Grade 9

Reading Material

Mathematics

Unit 12

Indices

Indices

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By Learning this lesson you will be able to,

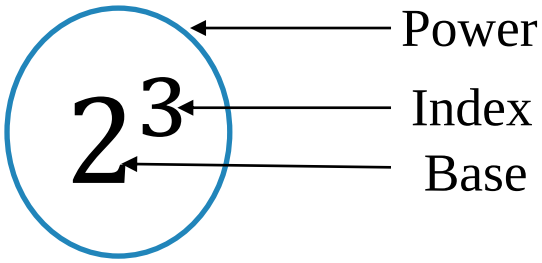
- identify the laws of indices on the product of powers, the quotient of powers and the power of a power
- simplify algebraic expressions using the above mentioned laws of indices
- identify the zero index and negative indices and simplify algebraic expressions containing these.

Introduction to power, base and index.

Let's write 8 as a product of prime numbers.

2 | 8
2 | 4
2 | 2
1

$2 \times 2 \times 2 = 2^3$



Write 81 as a product of prime numbers.

i. Write the above product as a power.

ii. Write is the base and the index.
Index: …… Base:……..

Revision

1) Write the below given products in Index notation.

i). $5 \times 5 \times 5 = 5^3$

ii). $(-5) \times (-5) \times (-5) \times (-5) = \cdots$

iii). $\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4}$ = ...

iv). $x \times x \times x \times x \times x = \cdots$

v). $pq \times pq \times pq \times pq$ = ...

2) Expand the below given and write as a product.

i). $5^4 = 5 \times 5 \times 5 \times 5$

ii). $(-2)^3 = \cdots$ ... ...

iii). $\left(\frac{5}{8}\right)^3$ = ...

iv). x^4 = ...

v). $(3y)^3$= .....

vi). $(3pq)^2$= .....

vi). $(-\frac{3}{4})^2$= .....

Power of a product and product of powers

$(x \times y)^5$ ← Power of a product

$x^5 \times y^5$ ← product of powers

Power of division and division of powers

$(\frac{a}{b})^3$ ← power of division

$\frac{a^3}{b^3}$ ← division of powers

3)

i. Expand the given power of a product and write it as a product of powers.

$$(xy)^3$$
$$= xy \times xy \times xy$$
$$= x \times x \times x \times y \times y \times y$$
$$= x^3 \times y^3$$

ii. Write the power of a product given below as a product of powers without expanding.

$(3pq)^2 =$……

4)

i. Expand the product of power given below and write it as a power of a product.

$$4x^2$$
$$= 2 \times 2 \times x \times x$$
$$= 2 \times x \times 2 \times x$$
$$= (2 \times x) \times (2 \times x)$$
$$= (2 \times x)^2$$

ii. Write the product of powers given below as a power of a product without expanding.

$27x^3y^3 =$…….

5)

i. Expand the power of a division given below and write it as a division of powers.

$\left(\frac{x}{y}\right)^3$

$= \frac{x}{y} \times \frac{x}{y} \times \frac{x}{y}$

$= \frac{x \times x \times x}{y \times y \times y}$

$= \frac{x^3}{y^3}$

ii. Expand the power of division given below and write it as a division of powers.

$\left(\frac{a}{b}\right)^5$ = ……

6)

i. Expand the division of powers given below and Write it as a power of division.

$\frac{a^2}{b^2}$

$= \frac{a \times a}{b \times b}$

$= \frac{a}{b} \times \frac{a}{b}$

$= (\frac{a}{b})^2$

ii. Write the division of powers given below and write it as a power of division without expanding.

$\frac{p^5}{q^5}$ =.....

- **Products of powers with the same base.**

Activity 1

Expand the powers given below and write them as a single power.

i. $2^3 \times 2^2$

$= 2 \times 2 \times 2 \times 2 \times 2 = 2^5$

ii. $p^3 \times p^5$

$= p \times p \times p \times p \times p \times p \times p \times p = p^8$

Simplify the expressions given below.

i). $3^2 \times 3^4 = 3^6$

ii). $9^5 \times 9^7$ = ...

iii). $y^4 \times y^7$= .....

iv). $p^4 \times p^2 \times p$= .....

v). $a^t \times a^t$= .....

vi). $x^4 \times x^p$= .....

vii). $b^p \times b^q$= .....

Laws of Indices - 1

$$a^m \times a^n = a^{m+n}$$

When multiplying powers with the same base; the indices are added and the base does not change.

Find out all the possible positive integers for m and n in $a^m \times a^n = a^6$.

i. m = 1 n= 5

ii. m = 2 n= …

iii. m = … n= 3

iv. m = … n= …

v. m = … n = …

Simplify the expressions given below using the laws of indices.

Example 1

$5a^4 \times 3a^7$
$= 5 \times 3 \times a^4 \times a^7$
$= 15 \times a^{4+7}$
$= 15a^{11}$

Example 2

$4p^2 \times 3p^6 \times p$
$= 4 \times 3 \times p^2 \times p^6 \times p$
$= 12 \times p^{2+6+1}$
$= 12p^9$

Exercise 1

Simplify the expressions given below using the laws of indices.

i. $5^3 \times 5^7$
ii. $7^2 \times 7^5 \times 7$
iii. $3x^4 \times 5x^2$

iv. $2y^2 \times 7y^4$
v. $5p^6 \times p^4$
vi. $5a^4 \times 3a^2 \times 2a$
vii. $3a^2 \times b^5 \times 5a^4 \times b^2$
viii. $2x^4 \times 3y^2 \times 2x \times 5y^3$

- **Quotients of powers with the same base.**

Activity 2

Expand the expressions given below, simplify and write the answer in index notation.

i. $3^5 \div 3^2$

$= \frac{3\times3\times3\times3\times3}{3\times3}$

$= 3 \times 3 \times 3$

$= 3^3$

ii. $y^6 \div y^2$

$= \frac{y \times y \times y \times y \times y \times y}{y \times y}$

$= y \times y \times y \times y$

$= y^4$

Simplify the expressions given below.

i. $5^7 \div 5^2 = 5^5$
ii. $11^9 \div 11^7 =$ ....
iii. $x^8 \div x^3 =$ ....
iv. $y^9 \div y^9 =$ ....
v. $p^5 \div p^{12} =$ ....
vi. $a^4 \div a^x =$ ....
vii. $b^y \div b^7 =$ .....
viii. $c^p \div c^q =$ .....

Laws of Indices - 2

$$a^m \div a^n = a^{m-n}.$$

When dividing powers with the same base; the indices get subtracted and the base does not change.

Find out all the possible positive integers which are less than 10 for m and n in $a^m \div a^n = a^6$.

i. m = 9 n= 3
ii. m = … n= 2
iii. m = 7 n=….

Simplify using the laws of indices.

$(4a^7 \times 3a) \div 6a^3$

$= \frac{12a^8}{6a^3}$

$= 2a^5$

$\frac{3p^6 \times 4p^4 \times 2p}{6p^2 \times p^4}$

$= \frac{24p^{11}}{6p^6}$

$= 4p^5$

Exercise 2

Simplify using the laws of indices.

i. $\mathbf{7^5 \div 7^4}$
ii. $\frac{\mathbf{10^7}}{\mathbf{10^4}}$
iii. $\frac{\boldsymbol{y^{12}}}{\boldsymbol{y^7}}$
iv. $\frac{\boldsymbol{a^6}}{\boldsymbol{a^6}}$
v. $\frac{y^5}{y^8}$
vi. $(x^4 \times x^7) \div x^5$
vii. $6y^5 \div 3y^2$
viii. $\frac{\boldsymbol{a^4 \times a^9}}{\boldsymbol{a^5}}$
ix. $\frac{\mathbf{3y^5 \times 4y^2 \times y^3}}{\mathbf{y^4 \times 6y}}$
x. $\frac{\mathbf{3a^4 \times 4b^5}}{\mathbf{a^3 \times 6b^2}}$

- **Negative Indices**

Activity 3

Simplify the following expressions using expansion and the use of the laws of indices.

$7^3 \div 7^5$

$= 7^{3-5}$

$= 7^{-2}$

$7^3 \div 7^5$

$= \frac{7 \times 7 \times 7}{7 \times 7 \times 7 \times 7 \times 7}$

$= \frac{1}{7 \times 7}$

$= \frac{1}{7^2}$

$\therefore 7^{-2} = \frac{1}{7^2}$

$x^4 \div x^7$

$= x^{4-7}$

$= x^{-3}$

$x^4 \div x^7$

$= \dfrac{\cancel{x \times x \times x \times x}}{\cancel{x \times x \times x \times x} \times x \times x \times x}$

$= \dfrac{1}{x \times x \times x}$

$= \frac{1}{x^3}$

$$\therefore x^{-3} = \frac{1}{x^3}$$

Laws of Indices – 3

$a^{-m} = \frac{1}{a^m}$ OR $\frac{1}{a^{-m}} = a^m$

AND

$\frac{a^{-m}}{b^{-n}} = \frac{b^n}{a^m}$

Denote the following expressions with positive indices.

i. $\frac{5^3}{2^{-5}} = 5^3 \times 2^5$

ii. $\frac{3^{-2}}{2^3} =$ …..

iii. $\frac{(5x)^{-3}}{2^{-4}} =$ …..

iv. $\frac{3^{-4}}{(3y)^{-2}} =$ .....

- **Zero Index**

Activity 4

Simplify the expressions given below using the laws of indices and by expansion.

i. $5^3 \div 5^3$

$= 5^{3-3}$

$= 5^0$

$5^3 \div 5^3$

$= \frac{\cancel{5 \times 5 \times 5}}{\cancel{5 \times 5 \times 5}}$

$= 1$

$\therefore 5^0 = 1$

ii. $p^2 \div p^2$

$= p^{2-2}$

$= p^0$

$p^2 \div p^2$

$\equiv \frac{\cancel{p \times p}}{\cancel{p \times p}}$

$= 1$

$\therefore p^0 = 1$

Laws of Indices - 4

$$a^0 = 1 \quad (a \neq 0)$$

When the Index of a power where the base is any number except 0 is 0; the value of that power is equals to 1.

Find out the value of the following powers.

i. 5^0
ii. 7^0
iii. $(-4)^0$
iv. $(5^2)^0$
v. 1^0
vi. $(2x)^0$

- **Power of a power**

Activity 5

Expand and simplify.

i. $(5^3)^2$

$= 5^3 \times 5^3$

$= 5^6$

ii $(x^2)^4$

$= x^2 \times x^2 \times x^2 \times x^2$

$= x^8$

Laws of Indices - 5

$$(a^m)^n = a^{m \times n}$$

The two indices should be multiplied.

Write the following power of powers as single powers.

i) • $(7^4)^2 = 7^8$

ii). $(11^3)^6$= ....

iii). $(a^4)^3$= .....

iv). $(y^5)^6$= .....

v). $(5^3)^x$= .....

vi). $(7^y)^2$= .....

vii).$(x^5)^a$= .....

viii. $(y^b)^4$= .....

ix). $(p^x)^y$= .....

Simplify.

i. $(3^2)^2$= .....

ii. $(2x^3)^4$= .....

iii. $(3^4y^2)^3$= .....

vi $(a^2b^6)^5 = a^{10}b^{30}$

v. $(5ax^3)^4$= .....

Find out the answer with positive indices by using the laws of indices.

Example 1

$(a^{-3})^4 \times (a^2)^{-1}$
$= a^{-12} \times a^{-2}$
$= a^{-14}$
$= \frac{1}{a^{14}}$

Example 1

$$\frac{(x^{-2})^2 \times (x^{-3})^{-3}}{(x^{-1})^{-3} \times x^7}$$
$$= \frac{x^{-4} \times x^9}{x^3 \times x^7}$$
$$\frac{x^5}{x^{10}}$$
$$= x^{-5}$$
$$= \frac{1}{x^5}$$

Exercise 1

Find out the answer with positive indices by using the laws of indices.

i. $a^4 \times (a^2)^3$

ii. $(b^{-2})^3 \times b^4$

iii. $(y^{-2})^3 \times (y^3)^{-1}$

iv. $(x^{3)^{-4}} \times (x^2)^0$

v. $\frac{(p^{-2})^3 \times (p^4)^2}{(p^{-3})^{-1}}$

vi. $\frac{(q^3)^{-1} \times (q^2)^{-2}}{(q^4)^2 \times (q^{-3})^2}$

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