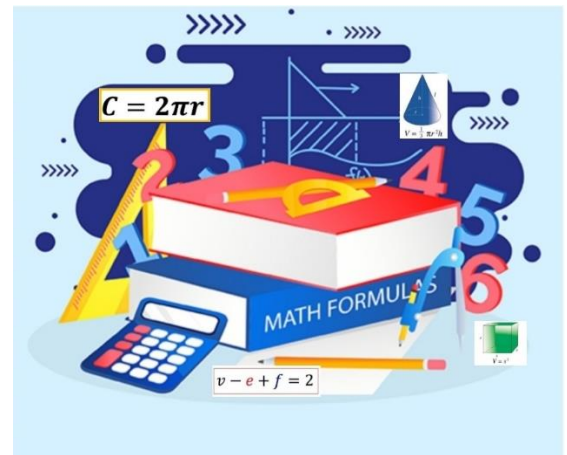


Grade 9

Unit 17

Formulae

Reading Material



17. Formula

Learning objectives

Through this lesson you will be able to,

- **define the term “ Formula”**
- **identify the subject of a formula**
- **change the subject of a formula**
- **find the value of a variable term in a formula, when the values of the other variables are given**

17.1 Introducing formulae

In grade 8, you have learnt the “Euler’s formula” that express the relationship among the number of edges, vertices and faces of solids.



You can find the above relationship below.

$$\text{Number of Edges} = \text{Number of Vertices} + \text{Number of faces} - 2$$

Let’s take:

Number of edges as : E

Number of vertices as : V

Number of faces as : F

Then we can write the above relationship as:

$$E = V + F - 2$$

This equation shows the relationship among three quantities; E,V and F.

This type of relationship is known as a “*Formula*”.

What is a formula ?

A mathematical relationship expressed by different quantities in symbols is known as a formula.

Following shows some formulae in mathematics and science:

1. $P = 2(a + b)$

2. $V = u + at$

3. $F = ma$

4. $C = 2\pi r$

5. $y = mx + C$

17.2 Introducing the Subject of a formula

The subject of a formula is the variable which is expressed in terms of the other variables. Usually it is on the left of the equal sign (“=”).

According to that:

The subject of the formula of $V = u + at$ is V .

17.3 Changing the subject of a formulae

Let’s see, how to change the subject E of the formula, $E = V + F - 2$ to V

Before that, we should concern how the right side of the formula is formed in related to V .

To the variable V :

- Variable F is added
- 2 is Subtracted

To change the subject E to V ;

- The inverse process of each steps should be followed.
- The Inverse process should start from the end to the beginning of the steps in which the expression with the variable required to change as the subject.

To make V as the subject, the inverse process of above steps is given below.

- Add 2 to both sides of the formula.
- Subtract F from both sides of the formula.

According to that,

Let's add 2 to both sides of the formula.

$$E + 2 = V + F - 2 + 2$$

$$E + 2 = V + F$$

Now, let's subtract F from both sides of the formula.

$$E + 2 - F = V + F - F$$

$$E + 2 - F = V$$

$$\underline{\underline{V = E + 2 - F}}$$

By referring the examples given below, you can understand how to change the subject of formulae further.

01. In the formula $y = mx + c$, change the subject to c .

Subtract mx from the both sides of the formula.

$$y = mx + c$$

$$y - mx = mx + c - mx$$

$$y - mx = c$$

$$\underline{\underline{c = y - mx}}$$

02. In the formula $y = mx + c$, change the subject to m .

Step 1 : Subtract c from both sides.

$$\begin{aligned}y &= mx + c \\y - c &= mx + c - c \\y - c &= mx\end{aligned}$$

Step 2 : Divide both sides of the formula by x and simplify.

$$\begin{aligned}\frac{y - c}{x} &= \frac{mx}{x} \\ \frac{y - c}{x} &= m \\ m &= \frac{y - c}{x}\end{aligned}$$

03. $S = \frac{n}{2}(a + l)$, Make the subject.

Step 1: Multiply both sides of the formula by 2

$$\begin{aligned}2 \times S &= \frac{n}{2}(a + l) \times 2 \\ 2S &= n(a + l)\end{aligned}$$

Step 2: Divide both sides of the formula by $(a + l)$

$$\begin{aligned}\frac{2s}{(a + l)} &= \frac{n(a + l)}{(a + l)} \\ \frac{2s}{(a + l)} &= n \\ n &= \frac{2s}{(a + l)}\end{aligned}$$

Make a the subject of the formula $v = u + at$

Step 1: Subtract u from both sides of the formula.

$$\begin{aligned}v &= u + at \\v - u &= u + at - u \\v - u &= at\end{aligned}$$

Step 2 : Divides both side of the formula by t and simplify .

$$\begin{aligned}\frac{v - u}{t} &= \frac{at}{t} \\ \frac{v - u}{t} &= a \\ \underline{\underline{a}} &= \underline{\underline{\frac{v - u}{t}}}\end{aligned}$$

04. In the formula $c = 2\pi r$, make r the subject.

Divide both sides of the formula by 2π

$$c = 2\pi r$$

$$\begin{aligned}\frac{c}{2\pi} &= \frac{\overset{1}{\cancel{2\pi}}r}{\cancel{2\pi} \underset{1}{}} \\ \frac{c}{2\pi} &= r \\ \underline{\underline{r}} &= \underline{\underline{\frac{c}{2\pi}}}\end{aligned}$$

05. Make n the subject, of the formula $l = a + (n - 1)d$.

Step 1: Subtract a from both sides.

$$\begin{aligned} l &= a + (n - 1)d \\ l - a &= a + (n - 1)d - a \\ l - a &= (n - 1)d \end{aligned}$$

Step 2: Divide both sides of the formula by d .

$$\begin{aligned} \frac{l - a}{d} &= \frac{(n - 1)\cancel{d}}{\cancel{d}} \quad 1 \\ \frac{l - a}{d} &= n - 1 \end{aligned}$$

Step 3: Add 1 to the both sides of the formula.

$$\begin{aligned} \frac{l - a}{d} + 1 &= n - 1 + 1 \\ n &= \frac{l - a}{d} + 1 \end{aligned}$$

06. In the formula $p = \frac{at}{a-t}$, make the subject a

First, take all the terms with a , in to one side of the formula.

$$\begin{aligned} (a - t) \times p &= \frac{at}{\cancel{a-t}} \times (\cancel{a-t}) \quad 1 \\ p(a - t) &= at \end{aligned}$$

$$pa - pt = at$$

$$pa - at = pt$$

$$a(p - t) = pt$$

$$\frac{a(\cancel{p-t})^1}{(\cancel{p-t})} = \frac{pt}{(p-t)}$$

$$\overset{1}{a} = \frac{pt}{(p-t)}$$

17.4 Substitution

The value of an unknown variable in a formula, can be found by substituting values for other variables.

❖ Find the number of edges of the solid with 6 vertices and 5 faces.

Substitute $V = 6$ and $F = 5$ to the formula $E = V + F - 2$,

$$E = V + F - 2$$

$$E = 6 + 5 - 2$$

$$E = 11 - 2$$

$$\underline{\underline{E = 9}}$$

There are two methods of substituting values of the variables in a formula to find the given variable.

Method 1 : Substitute the values to the variables by keeping the formula as it is given.

Method 2: Substitute the values, after making the unknown variable as the subject of the formula.

Let's see how to use above methods to find the value of an unknown variable.

❖ Find the number of vertices in a solid with 7 faces and 12 edges.

Let's take,

Number of Edges as E

Number of faces as F

Number of vertices as V

Method 1 : Substituting to the formula as it is given.

$$E = V + F - 2$$

$$12 = V + 7 - 2$$

$$12 = V + 5$$

$$12 - 5 = V + 5 - 5$$

$$7 = V$$

$$\underline{\underline{V = 7}}$$

∴ Number of vertices are 7.

Method 2 : Substituting the values after making the unknown variable as the subject.

- Make V as the subject

$$E = V + F - 2$$

$$E + 2 = V + F - 2 + 2$$

$$E + 2 = V + F$$

$$E + 2 - F = V + F - F$$

$$E + 2 - F = V$$

$$V = E + 2 - F$$

- Substitute values for E and F

$$V = E + 2 - F$$

$$V = 12 + 2 - 7$$

$$V = 14 - 7$$

$$\underline{\underline{V = 7}}$$

∴ Number of vertices are 7.

01. In the formula $l = a + (n - 1)d$, if $l = 22$, $a = -5$ and $n = 10$, find the value of d .

$$l = a + (n - 1)d$$

$$22 = (-5) + (10 - 1)d$$

$$22 + 5 = (-5) + (10 - 1)d + 5$$

$$27 = 9d$$

$$\frac{27}{9} = \frac{9d}{9}$$

$$3 = d$$

$$d = 3$$

02. If $s = -330$, $a=15$ and $l = -48$ in the formula $s = \frac{n}{2}(a + l)$, find the value of n

$$S = \frac{n}{2}(a + l)$$

$$(-330) = \frac{n}{2}(15 - 48)$$

$$(-330 = \frac{n}{2}(-33))$$

$$2 \times (-330) = \frac{n}{2}(-33) \times 2$$

$$2 \times \frac{(-\cancel{330})^{20}}{(-\cancel{33})^1} = \frac{n(-\cancel{33})^1}{(-\cancel{33})^{11}}$$

$$20 = n$$

$$\underline{\underline{n = 20}}$$

For further reference:

Grade 9 Mathematics text book.

Exercise 17.1 (Page 98)

Exercise 17.2 (Page 101)