01. $f(n) = 7^n$

$f(1) = 7 = 6\times1+1$ (5)

$f(p) = 7^p = 6k+1,\ k\in\mathbb{Z}^+$ (5)

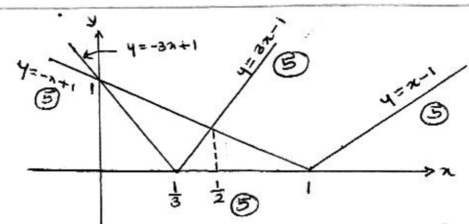
$f(p+1) = 7(6k+1)$ (5)

$= 6(7k+1)+1,\ 7k+1\in\mathbb{Z}^+$ (5)

Conclusion (5)

$\boxed{25}$

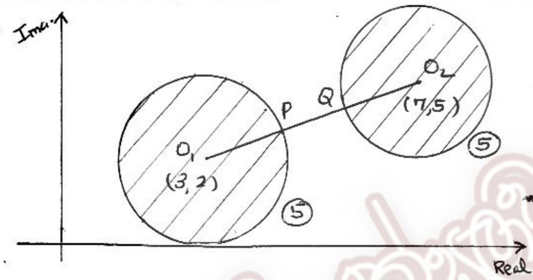
02.



Required solution $\{x \mid 0<x<\frac{1}{2}\}$ (5)

$\boxed{25}$

03.



$|z-\omega|_{min} = PQ$ (5)

$= 5-3$ (5)

$= 2$ (5)

$\boxed{25}$

04. $f(x) = k(x^2-4kx+4k^2)+k^2-2$ (5)

$= k(x-2k)^2+k^2-2$ (5)

$= a(x-b)^2+c$; where $a=k,\ b=2k,\ c=k^2-2$ (5)

$k>0$ and $f(x)|_{min} = k^2-2>2$ (5)

$\therefore k > 2$ (5)

$\boxed{25}$

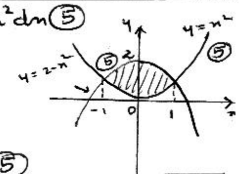
05. $$\lim_{x\to0}\frac{1+\sin4x-(\sin^2x+\cos^2x)}{x\{\sqrt{1+\sin4x-\sin^2x}+\cos x\}}$$ (10)

$$= 4\lim_{4x\to0}\frac{\sin4x}{4x}\cdot\lim_{x\to0}\frac{1}{\{\sqrt{1+\sin4x-\sin^2x}+\cos x\}}$$ (5) (5)

$$= 4\cdot1\cdot\frac{1}{\{\sqrt{1}+1\}}$$ (5)

$\boxed{25}$

06. $A = \int_{-1}^{1}(2-x^2)\,dx - \int_{-1}^{1}x^2\,dx$ (5)



$= \left\{2x-\frac{2x^3}{3}\right\}_{-1}^{1}$ (5)

$= \left(2-\frac{2}{3}\right)-\left(-2+\frac{2}{3}\right)$ (5)

$\boxed{25}$

07. $\frac{dx}{d\theta} = 3a\sec^3\theta\tan\theta$ (5)

$\frac{dy}{d\theta} = 3a\tan^2\theta\sec^2\theta$ (5)

$\frac{dy}{d\theta} = \frac{\tan\theta}{\sec\theta}$ (5)

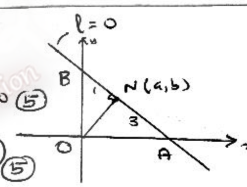
Equation of the tangent

$y-a\tan^3\alpha = \frac{\tan\alpha}{\sec\alpha}(x-a\sec^3\alpha)$ (5)

$x\tan\alpha - y\sec\alpha = a\sec\alpha\tan\alpha$ (5)

$\boxed{25}$

08. $M_{AB} = -a/b$ (5)



$l \equiv ax+by-(a^2+b^2)=0$ (5)

$A \equiv \left(\frac{a^2+b^2}{a},0\right),\ B\equiv\left(0,\frac{a^2+b^2}{b}\right)$ (5)

$AN:NB = 3:1$ (5)

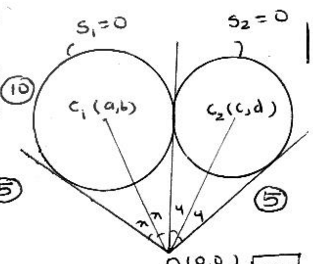
$a = \frac{3(0)+1\left(\frac{a^2+b^2}{a}\right)}{4}$ (5)

$\boxed{25}$

09. $x+y = 45^\circ$ (5)

$\tan C_1\hat{O}C_2 = \left|\frac{b/a-d/c}{1+b/a\cdot d/c}\right|$ (10)

$|bc-ad| = |ac+bd|$ (5)



$\boxed{25}$

10. $\cot A = -3/4$ (5)

$\cos A = -3/5$ (5)

$\sin A = 4/5$ (5)

$2\cot A - 5\cos A + \sin A$

$= 2(-3/4) - 5(-3/5) + 4/5$ (5)

$= \frac{23}{10}$ (5)

$\boxed{25}$

11. (a) (i) $P(-2)=0$ (5) $P(2)=-48$ (5)

$2a-b=5$ (5) $2a+b=-1$ (5)

$\Rightarrow a=1,\ b=-3$ (10)

(ii) $P(x) = (x+2)(x-6)(x+1)$ (10)

$\boxed{40}$

(b) $y = \dfrac{x^2+34x-71}{x^2+2x-7}$ (5)

$(y-1)x^2 + 2(y-17)x - 7y+71 = 0$ (5)

If $y=1$, $x=2$ (5)

Let $y \neq 1$, For all real values of x $\Delta \geq 0$ (5)

$\Rightarrow (y-5)(y-9) \geq 0$ (15)

$y \leq 5$ or $y \geq 9$ (5)

$\boxed{40}$

(c) (i) $\alpha+\beta=b$, $\alpha\beta=c$ (10)

$\dfrac{\alpha^2}{\beta} + \dfrac{\beta^2}{\alpha} = \dfrac{b(b^2-3c)}{c}$ (15)

$\dfrac{\alpha^2}{\beta} \cdot \dfrac{\beta^2}{\alpha} = c$ (10)

$\therefore$ Required equation $cx^2 - b(b^2-3c)x + c^2 = 0$ (5)

(ii) $\alpha+\alpha^2=b$, $\alpha^2-b\alpha=-c$ (10)

$\dfrac{b+c}{b+1} + \left(\dfrac{b+c}{b+1}\right)^2 = b$ (10)

$\Rightarrow b^3 = c(3b+c+1)$ (10)

$\boxed{70}$

12. (a) (i) ${}^9C_3 = 84$ (15)

(ii) ${}^5C_3 \cdot {}^4C_3 + {}^5C_3 \cdot {}^3C_3 + {}^4C_3 \cdot {}^3C_3 +$
${}^5C_3 \cdot {}^4C_2 \cdot {}^3C_1 + {}^5C_3 \cdot {}^4C_1 \cdot {}^3C_2 +$
${}^5C_2 \cdot {}^4C_3 \cdot {}^3C_1 + {}^5C_1 \cdot {}^4C_3 \cdot {}^3C_2 + {}^5C_1 \cdot {}^4C_2 \cdot {}^3C_3$
${}^5C_2 \cdot {}^4C_1 \cdot {}^3C_3 + {}^5C_2 \cdot {}^4C_2 \cdot {}^3C_2 = 784$ (60)

$\boxed{75}$

(b) If n is even Let $n=2m$

$S = 1^3 + 3\cdot 2^2 + 3^3 + \cdots + (2m-1)^3 + 3(2m)^2$ (5)

$= (1^3 + 3^3 + \cdots + (2m-1)^3) + 3(2^2+4^2+\cdots(2m)^2)$ (5)

$= \sum_{r=1}^{m}(2r-1)^3 + 3\cdot 4\sum_{r=1}^{m} r^2$ (10)

$= 8\sum_{r=1}^{m} r^3 + 6\sum_{r=1}^{m} r - \sum_{r=1}^{m} 1$ (10)

$= 8\,\dfrac{m^2(m+1)^2}{4} + 6\,\dfrac{m(m+1)}{2} - m$ (15)

$= \dfrac{n}{8}(n^3+4n^2+10n+8)$ (10)

If n is odd, $(n+1)$ is even

$S_n = S_{n+1} - T_{n+1}$ (10)

$= \dfrac{(n+1)}{8}\left[(n+1)^3 + 4(n+1)^2 + 10(n+1) + 8\right]$

$- 3(n+1)^2$ (10)

$\boxed{75}$

13. (i) (a) $|z_1| = \sqrt{26}$ (5) $|z_2| = \sqrt{13}$ (5)

$\therefore |z_1|^2 = 2|z_2|^2$ (5)

(b) $z_1 \cdot z_2 = -13 + 13i$ (5)

$= 13\sqrt{3}\left(\cos 3\pi/4 + i\sin 3\pi/4\right)$ (10)

$\therefore \arg(z_1 \cdot z_2) = 3\pi/4$ (5)

$\boxed{35}$

(ii) Let $\sqrt{16-30i} = x+iy$ (5)

$16-30i = x^2-y^2+2xyi$ (5)

$\Rightarrow x = -15/y$ (5)

$x^2+y^2 = 16$ (5)

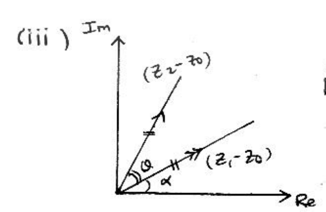
$\Rightarrow y = \pm 3$ (5)

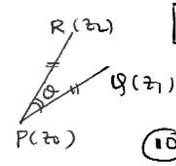
$\Rightarrow x = \mp 5$ (10)

$\therefore \sqrt{16-30i} = -5+3i,\ 5-3i$ (5)

$\boxed{40}$

(iii)





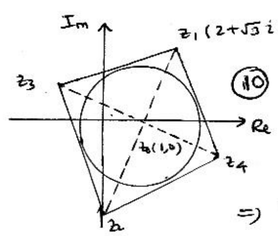
(10)

Let $z_1-z_0 = r(\cos\alpha + i\sin\alpha)$

$(z_2-z_0) = r[\cos(\theta+\alpha) + i\sin(\theta+\alpha)]$ (5)

$= (z_1-z_0)(\cos\theta + i\sin\theta)$ (10)

$\boxed{25}$



(10)

$z_2 = -\sqrt{3}i$ (10)

$(z_3-z_0) = (z_1-z_0)(\cos\pi/2 + i\sin\pi/2)$ (10)

$\Rightarrow z_3 = 1-\sqrt{3}i$ (5)

$(z_4-z_0) = (z_2-z_0)(\cos\pi/2 + i\sin\pi/2)$ (10)

$\Rightarrow z_4 = (1+\sqrt{3}) - i$ (5)

$\boxed{50}$

14. (a) $\frac{dy}{dx} = e^{\tan^{-1}x} \cdot \frac{1}{(1+x^2)} + e^{-\tan^{-1}x}\left(\frac{-1}{1+x^2}\right)$ (10)

$(1+x^2)\frac{dy}{dx} = e^{\tan^{-1}x} - e^{-\tan^{-1}x}$ (5)

$(1+x^2)\frac{d^2y}{dx^2} + \frac{dy}{dx}\cdot 2x = e^{\tan^{-1}x}\cdot\frac{1}{1+x^2} - e^{-\tan^{-1}x}\left(\frac{-1}{1+x^2}\right)$ (10)

$\boxed{25}$

(b) $f'(x) = \frac{(x^2-5x+4) - x(2x-5)}{(x^2-5x+4)^2}$ (10)

At the turning point $f'(x) = 0$ (5)

$\Rightarrow x = 2$ or $x = -2$ (5)

$x = -2-\delta \Rightarrow \frac{dy}{dx} < 0$ minimum point

$x = -2+\delta \Rightarrow \frac{dy}{dx} > 0$ (5) $(-2, -1/9)$ (5)

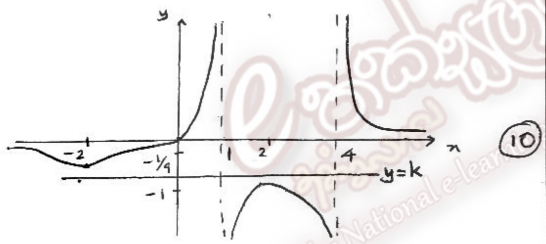
$x = 2-\delta \Rightarrow \frac{dy}{dx} > 0$ maximum point

$x = 2+\delta \Rightarrow \frac{dy}{dx} < 0$ (5) $(2, -1)$ (5)

$x = 1-\delta \Rightarrow y \to \infty$ (5) $x = 4-\delta \Rightarrow y \to -\infty$

$x = 1+\delta \Rightarrow y \to -\infty$ $x = 4+\delta \Rightarrow y \to \infty$ (5)

$x \to \pm\infty \Rightarrow y \to 0$ (5)



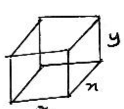
(10)

$\frac{x}{(x-1)(x-4)} = k$ (5)

If $-1 < k < -1/9$ both graphs will not intersect. (5)

$\boxed{75}$

(c)



$x^2 + 4xy = a^2$ (5)

$V = x^2 y$ (5)

$= \frac{1}{4}\{a^2 x - x^3\}$ (5)

$\frac{dV}{dx} = \frac{1}{4}\{a^2 - 3x^2\}$ (5)

For maximum or minimum $\frac{dV}{dx} = 0$ (5)

$\Rightarrow x = a/\sqrt{3}$ (5) ($\because x = -a/\sqrt{3}$ not valid) (5)

$x = a/\sqrt{3} - \delta \Rightarrow \frac{dV}{dx} > 0$ (5) At $x = a/\sqrt{3}$ V_{max} (5)

$x = a/\sqrt{3} + \delta \Rightarrow \frac{dV}{dx} < 0$

$\Rightarrow V_{max} = \frac{a^3}{6\sqrt{3}}$ (5)

$\boxed{50}$

15. (a) To show $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$ (15)

$I = \int_0^{\pi/2} \sin^4(\pi/2 - x)\cos^2(\pi/2 - x)\,dx$ (5)

$= \int_0^{\pi/2} \cos^4 x \sin^2 x\,dx$ (5)

$2I = I + I$

$= \int_0^{\pi/2} \sin^2 x \cos^2 x(\sin^2 x + \cos^2 x)\,dx$ (5)

$= \int_0^{\pi/2} \sin^2 x \cos^2 x\,dx$ (5)

$= \frac{1}{4}\int_0^{\pi/2} \sin^2 2x\,dx$ (5)

$= \frac{1}{4}\int_0^{\pi/2}\left(\frac{1-\cos 4x}{2}\right)dx$ (5)

$= \frac{1}{8}\left\{x - \frac{\sin 4x}{4}\right\}_0^{\pi/2}$ (5)

$= 1/8\left\{\pi/2 - \frac{\sin 2\pi}{4}\right\}$ (5)

$\boxed{55}$

(b) $t = x^2$ (5)

$\Rightarrow \int t^2 \cos t\,\frac{dt}{2}$ (5)

$= \frac{1}{2}\left\{t^2 \sin t - \int \sin t \cdot 2t\,dt\right\}$ (15)

$= \frac{1}{2}\, t^2 \sin t - \left\{t(-\cos t) - \int -\cos t\,dt\right\}$ (10)

$= \frac{1}{2}\, x^4 \sin x^2 + x^2 \cos x^2 - \sin x^2 + C$ (10)

$\boxed{45}$

(c) $\frac{x^2+2}{(x^2+1)(x^2+4)} = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{x^2+4}$ (10)

$\Rightarrow A = 0,\ B = \frac{1}{3},\ C = 0,\ D = \frac{2}{3}$ (20)

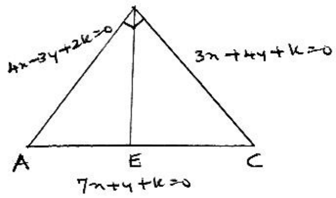
$\int \frac{x^2+2}{(x^2+1)(x^2+4)}\,dx = \int \frac{1/3}{(x^2+1)}\,dx + \int \frac{2/3}{(x^2+4)}\,dx$ (5)

$= \frac{1}{3}\tan^{-1}x + \frac{2}{3}\cdot\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C$ (15)

$\boxed{50}$

16. (a) To prove that $\dfrac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}} = \dfrac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}$ (20)

(i)



$M_{AB} = 4/3$
$M_{BC} = -3/4$ (5)
$M_{AB} \times M_{BC} = -1$ (5)

(ii) Equations of the bisectors

$$\frac{4x-3y+2k}{5} = \pm \frac{3x+4y+k}{5} \quad (10)$$

$x - 7y + k = 0$, $7x + y + 3k = 0$ (10)

(iii) Bisector of $A\hat{B}C$ $\quad x - 7y + k = 0$ (5)

$E \equiv (-4, 3)$ (10) $\quad k = 25$ (5)

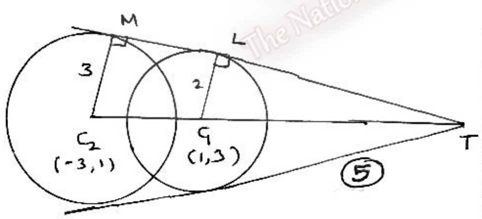
(iv) $D \equiv (3,4)$ (5)

$\boxed{75}$

(b) $C_1 \equiv (1,3)$ $\quad C_2 \equiv (-3,1)$
$r_1 = 2$ $\quad r_2 = 3$ (5)

$C_1C_2 = \sqrt{20}$ (5)

$3-2 < \sqrt{20} < 3+2$ (5)



(5)

$\dfrac{TC_1}{TC_2} = \dfrac{2}{3}$ (10) $\quad T \equiv (9,7)$ (10)

Let m be the gradient of tangent.
Equation of the tangent
$mx - y + 7 - 9m = 0$ (10)

Perpendicular distance = radius

$$\frac{|m(1) - 3 + 7 - 9m|}{\sqrt{m^2+1}} = 2 \quad (10)$$

$15m^2 - 16m + 3 = 0$ (10)

Equations $(8+\sqrt{19})x - 15y + 33 - 9\sqrt{19} = 0$ (5)
$(8-\sqrt{19})x - 15y + 33 + 9\sqrt{19} = 0$

17. (a) $\tan 3A = \dfrac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$ (15)

$\dfrac{\tan 3A}{\tan A} = \dfrac{3 - \tan^2 A}{1 - 3\tan^2 A} = k$ (5)

$\Rightarrow \tan^2 A = \dfrac{k-3}{3k-1}$ (5)

$\dfrac{\sin 3A}{\sin A} = \dfrac{3\sin A - 4\sin^3 A}{\sin A}$ (5)

$= 3 - 4/1+\cot^2 A$ (10)

$= \dfrac{2k}{k-1}$ (5)

$\boxed{45}$

(b) $\sin\alpha = x$, $\tan\beta = x$
$\cos(\alpha+\beta) = \cos \pi/2$ (5)

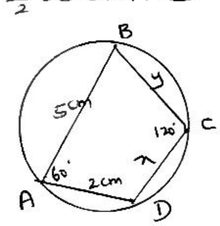
$\sqrt{1-x^2} \cdot \dfrac{1}{\sqrt{1+x^2}} = x \cdot \dfrac{x}{\sqrt{1+x^2}}$ (10)

$x^4 + x^2 - 1 = 0$ (5)

$x^2 = \dfrac{-1+\sqrt{5}}{2}$ (10)

$\boxed{30}$

(c) State cosine rule (5)
Prove $\quad$ Area $= \frac{1}{2} bc \sin A$ (10)



$\triangle ABD = \dfrac{5\sqrt{3}}{2}$ (10)

$\triangle BCD = \dfrac{3\sqrt{3}}{2}$ (5)

$\triangle BCD = \dfrac{\sqrt{3}}{4} xy$ (5)

$\therefore xy = 6$ (5)

In $\triangle ABD$ $\quad$ cos rule

$\frac{1}{2} = \dfrac{2^2 + 5^2 - BD^2}{2 \cdot 2 \cdot 5}$ (10)

$BD^2 = 19$ (5)

In $\triangle BCD$ $\quad -\frac{1}{2} = \dfrac{x^2+y^2-19}{2xy}$ (5) $\Rightarrow x^2+y^2 = 13$ (5)

$\Rightarrow x = 3, y = 2$
OR (10)
$x = 2, y = 3$

$\boxed{75}$