(ii) $\frac{2-\sqrt{3}}{2+\sqrt{3}} + \frac{2+\sqrt{3}}{2-\sqrt{3}} = \frac{2-\sqrt{3}}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} + \frac{2+\sqrt{3}}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$

$= \frac{(2-\sqrt{3})^2}{4-3} + \frac{(2+\sqrt{3})^2}{4-3}$

$= \frac{4-4\sqrt{3}+3}{1} + \frac{4+4\sqrt{3}+3}{1}$

$= 14$

(iii) $N = 3.456\ .\ 456\ 456\ .....$ ———①

$1000N = 3456.456\ 456.....$ ———②

② - ① $999N = 3453$

$N = \frac{3453}{999} = \frac{1151}{333}$

(iv) $2^{2(3x-1)} = 2^{-x+1}$

$2(3x-1) = -x + 1$

$6x - 2 = -x + 1$

$7x = 3$

$x = \frac{3}{7}$

(v) $\frac{2^{3x\frac{1}{3}}\, x^{-\frac{1}{3}} \times x^{2x\frac{1}{3}}\, y^{\frac{1}{3}}}{x^{3x\frac{1}{9}} \times y^{6x\frac{1}{9}}}$

$= \frac{2x^{\frac{1}{3}}\, y^{\frac{1}{3}}}{2x^{\frac{1}{3}}\, y^{\frac{1}{3}}} = 2y^{\frac{1}{3}}$

සංයුක්ත ගණිතය II කොටස
A කොටස (පිළිතුරු පත්‍රය)

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01. $\overrightarrow{AB} = \underline{a}$, $\overrightarrow{BC} = \underline{b}$, $\overrightarrow{CD} = -a$, $\overrightarrow{DA} = -b$.

තවද $\overrightarrow{AP} = k\underline{a}$, $\overrightarrow{BQ} = k.\underline{b}$, $\overrightarrow{CR} = -k\underline{a}$ හා $\overrightarrow{DS} = k\underline{b}$

$\overrightarrow{PB} = \overrightarrow{AB} - \overrightarrow{AP} = \underline{a} - k\underline{a} = (1-k)\underline{a}$

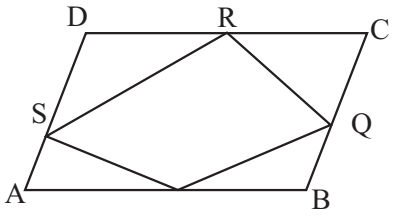
එලෙසම $\overrightarrow{DR} = (1-k)\underline{a}$

$\overrightarrow{PQ} = \overrightarrow{PB} + \overrightarrow{BQ} = (1-k)\underline{a} + k\underline{b}$ සහ $\overrightarrow{SR} = \overrightarrow{SD} + \overrightarrow{DR}$

$= k\underline{b} + (1-k)\underline{a}$

$\therefore \overrightarrow{PQ} = \overrightarrow{SR}$

$\therefore$ PQRS සමාන්තරාස්‍රයකි.



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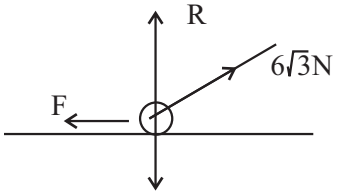
02. ලම්බ ප්‍රතික්‍රියාව R ද ඝර්ෂණ බලය F ද ලෙස නම්,

$F - 6\sqrt{3}\ \text{Cos}\ 60 = 0$

$\leftarrow F = 3\sqrt{3}$ N

$\uparrow \quad R + 6\sqrt{3}\ \text{Sin}\ 60 - 20 = 0$

$R = 11$N



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03. $-aP + 3P = 0 \qquad \uparrow\ 2P + bP = P$

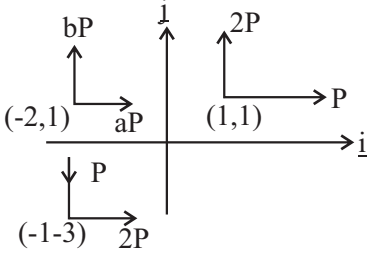
$a = -3 \qquad b = -1$

$O\circlearrowleft M = P \times 1 - 2P \times 1 - P \times 1 - 2P \times 3 + aP \times 1 + bP \times 2$

$= P - 2P - P - 6P - 3P - 2P$

$= -13P$

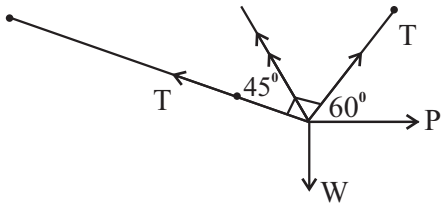
$M\circlearrowleft = 13P$



04. R යනු T ආනතිවල සම්ප්‍රයුක්තය නම්,

$$\frac{W}{\text{Sin } (60 + 45)} = \frac{P}{\text{Sin } (180 - 15)} = \frac{R}{\text{Sin } 90}$$

$$P = \frac{W \text{ Sin } 15}{\text{Cos } 15} = W \tan 15$$



05. $\tan \alpha = f$

$\tan \beta = 2f$

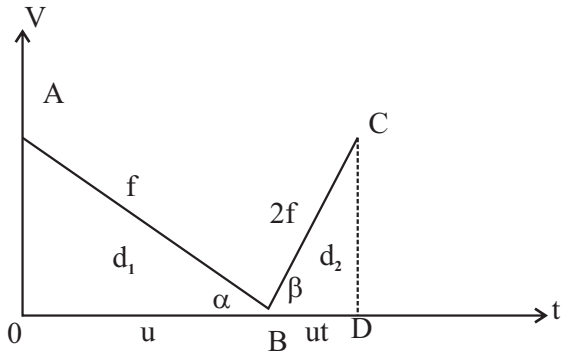
$\tan \alpha = \frac{u - 0}{T_1}$

$\tan \beta = \frac{u - 0}{T_2}$

$f = \frac{u}{T_1}$ $\quad 2f = \frac{u}{T_4}$

$f = \frac{u}{f}$ $\quad T_2 = \frac{u}{2t}$

මුළු කාලය $T_1 + T_2 = \frac{u}{f} + \frac{u}{2f} = \frac{3u}{2f}$



ගමන් කළ දුර = $d_1 + d_2$ = OAB ත්‍රි. + BCD ත්‍රි.

$= \frac{1}{2} u \cdot T_1 - \frac{1}{2} u \cdot T_2$

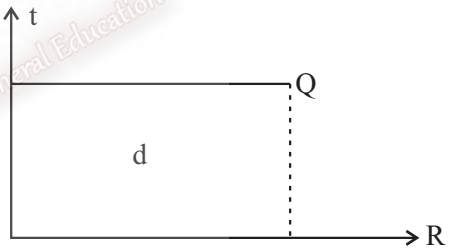
$= \frac{u}{2} \times \frac{3u}{2f} = \frac{3u^2}{4f}$

ප්‍රමාද නොවන විට ප්‍රස්තාරය

OPQR ත්‍රි. = d

$u \cdot T_3 = \frac{3u^2}{4f}$

$T_3 = \frac{3u}{4f}$



∴ ප්‍රමාද කාලය = $(T_1 + T_2) = T_3$

$= \frac{3u}{2f} - \frac{3u}{4f} = \frac{3u}{4f}$

06. $|\underline{a}| = |\underline{b}| = 1$ $\theta = \frac{\pi}{3}$

$\underline{a} \cdot \underline{b} = |\underline{a}| = |\underline{b}| \text{ Cos } \frac{\pi}{3} = \frac{1}{2}$

$(\underline{a} - \underline{b})(\underline{a} - 5\underline{b}) = 3\underline{a} \cdot \underline{a} - 3\underline{a} \cdot 5\underline{b} - \underline{b} \cdot \underline{a} + \underline{b} \cdot 5\underline{b}$

$= 3 \times 1^2 \times 15 \times \frac{1}{2} - \frac{1}{2} + 15 \times 1^2$

$= 3 - \frac{15}{2} - \frac{1}{2} + 5 = 8 - 8 = 0$

$3\underline{a} \cdot \underline{b} = 0$, $\underline{a} - 5\underline{b} = 0$

$\underline{a} = \frac{1}{3}\underline{b}$, $\underline{a} = 5\underline{b}$

$\underline{a} = \lambda \underline{b}$ විට $\lambda \in \mathbb{R}$, $\underline{a}$ හා $\underline{b}$ සමාන්තර වේ.

නමුත් $\underline{a}$, $\underline{b}$ $\frac{\pi}{3}$ කින් ආනත වේ.

∴ $3\underline{a} - \underline{b} \neq 0$, $\underline{a} - 5\underline{b} \neq 0$

∴ $|3a - b| \neq 0$

$|\underline{a} - 5\underline{b}| \neq 0$

∴ වියහැකි එකම විසඳුම $(3\underline{a} - \underline{b})$ හා $(\underline{a} - 5\underline{b})$ ආනත කෝණය $\frac{\pi}{2}$ වීමයි.

$3\underline{a} - \underline{b}$ හා $\underline{a} - 5\underline{b}$ එකිනෙකට ලම්බක වේ.

07. AB දණ්ඩට

B↺

$X \cdot 2a \cos\alpha = W\, a \sin\alpha$

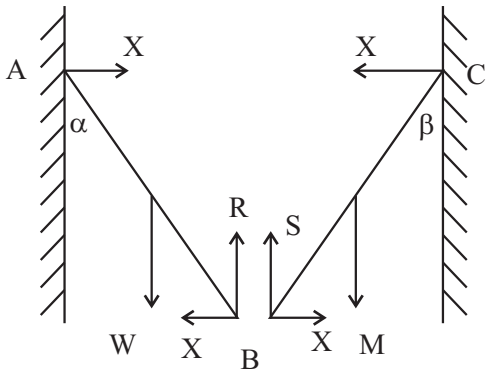
$X = \frac{W}{2} \cdot \tan\alpha$

BC B↻ $X \cdot 2b \cos\alpha = Mb \sin\beta$

$X = \frac{M}{2} \cdot \tan\beta$

සමතුලිතතාව සඳහා, $\frac{W}{2}\tan\alpha = \frac{M}{2}\tan\beta$ විය යුතුය.

$W \tan\alpha = M \tan\beta$



08. පද්. B↻ $Pa = kw \cdot \frac{a}{2}$

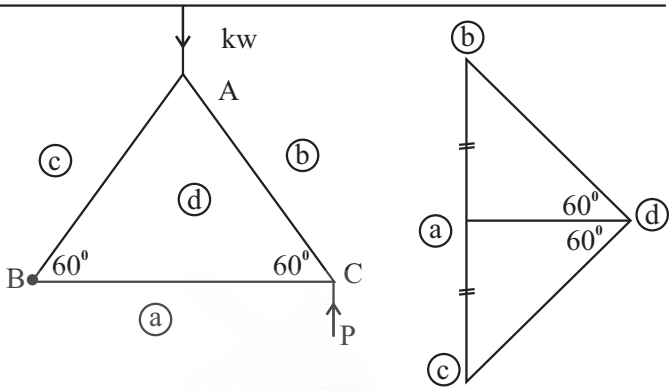
$P = \frac{kw}{2}$

AC දණ්ඩේ ප්‍රත්‍ය බලය $bd = 5\sqrt{3}w$

$\text{Sin } 60 = \frac{1}{2} \times \frac{kw}{5\sqrt{3}w}$

$k = 5\sqrt{3} \times \frac{\sqrt{3}}{2} \times 2$

$k = 15$



09. බලයක විශාලත්වය x නම්, හා බල දෙක අතර ∢ය θ නම්,

$R^2 = P^2 + Q^2 + 2PQ \cos\theta$

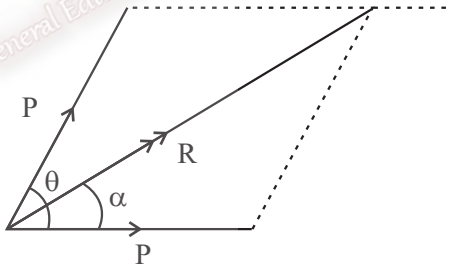
$x^2 = x^2 + x^2 + 2x \cos\theta$

$-x^2 = 2x^2 \cos\theta$

$\cos\theta = -\frac{1}{2}$

$\theta = \frac{2\pi}{3}$

බල දෙක අතර ∢ $\frac{2\pi}{3}$ වේ.



$$\tan\alpha = \frac{Q \sin\theta}{P + Q\cos\theta} = \frac{x \sin\frac{2\pi}{3}}{x + x\cos\frac{2\pi}{3}} = \frac{x \sin(\pi - \frac{\pi}{3})}{x + x\cos(\pi - \frac{\pi}{3})}$$

$$= \frac{\sqrt{3}\,x}{2x - x} = \sqrt{3}$$

$\therefore \tan\alpha = \sqrt{3}$

$\alpha = \frac{\pi}{3}$

10. $\overrightarrow{X} = 1 + 4 = 5$

$Y\uparrow = 2 + 3 = 5$

සම්ප්‍රයුක්තය R නම්, $R^2 = x^2 + y^2$

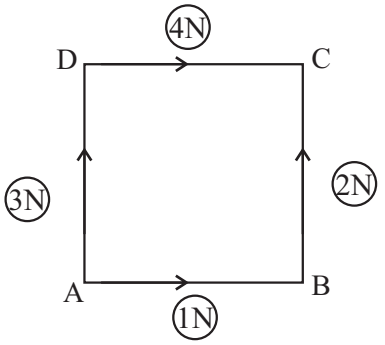
$R = 5\sqrt{2}$

සම්ප්‍රයුක්තය තිරස සමඟ සාදන ∢ θ නම්,

$\tan\theta = \frac{Y}{X} = \frac{5}{5} = 1$

$\theta = 45$

$\therefore$ සම්ප්‍රයුක්තය විකර්ණයට සමාන්තර වේ.



සංයුක්ත ගණිතය II කොටස B කොටස (පිළිතුරු පත්‍රය)

11. (i) අනුක්‍රමණය = මන්දනය

$\tan \theta = g$

$\frac{u}{t} = g \Rightarrow t = \frac{u}{g}$

උපරිම උස H විට විස්ථාපනය = යට වූ වර්ගඵලය

$H = OAB\ \Delta = \frac{u}{2} \cdot \frac{u}{g} \Rightarrow H = \frac{u^2}{2g}$

නැවත O ට කාලය T නම්,

අනුක්‍රමණය = මන්දනය

$\tan \alpha = g \Rightarrow \frac{V}{T - t} = g$

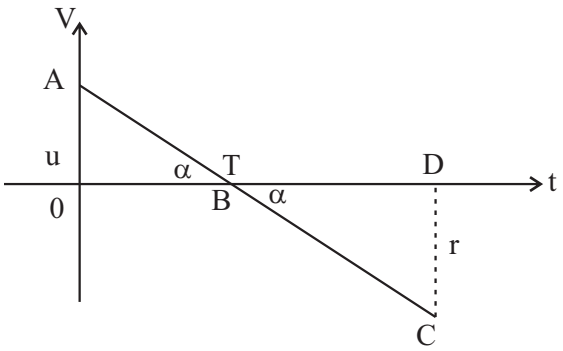
$V = g\ (T - t)$

විස්ථාපනය = යට වූ වර්ගඵලය

$O = \frac{ut}{2} - \frac{(T-t)\ g\ (T-t)}{2}$

$(T-t)^2 = \frac{u}{g} \cdot \frac{u}{g} \Rightarrow T - t = \frac{u}{g}$

$T = \frac{u}{g} + \frac{u}{g} = \frac{2u}{g}$



(ii) O සිට A ට කාලය t_1 ද A සිට O ට කාලය t_2 ද වේ.

පියාසර කාලය T නම් $T = t_1 + t_2$

අනුක්‍රමණය = මන්දනය

$\tan \alpha = g$

$\frac{u - V}{t_1} = g \Rightarrow V = u - gt_1$

විස්ථාපන = වර්ගඵලය

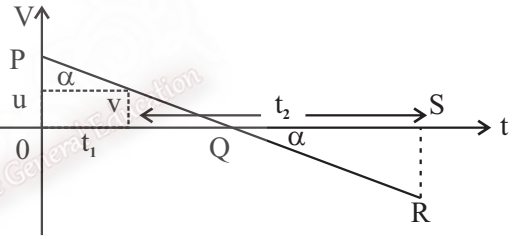
$h = \left(\frac{u + V}{2}\right) t_1 \Rightarrow (u + u - gt_1) = ut_1 - \frac{1}{2} gt_1^2$

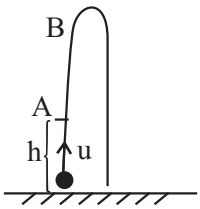
$T = \frac{2u}{g} \Rightarrow u = \frac{2u}{g} (t_1 + t_2)$ බව ආදේශයෙන්,

$h = t_1\ (t_1 + t_2) \frac{g}{2} - \frac{1}{2} gt_1^2$

$2h = gt_1t_2$

උපරිම උස $= \frac{u^2}{2g} = \frac{g^2}{4} (t_1 + t_2)^2 \times \frac{1}{2g} = \frac{g}{8} (t_1 + t_2)^2$





12. (i) P සිට O ට කාලය t නම්,

$\overrightarrow{S} = ut$

$\frac{130V^2}{g} = 13V \cos \theta \cdot t$

$= 13V \times \frac{5}{13} t$

$t = \frac{26}{g} V$

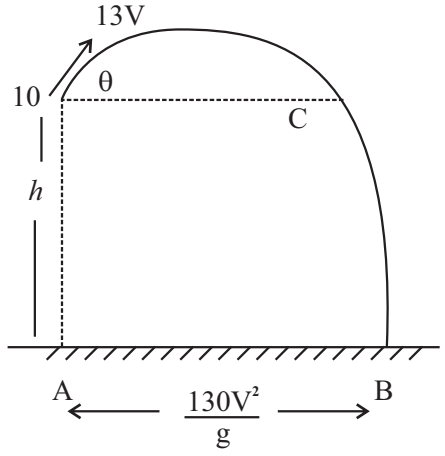
$\uparrow S = ut + \frac{1}{2} at^2$

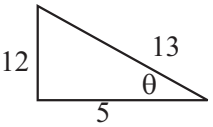
$-h = 13V \sin \theta - \frac{1}{2} gt^2$

$-h = 13V \cdot \frac{12}{13} \times \frac{260}{g} - \frac{1}{2} g \left(\frac{26V}{g}\right)^2$

$-h = \frac{26V}{g} - \left(12V - \frac{1}{2} g \times \frac{26V}{g}\right)$

$\therefore h = \frac{26V^2}{g}$





(ii) P සිට C ට කාලය T නම් $\uparrow S = uT + \frac{1}{2} a + 2$

$0 = 13V \sin\alpha - \frac{1}{2} gt^2$

$t = \frac{26V \sin\alpha}{V} = \frac{26}{g} \times \frac{12}{13} = \frac{24V}{g}$

$\overrightarrow{S = ut}$

$OC = 13V \cos\alpha \cdot t$

$= 13 \times \frac{5}{13} \times \frac{24V}{g} \times \frac{120V^2}{g}$

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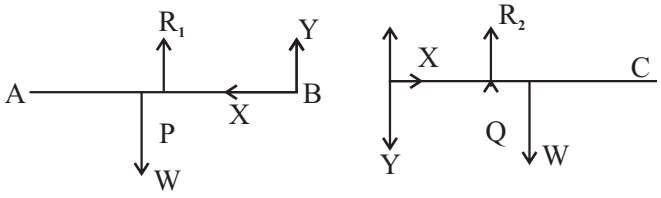
13. (i) BC ට $B\curvearrowleft = O$

$R_2 y = Wa = 0$

$R_2 = \frac{Wa}{y}$



පද්ධතිය . $P\curvearrowleft = O$

$R_2 (x + y) - w(x + a) + w(a - x) = 0$

R_2 හි අගය ආදේශයෙන්,

$\frac{Wa}{y}(x + y) - 2Wx = 0 \Rightarrow a(x + y) = 2xy$

$x(2y - a) = ay$

$\Rightarrow x = \frac{ay}{2y - a}$

සමතුලිතව පැවතීමට නම් $x \leq 2a$

$\frac{ay}{2y - a} \leq 2a \Rightarrow y \leq 4y - 2a$

$3y \geq 2a \Rightarrow y \geq \frac{2ay}{3}$

පද්ධතිය $\uparrow$ විභේදනයෙන් $= 0$

$R_1 + R_2 - 2w = 0$

$R_1 = 2W - \frac{Wa}{y} \Rightarrow \frac{W}{y}(2y - a)$

$AB\Delta \rightarrow = 0 \quad X = 0$

$AB\Delta \uparrow = 0$

$Y + R_1 - W = 0$

$Y = W - \frac{W}{y}(2y - a) = \frac{W}{y}(a - y)$

$R_1 \leq \frac{5w}{4}$

$\Rightarrow \frac{W}{y}(2y - a) \leq \frac{5w}{4}$

$8y - 4a \leq 5y$

$3y \leq 4a \Rightarrow y \leq \frac{4a}{3}$

---

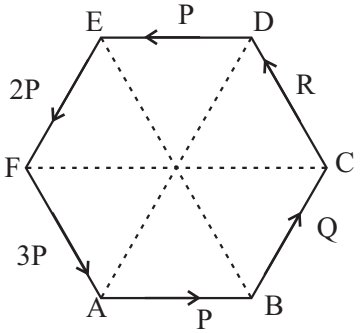
14. (i) $\overrightarrow{X} = P + Q \cos 60 - R \cos 60 - P - 2P \cos 60 + 3P \cos 60$

$= P + (Q - R)\frac{1}{2} - P + \frac{3P}{2}$

$= \frac{1}{2}(P + Q - R)$

$Y\uparrow = Q \sin 60 + R \sin 60 - 2P \sin 60 - 3P \sin 60$

$= (Q + R - 5P)\frac{\sqrt{3}}{2}$



බල පද්ධතිය යුග්මයකට තුල්‍ය වේ නම් X = 0 , Y = 0 නිසා

X = 0 විට P + Q - R = 0

Y = 0 විට Q + R - 5P = 0

R - Q = P

R - Q = 5P

$\therefore$ R = 3P, Q = 2P

O වට යුග්මයේ සුර්ණය $\curvearrowright$ = (P + Q + R + P + 2P + 3P)d

= 12P + a$\sqrt{3}$ = 12$\sqrt{3}$ Pa

පද්ධතිය AD දිගේ තනි බලයකට තුල්‍ය නම්, A හා D ඔස්සේ පූර්ණ ශූන්‍ය වේ.

A වටා a$\sqrt{3}$ (Q + 2P) + 2a$\sqrt{3}$ (R + P) = 0 $\Rightarrow$ Q + 2R = -4P

D වටා a$\sqrt{3}$ (Q + 2P) + 2a$\sqrt{3}$ (3P + P) = 0 $\Rightarrow$ Q + 10P = 0

Q = -10P

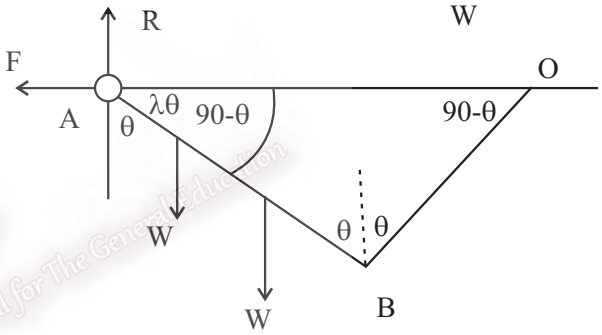
R = 3P

15. (i) AB විට A$\curvearrowright$ = 0

T . Sin 2θ x 2*a* - Wa Sin θ - $\frac{W}{2}$ λ*a* Sin θ = 0

4T Cos θ = W + $\frac{W}{2}$ λ

T = $\frac{W(2 + \lambda)}{8 \text{ Cos } \theta}$



AB ට → විභේදනය = 0

T Sin θ - F = 0

F = $\frac{W(2 + \lambda) \text{ Sin } \theta}{8 \text{ Cos } \theta}$ = $\frac{W}{8}$ (2 + λ)

AB ට ↑ විභේදනය = 0

R - $\frac{W}{2}$ - W + T Cos θ = 0

R = $\frac{W}{2}$ + W - $\frac{W(2 + \lambda)}{8}$

= $\frac{W}{8}$ (10 - λ)

$\frac{F}{R}$ = $\frac{W(2 + \lambda) \tan \theta}{8 \text{ x } \frac{W}{8}(10 - \lambda)}$ = $\frac{(2 + \lambda) \tan \theta}{10 - \lambda}$

16. (i) B ට ලාමිගේ ප්‍රමේයයෙන්,

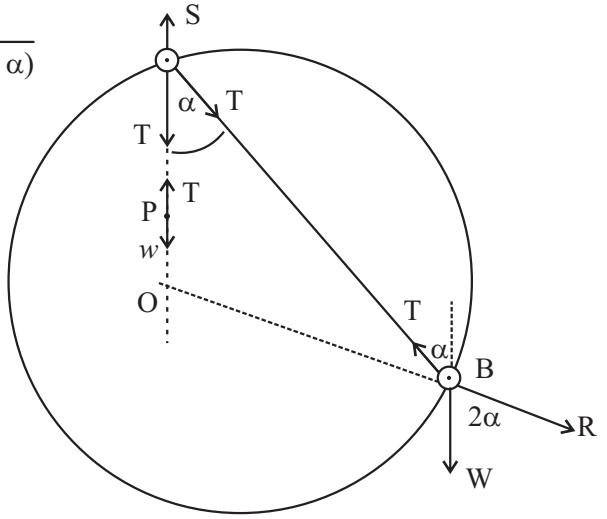
$\frac{R}{\text{Sin } (180 - \alpha)}$ = $\frac{T}{\text{Sin } 2\alpha}$ = $\frac{W}{\text{Sin } (180 - \alpha)}$

$\frac{R}{\text{Sin } \alpha}$ = $\frac{T}{\text{Sin } 2\alpha}$ = $\frac{W}{\text{Sin } \alpha}$

R = W

$\frac{T}{\text{Sin } 2\alpha}$ = $\frac{W}{\text{Sin } \alpha}$

T = 2W Cos α



T = w බව ආදේශයෙන් w = 2W Cos α

$\Rightarrow$ Cos α = $\frac{w}{2W}$

α තාත්වික විට Cos α < 1, $\frac{w}{2W}$ < 1

$\Rightarrow$ *w* < 2W

AB = *a* Cos α + *a* Cos α = 2*a* Cos α = $\frac{2a \text{ x } w}{2W}$ = $\frac{aw}{W}$

A මත ක්‍රියාව S නම්, $S^2 = T^2 + T^2 + 2TT \cos \alpha$

$= 2T^2 \left(1 + \frac{w}{2W}\right)$

$= 2w^2 \left(\frac{2W + w}{2W}\right)$

$S = W\sqrt{\frac{2W + w}{w}}$

17. (i) $A\circlearrowleft = 0$

$S \cdot 2a \cos 30 - W \cdot 2a = 0$

$\Rightarrow S = \frac{W}{\cos 30} = \frac{W}{\frac{\sqrt{3}}{2}} = \frac{2W}{\sqrt{3}} = \frac{2W\sqrt{3}}{3}$

$\rightarrow X - S = 0$

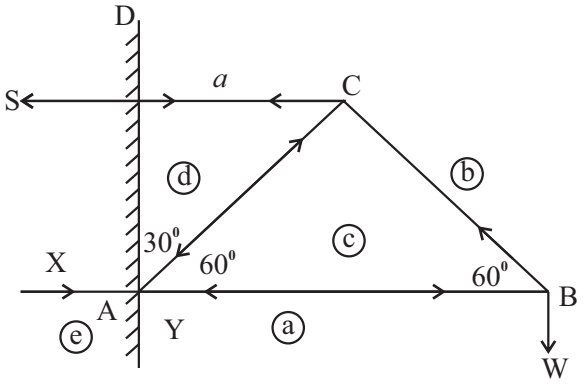
$\Rightarrow X = \frac{2W\sqrt{3}}{3}$

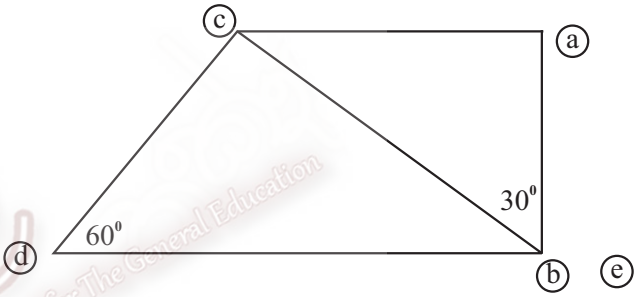
$\uparrow Y - W = 0$

$Y = W$

දණ්ඩ	ප්‍රත්‍යාබලය	ආනති	තෙරපුම
AB(ac)	$W / \sqrt{3}$		✓
BC (bc)	$2W / \sqrt{3}$	✓	
AC (cd)	$2W / \sqrt{3}$		✓
CD (bd)	$2W\sqrt{3} / 3$	✓	

$ad = \frac{W}{\frac{\sqrt{3}}{2}} = \frac{2W}{\sqrt{3}}$





A කොටසේ එක් ප්‍රශ්නයකට ලකුණු 25 බැගින් ප්‍රශ්න 10 ට ලකුණු = 10 x 25 = 250

B කොටසේ එක් ප්‍රශ්නයකට ලකුණු 150 බැගින් ප්‍රශ්න 05 කට ලකුණු = 5 x 150 = 750

මුළු ලකුණු = 750 + 250 = 1000