=k\in\mathbb{Z}^+$ (5)

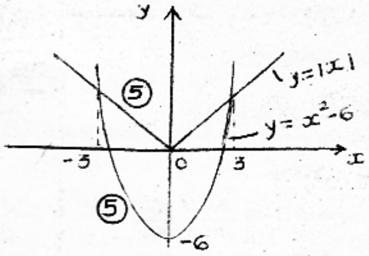
$f(p+1)=\frac{1}{3}(p^3+3p^2+3p+1)+\frac{2}{3}(p+1)$ (5)

$=k+p^2+p+1\in\mathbb{Z}^+$ (5)

$\therefore$ By the principle of mathematical induction, the result is true for all $n\in\mathbb{Z}^+$. (5)

$\boxed{25}$

2.



Points of intersection (5)

$x^2-6<|x|$ (5)

Solutions are $-3<x<3$ (5)

$\boxed{25}$

3. $x^5+kx^2\equiv(x-k)(x+k)\phi(x)+16x-8$ (5)

when $x=k$ $k^5+k^3=16k-8$ ---(1) (5)

when $x=-k$ $-k^5+k^3=-16k-8$ ---(2) (5)

(1),(2) $\Rightarrow$ $k^3=-8$ (5) $k=-2$ (5)

$\boxed{25}$

4. $g(x)=3(x^2+\frac{5}{3}x)+7$ (5)

$=3(x+\frac{5}{6})^2+\frac{59}{12}$ (5)

$(g(x))_{min}=\frac{59}{12}$ (5)

$\left(\frac{1}{g(x)}\right)_{min}=\frac{12}{59}$ (5) (5)

$\boxed{25}$

5. $\lim\limits_{x\to 0}\frac{\sin x(1-\cos x)}{x^2\cos x\sin 5x}$ (5)

$=\lim\limits_{x\to 0}\frac{\sin^3 x}{x^2\sin 5x\cos x(1+\cos x)}$ (5)

$=\lim\limits_{x\to 0}\frac{\left(\frac{\sin x}{x}\right)^3}{5\left(\frac{\sin 5x}{5x}\right)}$ (5) (5) $\cdot\lim\limits_{x\to 0}\frac{1}{\cos x(1+\cos x)}$

$=\frac{1^3}{5\cdot 1}\cdot\frac{1}{1(1+1)}$ (5)

$\boxed{25}$

6. $\frac{dx}{d\theta}=3\sec^3\theta\tan\theta$ (5)

$\frac{dy}{d\theta}=3\tan^2\theta\sec^2\theta$ (5)

$\left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{4}}=\sqrt{2}$ (5)

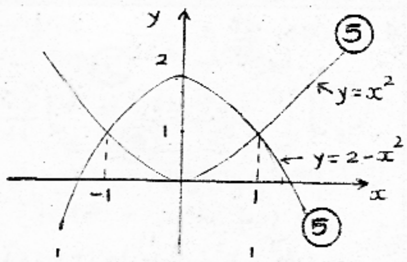
Equation of the tangent is

$y-\tan^3\frac{\pi}{4}=\sqrt{2}\left(x-\sec^3\frac{\pi}{4}\right)$ (5)

$y-1=\sqrt{2}(x-2\sqrt{2})$ (5)

$\boxed{25}$

7.



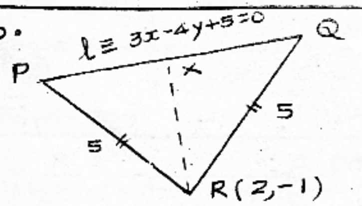
$A=\int_{-1}^{1}(2-x^2)\,dx-\int_{-1}^{1}x^2\,dx$ (5)

$=2\left(x-\frac{x^3}{3}\right)_{-1}^{1}$ (5)

$=2\left(\left(1-\frac{1}{3}\right)-\left(-1+\frac{1}{3}\right)\right)$ (5)

$\boxed{25}$

8.



$RX=\frac{|3\cdot 2-4(-1)+5|}{\sqrt{3^2+4^2}}$ (5)

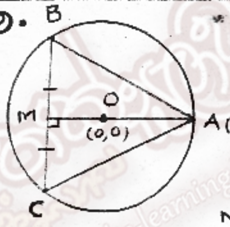
$=3$ units (5)

$QX=4$ units (5)

Area of $\triangle PQR=\frac{1}{2}\times\overbrace{(2\times 4)}^{(5)}\times 3$ (5)

$\boxed{25}$

9.



AM is altitude of $\triangle ABC$. (5)

$MO:OA=1:2$ (5)

$M\equiv(x_m,y_m)$

$\frac{2x_m+1\cdot(a)}{2+1}=0$ (5) $x_m=-\frac{a}{2}$, $y_m=0$ (5)

Equation of BC is $x=-\frac{a}{2}$ (5)

$\boxed{25}$

10. $2\cos\frac{3x}{2}\sin\frac{x}{2}=2\sin\frac{3x}{2}\sin\frac{x}{2}$ (10)

$\sin\frac{x}{2}\left(\cos\frac{3x}{2}-\sin\frac{3x}{2}\right)=0$ (5)

$\frac{x}{2}=n_1\pi$ $\frac{3x}{2}=n_2\pi+\frac{\pi}{4}$ (5)

$x=\frac{\pi}{6}$ (5)

$\boxed{25}$

11. (a) $(\lambda+1)\alpha=-\frac{b}{a}$ (5)

$\lambda\alpha^2=\frac{c}{a}$ (5)

$\frac{(\lambda+1)^2\alpha^2}{\lambda\alpha^2}=\frac{\left(-\frac{b}{a}\right)^2}{\frac{c}{a}}$ (10)

$ac(\lambda^2+2\lambda+1)=\lambda b^2$ (5)

$(ac)\frac{9}{16}+(2ac-b^2)\frac{3}{4}+ac=0$ (5)

$9ac+24ac-12b^2+16ac=0$ (10)

$12b^2=49ac$ (5)

$12(49k)^2=49(49)(k-1)(48)$ (10)

$k^2-4k+4=0$ (5)

$k=2$ (5)

$\boxed{65}$

(b) $\Delta=p^2-4(p-q-r)(q+r)$ (10)

$=p^2+4q^2+4r^2-4pq-4pr+8qr$

$=(p-2q-2r)^2$ (5) (10)

If the roots are coincided,

$\Delta=0$ (5)

$p-2q-2r=0$ (5)

$\boxed{35}$

(c) $4\left(\frac{1}{2}\right)^3+2\left(\frac{1}{2}\right)^2+a\left(\frac{1}{2}\right)+1=0$ (10)

$a=-4$ (5)

$4x^3+2x^2-4x+1\equiv(x+4)(x-1)\phi(x)+Ax+B$ (10)

$x=1\Rightarrow 3=A+B$ (5)

$x=-4\Rightarrow -207=-4A+B$ (5)

$A=42$ (5), $B=-39$ (5)

Remainder is $42x-39$. (5)

$\boxed{50}$

12. (a) Let $E=\frac{1}{x^2-5x+9}$. (5)

$Ex^2-5Ex+9E-1=0$ (5)

$\Delta\geq 0$ (5)

$25E^2-4E(9E-1)\geq 0$ (10)

$E(11E-4)\leq 0$ (10)

$+++\;---\;+++$; 0 ; $\frac{4}{11}$ (10) $E\neq 0$ (5)

$\boxed{50}$

(b)(i) $\frac{a^2+b^2-2ab}{ab}$ (5)

$=\frac{(a-b)^2}{ab}\geq 0$ (5)

$\Rightarrow \frac{a}{b}+\frac{b}{a}-2\geq 0$ (5)

(ii) $(a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$

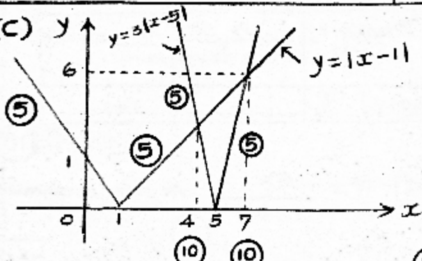
$=1+\frac{a}{b}+\frac{a}{c}+\frac{b}{a}+1+\frac{b}{c}+\frac{c}{a}+\frac{c}{b}+1$ (10)

$=3+\left(\frac{a}{b}+\frac{b}{a}\right)+\left(\frac{a}{c}+\frac{c}{a}\right)+\left(\frac{b}{c}+\frac{c}{b}\right)$ (10)

$\geq 3+2+2+2$ (10)

$\boxed{45}$

(c)



$\therefore$ Set of values of x is $\{x: x\in\mathbb{R}, 4<x<7\}$.

$\boxed{55}$

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13. (a) $f'(x) = \dfrac{(x^2-5x+4)\cdot 1 - x(2x-5)}{(x-1)^2(x-4)^2}$ (10)

At the turning points $f'(x)=0$ (5)

$\Rightarrow x = -2, 2$ (5)

$x=1$ and $x=4$ are vertical asymptotes. (5)

Range of x	$-\infty<x<-2$	$-2<x<1$	$1<x<2$	$2<x<4$	$4<x<\infty$
Sign of dy/dx	$(-)$	$(+)$	$(+)$	$(-)$	$(-)$

(15)

Minimum at $x=-2$ (5)

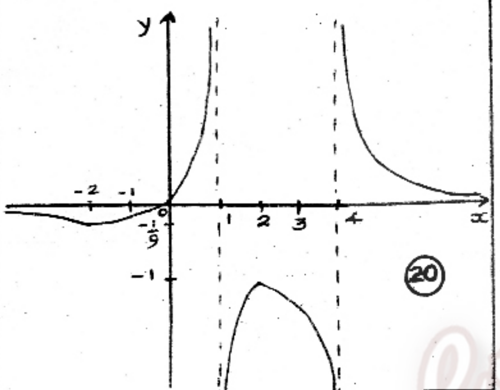
Minimum point $\equiv (-2, -\frac{1}{9})$ (5)

Maximum at $x=2$ (5)

Maximum point $\equiv (2,-1)$ (5)

$x=0 \Rightarrow y=0$ $(0,0)$ (5)

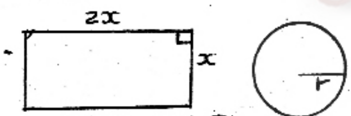
$x\to\pm\infty \Rightarrow y\to 0$ (5)



(20)

90

(b)



$6x + 2\pi r = 8\pi$ (10)

$x = \dfrac{4\pi - \pi r}{3}$ (5)

$A = 2x^2 + \pi r^2$ (5)

$= 2\left(\dfrac{4\pi-\pi r}{3}\right)^2 + \pi r^2$ (5)

$= \dfrac{\pi}{9}\left\{2\pi(16-8r+r^2)+9r^2\right\}$ (5)

$\dfrac{dA}{dr} = \dfrac{\pi}{9}\left\{(2\pi+9)2r - 16\pi\right\}$ (5)

For maximum or minimum $\dfrac{dA}{dr}=0$ (5)

$\Rightarrow r = \dfrac{8\pi}{2\pi+9}$ (5)

Range of x	$0<r<\frac{8\pi}{2\pi+9}$	$\frac{8\pi}{2\pi+9}<r<8\pi$
Sign of $\frac{dy}{dr}$	$(-)$	$(+)$

(10)

$\therefore$ At $r = \dfrac{8\pi}{2\pi+9}$ A is minimum. (5)

60

14. (a) $\dfrac{dt}{dx} = \dfrac{1}{4t^3}$ (5) $t: 0\to 2$ (5)

$\displaystyle\int_0^2 \frac{4t^3\,dt}{1+t}$ (5) $= 4\displaystyle\int_0^2 \frac{(t^3+1)-1}{1+t}dt$ (5)

$= 4\displaystyle\int_0^2 \left(t^2-t+1-\frac{1}{t+1}\right)dt$ (10)

$= 4\left(\dfrac{t^3}{3}-\dfrac{t^2}{2}+t-\ln|t+1|\right)$ (10)

$= \dfrac{32}{3} - 4\ln 3$ (10)

50

(b) $-x^2\cos x + \int \cos x \cdot 2x\,dx$ (15)

$= -x^2\cos x + 2\left(x\sin x - \int \sin x\,dx\right)$ (10)

$= -x^2\cos x + 2x\sin x + 2\cos x + C$ (5) (5)

35

(c) $\dfrac{1}{(x-1)(x+1)(x^2+4)} = \dfrac{A}{x-1} + \dfrac{B}{x+1} + \dfrac{Cx+D}{x^2+4}$ (15)

$\Rightarrow A = \frac{1}{10}$ $B = -\frac{1}{10}$ $C=0$

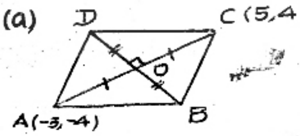
$D = -\frac{2}{10}$ (20)

$\displaystyle\int \frac{dx}{(x-1)(x+1)(x^2+4)} = \frac{1}{10}\int\frac{dx}{x-1} - \frac{1}{10}\int\frac{dx}{x+1} - \frac{2}{10}\int\frac{dx}{x^2+4}$ (5)

$= \dfrac{1}{10}\ln\left|\dfrac{x-1}{x+1}\right| - \dfrac{1}{10}\tan^{-1}\dfrac{x}{2} + C$ (15) (5) (5)

65

15. (a)



$m_{AC} = 1$ (5) $O \equiv (1,0)$ (10)

$m_{BD} = -1$ (5)

$BD: x+y-1=0$ (10)

$BC: 2x-y-6=0$ (10)

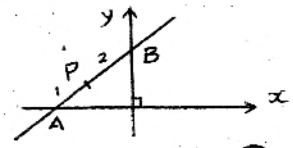
$B \equiv \left(\frac{7}{3}, -\frac{4}{3}\right)$ (10) $D \equiv \left(-\frac{1}{3}, \frac{4}{3}\right)$ (10)

Area of $ABCD = 2\times\frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ -3 & 5 & \frac{7}{3} \\ -4 & 4 & -\frac{4}{3}\end{vmatrix}$ (10)

$= \dfrac{64}{3}$ sq. un (5)

75

(b)



$AB: y-1 = m(x-1)$ (10)

$A \equiv \left(\dfrac{m-1}{m}, 0\right)$ (10) $B \equiv (0, 1-m)$ (10)

$P \equiv \left(\dfrac{2(m-1)}{3m}, \dfrac{1-m}{3}\right)$ (20)

$P \equiv (x_0, y_0)$

$x_0 = \dfrac{2(1-3y_0-1)}{3(1-3y_0)}$ (10)

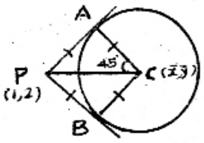
$x_0 - 3x_0y_0 + 2y_0 = 0$ (10)

$x_0 \to x$, $y_0 \to y$ (5)

75

16. Circle $x^2+y^2-a^2+m(y-lx)=0$ bisects the circumference of circle $x^2+y^2=a^2$. (45)

$S \equiv x^2+y^2-\frac{5}{3}+m(y-lx)=0$



PACB is a square (10)

$A\hat{C}P = 45^\circ$ (5)

$\cos 45 = \dfrac{AC}{PC}$ (10) $PC^2 = 2AC^2$ (10)

$(\bar{x}-1)^2 + (\bar{y}-2)^2 = 2\left(\bar{x}^2+\bar{y}^2+\frac{5}{3}\right)$ (20)

$3\bar{x}^2 + 3\bar{y}^2 + 6\bar{x} + 12\bar{y} - 5 = 0$ (25)

$\bar{x}\to x$, $\bar{y}\to y$ (10)

150

17. (a) (i) $2\sin 4\theta\cos 3\theta - \sin 4\theta = 0$

$\sin 4\theta(2\cos 3\theta - 1) = 0$ (10)

$\theta = \dfrac{n_1\pi}{4}$ (5), $\theta = \dfrac{2n_2\pi}{3} \pm \dfrac{\pi}{9}$ (5)

(ii) $\cos 2\theta - \sin 2\theta = \sqrt{2}\left((\cos 2\theta - \sin 2\theta)(\cos 2\theta + \sin 2\theta)\right\}$

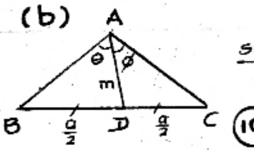
$(\cos 2\theta - \sin 2\theta)\left(1 - \sqrt{2}(\cos 2\theta + \sin 2\theta)\right) = 0$ (10)

$\theta = \dfrac{n_1\pi}{2} + \dfrac{\pi}{8}$ (10) $\cos\left(2\theta - \frac{\pi}{4}\right) = \frac{1}{2}$ (10)

$\theta = n_2\pi + \frac{\pi}{8} \pm \frac{\pi}{6}$ (5)

55

(b)



(10)

$\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c} = k$ (5)

By sin rule in ΔABD; in ΔADC

$\dfrac{a/2}{\sin\theta} = \dfrac{m}{\sin B}$ (10) $\dfrac{a/2}{\sin\phi} = \dfrac{m}{\sin C}$ (10)

$\Rightarrow a(bk - ck) = 2m\left\{2\cos\left(\dfrac{\theta+\phi}{2}\right)\sin\left(\dfrac{\theta-\phi}{2}\right)\right\}$ (10)

$2\sin\frac{A}{2}\cos\frac{A}{2}(b-c) = 4m\cos\frac{A}{2}\sin\left(\frac{\theta-\phi}{2}\right)$ (10)

55

(c) Let $\alpha = \tan^{-1}\frac{1}{2}$, $\beta = \tan^{-1}\frac{1}{3}$, $\gamma = \sin^{-1}x$. (5)

$\tan(\alpha-\beta) = \tan\gamma$ (5)

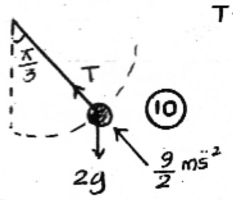
$\dfrac{1/2 - 1/3}{1 + 1/2\cdot 1/3} = \tan\gamma$ (10)

$\tan\gamma = 1/7$ (5) $\Rightarrow \sin\gamma = 1/5\sqrt{2}$ (10)

$x = 1/5\sqrt{2}$ (5)

40

1.



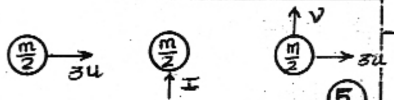
(10)

$T - 2g\cos 60^\circ = 2 \times \frac{9}{2}$ (10)

$T = 19N$ (5)

25

2.



(5)

$v = 4u$ (5)

$\uparrow I = \Delta mv$

$I = \frac{m}{2}v$ (10)

$= 2mu$ (5)

25

3. $y = \sqrt{3}x - \frac{4gx^2}{8ag}$ (10)

when $x = \sqrt{3}a$ $y = \frac{3a}{2} > a$ (5)

when $x = 2\sqrt{3}a$ $y = 0$ (5)

$\therefore$ The particle will hit on CD but not on DE. (5)

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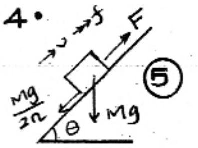
4.



(5)

$F - Mg\sin\theta - \frac{Mg}{2n} = Mf$ (10)

$F = M\left(f + \frac{5Mg}{2n}\right)$ (5)

$\text{Power} = Mv\left(f + \frac{5Mg}{2n}\right)$ (5)

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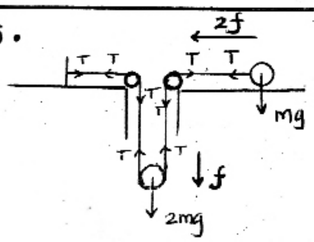
5.



(5)

(5)

$T = m \times 2f$ (5) $2mg - 2T = 2mf$

$f_P = \frac{2g}{3}$ (5) $f_Q = \frac{g}{3}$ (5)

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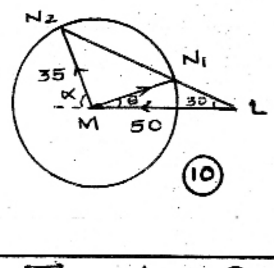
6. $V_{BS} = V_{BE} + V_{ES}$ (5)

30° 35 $\leftarrow 50$



$\theta = \sin^{-1}\frac{5}{7} - 30^\circ$ (5)

$\alpha = \sin^{-1}\frac{5}{7} + 30^\circ$ (5)

(10)

25

7. $\overrightarrow{PQ} = 2\mathbf{b} - \mathbf{a}$ (5)

$\overrightarrow{QR} = 3(\mathbf{a} - 2\mathbf{b})$ (5)

$\overrightarrow{RQ} = 3\,\overrightarrow{PQ}$ (5)

$\therefore$ PQ // PQ (5)

$\therefore$ P, Q and R are on the same straight line. (5)

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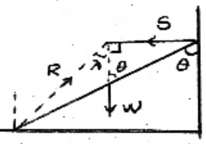
8.



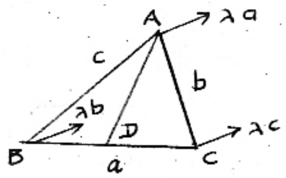
(5)

$2\cot\theta = \cot\lambda - \cot 90$ (10)

$\tan\theta = 2\mu$ (10)

25

9.



(5)

λB at B $+ \lambda C$ at C $= \lambda(b+c)$ at D (5)

AD is a bisector of A. (5)

Resultant of $\lambda a, \lambda b, \lambda c$ act at a point on the angular bisector of A. (5)

$\Rightarrow$ Incentre of ΔABC (5)

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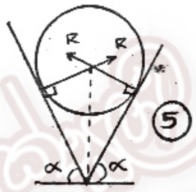
10.



(5)

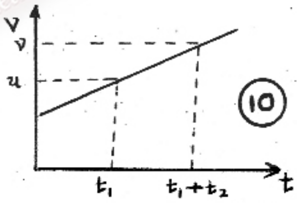
$\uparrow$ $2R\cos\alpha = W$ (5)

$\frac{W}{2}\sec\alpha = W$ (5)

$\alpha = 60^\circ$ (10)

25

11. (a)



(10)

$\frac{v-u}{t_1} = g\sin\alpha$ (10), $\left(\frac{v+u}{2}\right)t_1 = s$ (10)

$v = \frac{s}{t_1} + \frac{gt_1\sin\alpha}{2}$ (5), $u = \frac{s}{t_1} - \frac{gt_1\sin\alpha}{2}$ (5)

In the motion of B–C,

$v = \frac{s}{t_2} - \frac{gt_2\sin\alpha}{2}$ (10)

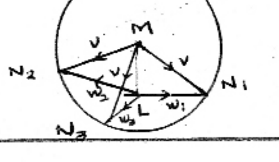
Equating v, $\sin\alpha = \frac{2s(t_1 - t_2)}{gt_1t_2(t_1+t_2)}$ (10)

60

(b) $V_{ME} = V_{MW} + V_{WE}$

B→C	→	v	↑ u
C→A	60°	v	↑ u
A→C	60°	v	↑ u

(20)



(30)

$w_1 = v^2 - u^2$ (10)

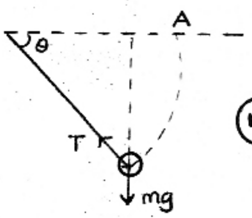
$w_2 = v^2 - u^2\sin^2 30 + u\cos 30$ (10)

$w_3 = v^2 - u^2\sin^2 30 - u\cos 30$ (10)

$T = \frac{a}{\sqrt{v^2 - u^2}} + 2\frac{\sqrt{v^2 - u^2\sin^2 30}\; a}{v^2 - u^2}$ (10)

90

12.



(10)

By the principle of conservation energy

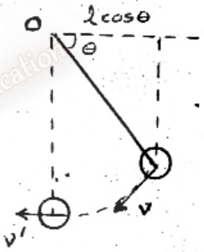
$0 = \frac{1}{2}m(a\dot{\theta})^2 - mga\sin\theta$ (10)

$a\dot{\theta}^2 = 2g\sin\theta$ (5)

$a\ddot{\theta} = g\cos\theta$ (10)

Acceleration of the particle is

$g\sqrt{1 + 3\sin^2\theta}$ (10)



$v = \sqrt{2gl\sin\theta}$ (5)

$w = \sqrt{2gl\sin\theta}\,\cos\theta$ (10)

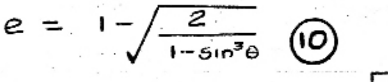
By the principle of conservation energy

$-mgl\sin\theta + \frac{1}{2}mw^2 = -mgl + \frac{1}{2}mv'^2$ (15)

$v' = \sqrt{2gl(1 - \sin^3\theta)}$ (10)

$T - mg = \frac{mv'^2}{l}$ (5)

$T = mg(3 - 2\sin^3\theta)$ (5)



$mv' = mv'' + mu'$, $u' - v'' = ev'$ (10)

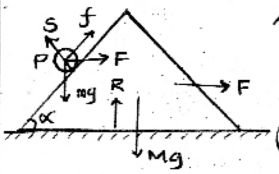
(10) $v'' = \frac{v'}{2}(1 - e)$ (10)

$\frac{1}{2}mv''^2 = mg\frac{l}{2}$ (10)

$\Rightarrow e = 1 - \sqrt{\frac{2}{1 - \sin^3\theta}}$ (10)

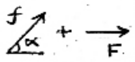
150

13.



(15)

$A_{PE} = A_{PW} + A_{WE}$



(15)

For the system $\rightarrow F = ma$

$0 = MF + m(F + f\cos\alpha)$ (25)

For the particle $\nearrow$

$-mg\sin\alpha = m(f + F\cos\alpha)$ (15)

$F = \dfrac{mg\sin\alpha\cos\alpha}{M + m\sin^2\alpha}$ (10)

$f = -\dfrac{(M+m)g\sin\alpha}{M + m\sin^2\alpha}$ (10)

For the particle relative to wedge

$\nearrow\ v^2 = u^2 + 2fs$

$0 = v^2 + 2fh\,\mathrm{cosec}\,\alpha$ (10)

$v = \sqrt{\dfrac{2(M+m)gh}{M + m\sin^2\alpha}}$ (10)

$\nearrow\ s = ut + \frac{1}{2}ft^2$

$0 = vt + \frac{1}{2}ft^2$ (10)

$t = -\dfrac{2v}{f}$ (10)

For the wedge $\rightarrow s = ut + \frac{1}{2}ft^2$

$s = \frac{1}{2}Ft^2$ (10)

$= \dfrac{4mh\cot\alpha}{M+m}$ (10)

150

14.(a) $y = x\tan\theta - \dfrac{gx^2}{2v^2\cos^2\theta}$ (30)

$b = a\tan\theta - \dfrac{ga^2}{2v^2\cos^2\theta}$ (10)

$ga^2\sec\theta = 2v^2(a\sin\theta - b\cos\theta)$ (10)

$v^2 = \dfrac{ga^2}{a\sin2\theta - b(1+\cos2\theta)}$ (10)

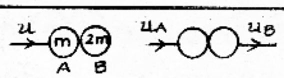
$= \dfrac{ga^2}{\sqrt{a^2+b^2}\left(\frac{a\sin2\theta}{\sqrt{a^2+b^2}} - \frac{b\cos2\theta}{\sqrt{a^2+b^2}}\right) - b}$ (10)

$= g\left(\sqrt{a^2+b^2} + b\right)$ (5)

$\tan2\theta = -\cot\alpha = -\dfrac{a}{b}$ (5)

80

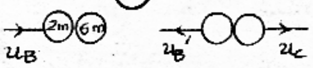
(b)



$2mu_B + mu_A = mu$ (10)

$u_B - u_A = \frac{2}{3}u$ (10)

$u_B = \dfrac{5u}{9}$ (10), $u_A = -\dfrac{u}{9}$ (5)



$6mu_C - 2mu_B' = 2mu_B$ (5)

$u_C + u_B' = eu_B$ (5)

$u_B' = \dfrac{u_B}{4}(3e-1)$ (5)

For A, B to collide again

$u_B' > \dfrac{u}{9}$ (10)

$\Rightarrow e > \dfrac{3}{5}$ (10)

70

15.(a) $d = \dfrac{b+2c}{3}$ (5)

$m = \dfrac{3a+b+2c}{6}$ (10)

$m = \dfrac{2n+c}{3}$ (5)

$n = \dfrac{3a+b}{4}$ (10)

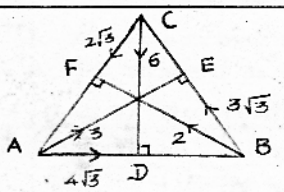
$\overrightarrow{AN} = \dfrac{b-a}{4}$ (5) $\overrightarrow{AB} = b - a$ (5)

$\therefore$ N lies on AB. (10)

$\dfrac{AN}{NB} = \dfrac{1}{3}$ (10)

60

(b)



(10)

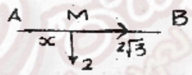
$\overrightarrow{AB}$ $X = 4\sqrt3 - 3\sqrt3\cos60 - 2\sqrt3\cos60 + 3\cos30 - 2\cos30$ (10)

$= 2\sqrt3$ (5)

$CD\downarrow$ $Y = 6 - 3\sqrt3\sin60 + 2\sqrt3\sin60 - 3\sin30 - 2\sin30$ (10)

$= 2$ (5)

$R = 4$ (5) $\theta = 30°$ (5) $R \parallel FB$ (5)



$A\circlearrowleft$ $2x = -5a$ (10)

$x = \dfrac{5a}{2}$ from A (5)

in extended BA

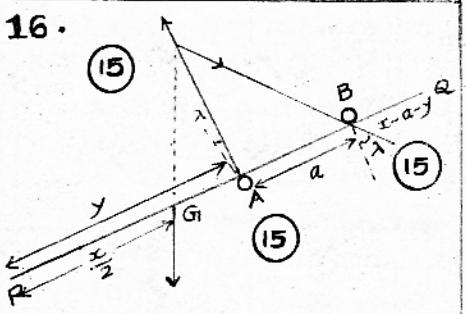
R at B = R at M + G $\circlearrowleft$

$G = R\cdot\dfrac{9a}{2}\sin30$ (10)

$= 9a$ $\circlearrowleft$ (10)

90

16.



(15) (15) (15)

$\left(a + y - \frac{x}{2}\right)\tan\lambda = a\tan\alpha - \left(y - \frac{x}{2}\right)\tan\lambda$ (15)

$2y = x - a(1 - \tan\alpha\cot\lambda)$ (15)

$y \geq 0 \Rightarrow x \geq a(1 - \tan\alpha\cot\lambda)$ (15)

$y \leq x - a$ (15)

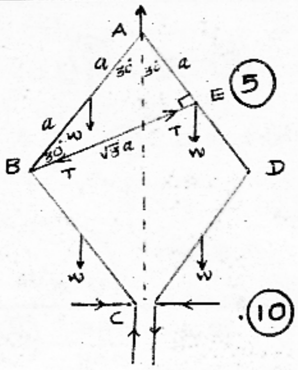
$\dfrac{x - a(1-\tan\alpha\cot\lambda)}{2} \leq x - a$ (15)

$x \geq a(1 + \tan\alpha\cot\lambda)$ (15)

$\therefore$ Minimum length of the rod is $a(1 + \tan\alpha\cot\lambda)$. (15)

150

17.(a)



(5) (10)

BC : B$\circlearrowleft$ $X\cdot2a\cos30 + Y\cdot2a\sin30 = W\cdot a\sin30$ (10)

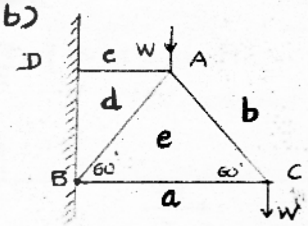
AB+BC : A$\circlearrowleft$ $X\cdot4a\cos30 + 2W\cdot a\sin30 = T\cdot a$ (15)

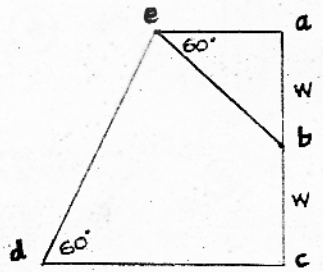
$X = \dfrac{T-W}{2\sqrt3}$, $Y = \dfrac{2W-T}{2}$ (10)

$\dfrac{1}{\sqrt3} = \dfrac{(2W-T)/2}{(T-W)/2\sqrt3}$ (10) $T = \dfrac{7W}{4}$ (10)

70

(b)





(20)

Rod	Tension	Thrust
AB	–	$4W/\sqrt3$
BC	–	$W/\sqrt3$
CA	$2W/\sqrt3$	–
AD	$\sqrt3\,W$	–

(40)

Rod	New Tension	New Thrust
AB	–	$2W/\sqrt3$
AD	$2W/\sqrt3$	–

(20)

80