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Provincial Department of Education - NWP

10 E I

Third Term Test - Grade 13 - 2016

Index No : ....

Combined Mathematics I

Three hours only

Instructions:

* *This question paper consists of two parts.*
  Part A (Question 1 - 10) and **Part B** (Question 11 - 17)
* **Part A**
  *Answer **all** questions. Write your answers to each question in the space provided. you may use additional sheets if more space is needed.*
* **Part B**
  Answer five questions only. Write your answers on the sheets provided.
* *At the end of the time allocated, tie the answers of the two parts together so that* **Part A** *is on top of* **part B** *before handing them over to the supervisor.*
* *You are permitted to remove only* **Part B** *of the question paper from the Examination Hall.*

For Examiner's Use only

(10) Combined Mathematics I		
Part	Question No	Marks Awarded
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
	Total	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
	Total	
Paper 1 total		
Percentage		

Paper I	
Paper II	
Total	
Final Marks	

Final Marks

In Numbers	
In Words	

Marking Examiner	
Marks Checked by 1	
2	
Supervised by	

Combined Mathematics 13 -I (Part A)

(01) Using the **Principle of Mathematical Induction**, prove that when $f(n) = 4.6^n + 5^{n+1}$ is divided by 20, remainder is 9 for all $n \in z^+$

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(02) Find all real values of x, satisfying the inequality $\frac{|x-3|}{(x-2)} > 5$.

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(03) Sketch the loci of the points represented by the complex numbers satisfying $|z-2| = 2$ and $|z-4| = 2$ on the same diagram and find the complex number represented by their points of intersection.

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(04) Find the range of values of x for which the greatest numerical value of the expansion $\left(2+\frac{3x}{8}\right)^{10}$ is only at the fourth term.

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(05) Prove that. $\lim_{x \to \frac{\pi}{4}} \left[\frac{\sin\left(\frac{\pi}{4}-x\right)}{\sqrt{2}Cosx-1}\right] = 1$

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(06) Find the area of the region enclosed by the curves $y = x^2 - 4x$ and $y = 2x$.

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(07) Let C be the curve given by $x = t,\ \ y = \frac{1}{t}$ where "t" is a parameter. Prove that the normal to the curve C at the point corresponding to x = 2; meets the curve again at point $\left(-\frac{1}{8}\,, -8\right)$

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(08) Equations of sides AB, BC, CA of triangle ABC are $2y - x + 4 = 0,$, $y - 2x - 1 = 0$ and $y + x - 1 = 0$ respectively. Find the equation of the interior angle bisector of the angle B.

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(09) Two circles S_1 and S_2 are given by; $S_1 \equiv x^2 + y^2 - 5x + 5y + k = 0$ and $S_2 \equiv x^2 + y^2 + 2y - 3 = 0$. If the circumference of the circle S_2 is bisected by the circle S_1, find the value of k.

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(10) $\sin A = \frac{3}{5}$, where A is an obtuse angle and $\sin B = \frac{12}{13}$; where B is an acute angle. Find $\cos(A + B)$ and $\tan(A - B)$.

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Combined Mathematics 13 -I (Part B)

(11)

a) If roots of the equation $ax^2 + bx + c = 0$: $(a, c \neq 0)$ are in the ratio λ:1; prove that : $ac\lambda^2 + (2ac - b^2)\lambda + ac = 0$.

If ratio between the roots of the equation $lx^2 + mx + n = 0$; $(l, n \neq 0)$and the ratio between the roots of the equation $px^2 + qx + r = 0$ $(p, r \neq 0)$are equal, deduce that $m^2rp = q^2nl$.

b) Prove that $f(x) = ax^2 + bx + c$: $(a \neq 0)$ where a, b, c are real numbers, can be represented either in the form of $a[(x-p)^2 + q^2]$ or $a[(x-p)^2 - r^2]$. Such that p, q, r are real numbers. Explain the difference between two instances. What happens when $b^2 - 4ac = 0$.

Represent $f_1(x) = -x^2 + 2x + 3$ in one of the above form, and hence sketch a rough graph of $y = f_1(x)$

(12)

a) In the word "DEFEATED", Find the number of permutations which can be made by not keeping letter E together.

b) A committee consisting of four members is to be selected from six boys and six girls. Find the number of different ways in which this can be done,

i. If the committee must consist of two boys and two girls.
ii. If the committee including at least one girl and one boy.
iii. Although either the oldest boy or the oldest girl should be included in to the committee, both should not included in to the committee together.

c) Find U_r of the series; $1 + \frac{1}{1+2} + \frac{1}{1+2+3} + \cdots$

Find $f(r)$ and A such that: $U_r = A\,[\,f(r) - f\,(r+1)]$ Hence find $\sum_{r=1}^{n} \mathrm{U_r}$ for $r \in z^+$

Is the above series convergent?

Find $\sum_{r=1}^{\alpha} \mathrm{U_r}$

Further find $\sum_{r=1}^{\alpha} 4U_r$

(13)

a) Two matrices A and B are given as $A \equiv \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$, $B \equiv \begin{pmatrix} 1 & -1 \\ -1 & -1 \\ 0 & 0 \end{pmatrix}$

i. Find matrix C: such that: $AB = C$

ii. Is $(AB)^T = B^T A^T$?

iii. Find the inverse matrix of C.

iv. Hence solve

-x - 3y = 8

-x - 9y = -4

b) If $z = x + iy$:

i. Find $Re\left(Z + \frac{1}{z}\right)$ and $Im\left(Z + \frac{1}{z}\right)$

Find the locus of the points of Z such that $Im\left(Z + \frac{1}{z}\right) = 0$.

ii. Two complex numbers Z_1 and Z_2 are such that $Z_1 + Z_2 = 1$: where $Z_1 = \frac{a}{1+i}$ and $Z_2 = \frac{b}{1+2i}$; a, b are real.

Find the values of a and b.

For these values of a and b; represents Z_1 and Z_2 on the Argand diagram and find the distance between relevant points.

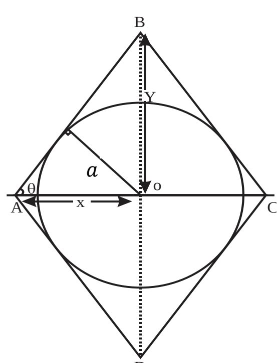
(14)

a) If $y = e^{\sin x}$, show that $y\frac{d^2y}{dx^2} + y^2 \sin x - \left[\frac{dy}{dx}\right]^2 = 0$.

b) It is given that $f(x) = \frac{2x^2-4x}{(x-3)(x+1)}$; $for\ x \neq 3, -1$. Show that $f'(x) = \frac{-12\,(x-1)}{(x-3)^2\,(x+1)^2}$

Sketch the graph of the function y = f(x) indicating the turning points and asymptotes.

c) The figure represents a rhombus, touching its four sides of a circle of centre 0 and radius a Here $OB = y,\ OA = x$ and $B\hat{A}O = \theta$



i. Show that $y = \frac{ax}{\sqrt{x^2-a^2}}$

ii. When the area of the above rhombus takes its minimum value, show that rhombus becomes a square and the minimum area of it is $4a^2$.

(15)

a) F(x) is a continuous and differentiable function in the range $[a, b]$. Where $a < c < b$; prove that; ${}_a^b\int f(x)\,dx = {}_a^c\int f(x)dx + {}_c^b\int f(x)\,dx$.

Find ${}_{-2}^{5}\int |x+1|dx$.

b) Prove that: $\frac{1}{(x-p)(x+p)} = \frac{1}{2p(x-p)} - \frac{1}{2p(x+p)}$

Hence: find a and b such that;

$$\int \frac{2x^2+3}{(x^2-1)\,(x^2+4)}\,dx = a\ \ln\left|\frac{x-1}{x+1}\right| + b\ \tan^{-1}\frac{x}{2} + C \qquad .$$

where c is an arbitrary constant.

c) Using integration by parts: Evaluate

$\int x^{1/2}(\ln x)^2\,dx$.

d) Prove that: . $\int \sqrt{1+\sin\frac{x}{4}}\ dx = 8\left(\sin\frac{x}{8} - \cos\frac{x}{8}\right) + C,$ c is an arbitrary constant.

16) Let $S_1 \equiv\ x^2 + y^2 - 25 = 0$ and $l_1 \equiv y - x + 1 = 0$. Two circles S and S' are drawn through the points of intersection of the circle S_1 and line l_1, so that both S and S' touch the line

$l_2 \equiv x + y - 25 = 0$. Find the equations of S and S'.

Show that two common tangents can be drawn to S and S' and, further show that the common tangents do not intersect.

17) (a) State the Cosine rule for a triangle ABC, in the usual notation.

Prove that,

$a^2 = (b + c)^2 - 4bcCos^2\left(\frac{A}{2}\right) = (b - c)^2 + 4bcSin^2\left(\frac{A}{2}\right)$ Hence,

deduce that, $tan^2 \frac{A}{2} = \frac{(a+b-c)(a+c-b)}{(a+b+c)\,(b+c-a)}$

(b) Sketch the graphs of the functions y = 2 sin x + 1 and y = $\sqrt{2}\,(cos\ x + sin\ x)$ on the same co ordinate plane for $0 \leq x \leq 2\pi$

Hence find the number of real solutions of the equation,

$2\ sin\ x + 1 = \sqrt{2}\,|\ cos\ x + sin\ x\ |$ in the range given above.

(c) Solve for x;

$\tan^{-1}(\frac{1 - x}{1 + x}) = \frac{1}{2}\tan^{-1} x,$ Where $x > 0$